CMSC 330: Organization of Programming Languages

Context-Free Grammars

Review

- Why should we study CFGs?
- What are the four parts of a CFG?
- How do we tell if a string is accepted by a CFG?
- What's a parse tree?

Review

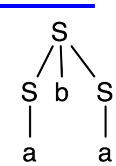
A *sentential form* is a string of terminals and non-terminals produced from the start symbol

Inductively:

- The start symbol
- If $\alpha A \delta$ is a sentential form for a grammar, where (α and $\delta \in (N|\Sigma)^*$), and $A \rightarrow \gamma$ is a production, then $\alpha\gamma\delta$ is a sentential form for the grammar
  - In this case, we say that $\alpha A \delta$ *derives* $\alpha\gamma\delta$ in one step, which is written as $\alpha A \delta \Rightarrow \alpha\gamma\delta$

Leftmost and Rightmost Derivation

- Example: $S \rightarrow a \mid SbS$ String: aba



Leftmost Derivation

$S \Rightarrow SbS \Rightarrow abS \Rightarrow aba$

At every step, apply production to leftmost non-terminal

Rightmost Derivation

$S \Rightarrow SbS \Rightarrow Sba \Rightarrow aba$

At every step, apply production to rightmost non-terminal

- Both derivations happen to have the same parse tree
- A parse tree has a unique leftmost and a unique rightmost derivation
- Not every string has a unique parse tree
- Parse trees don't show the order productions are applied

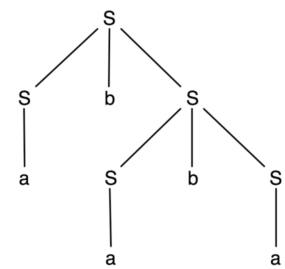
Another Example (cont'd)

$S \rightarrow a \mid SbS$

- Is ababa in this language?

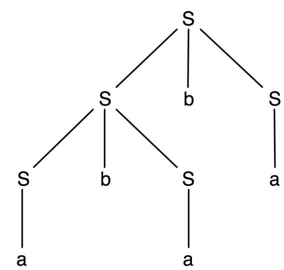
A leftmost derivation

$S \Rightarrow SbS \Rightarrow abS \Rightarrow abSbS \Rightarrow ababS \Rightarrow ababa$



Another leftmost derivation

$S \Rightarrow SbS \Rightarrow SbSbS \Rightarrow abSbS \Rightarrow ababS \Rightarrow ababa$



Ambiguity

- A string is *ambiguous* for a grammar if it has more than one parse tree
  - Equivalent to more than one leftmost (or more than one rightmost) derivation
- A grammar is *ambiguous* if it generates an ambiguous string
  - It's can be hard to see this with manual inspection
- Exercise: can you create an unambiguous grammar for $S \rightarrow a \mid SbS$?

Are these Grammars Ambiguous?

(1) S → aS | T
T → bT | U
U → cU | ε

(2) S → T | T
T → Tx | Tx | x | x

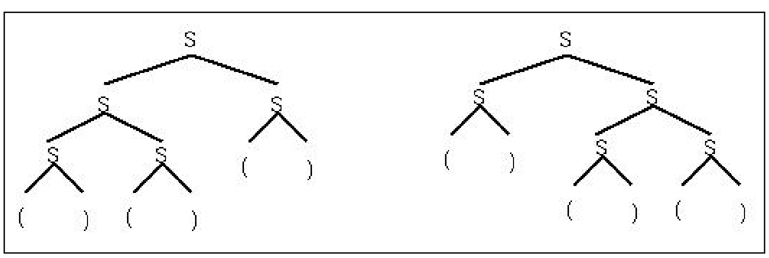
(3) S → SS | () | (S)

Ambiguity of Grammar (Example 3)

- 2 different parse trees for the same string: ()()()
- 2 distinct leftmost derivations :

$S \Rightarrow SS \Rightarrow SSS \Rightarrow ()SS \Rightarrow ()()S \Rightarrow ()()()$

$S \Rightarrow SS \Rightarrow ()S \Rightarrow ()SS \Rightarrow ()()S \Rightarrow ()()()$



- We need unambiguous grammars to manage programming language semantics

More on Leftmost/Rightmost Derivations

- Is the following derivation leftmost or rightmost?

  $S \Rightarrow aS \Rightarrow aT \Rightarrow aU \Rightarrow acU \Rightarrow ac$

  – There's at most one non-terminal in each sentential form, so there's no choice between left or right non-terminals to expand

- How about the following derivation?
  – $S \Rightarrow SbS \Rightarrow SbSbS \Rightarrow SbabS \Rightarrow ababS \Rightarrow ababa$

Tips for Designing Grammars

1. Use recursive productions to generate an arbitrary number of symbols

   $A \rightarrow xA \mid \varepsilon$ Zero or more x's

   $A \rightarrow yA \mid y$ One or more y's

2. Use separate non-terminals to generate disjoint parts of a language, and then combine in a production

   $G = S \rightarrow AB$

   $A \rightarrow aA \mid \varepsilon$

   $B \rightarrow bB \mid \varepsilon$

   $L(G) = a^*b^*$

Tips for Designing Grammars (cont'd)

3. To generate languages with matching, balanced, or related numbers of symbols, write productions which generate strings from the middle

$\{a^n b^n \mid n \geq 0\}$ (not a regular language!)

$S \rightarrow aSb \mid \varepsilon$

Example: $S \Rightarrow aSb \Rightarrow aaSbb \Rightarrow aabb$

$\{a^n b^{2n} \mid n \geq 0\}$

$S \rightarrow aSbb \mid \varepsilon$

Tips for Designing Grammars (cont'd)

$\{a^n b^m \mid m \geq 2n, n \geq 0\}$

$S \rightarrow aSbb \mid B \mid \varepsilon$

$B \rightarrow bB \mid b$

The following grammar also works:

$S \rightarrow aSbb \mid B$

$B \rightarrow bB \mid \varepsilon$

How about the following?

$S \rightarrow aSbb \mid bS \mid \varepsilon$

Tips for Designing Grammars (cont'd)

$\{a^n b^m a^{n+m} \mid n \geq 0, m \geq 0\}$

Rewrite as $a^n b^m a^m a^n$, which now has matching superscripts (two pairs)

Would this grammar work?

$S \rightarrow aSa \mid B$

$B \rightarrow bBa \mid ba$

Doesn't allow m = 0

Corrected:

$S \rightarrow aSa \mid B$

$B \rightarrow bBa \mid \varepsilon$

The outer $a^n a^n$ are generated first, then the inner $b^m a^m$

Tips for Designing Grammars (cont'd)

4. For a language that's the union of other languages, use separate nonterminals for each part of the union and then combine

$\{ a^n(b^m|c^m) \mid m > n \geq 0\}$

Can be rewritten as

$\{ a^n b^m \mid m > n \geq 0\} \cup$

$\{ a^n c^m \mid m > n \geq 0\}$

Tips for Designing Grammars (cont'd)

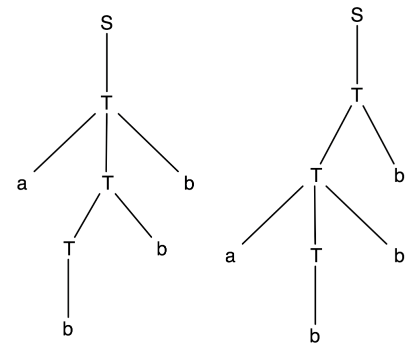
{ $a^n b^m$ | m > n ≥ 0} ∪ { $a^n c^m$ | m > n ≥ 0}

S → T | U

T → aTb | Tb | b — T generates the first set

U → aUc | Uc | c — U generates the second set

- What's the parse tree for string abbb?
  - Ambiguous!



Tips for Designing Grammars (cont'd)

{ $a^n b^m$ | m > n ≥ 0} ∪ { $a^n c^m$ | m > n ≥ 0}

Will this fix the ambiguity?

S → T | U

T → aTb | bT | b

U → aUc | cU | c

- It's not amgiguous, but it can generate invalid strings such as babb

Tips for Designing Grammars (cont'd)

$\{ a^n b^m \mid m > n \geq 0\} \cup \{ a^n c^m \mid m > n \geq 0\}$

Unambiguous version

$S \rightarrow T \mid V$

$T \rightarrow aTb \mid U$

$U \rightarrow Ub \mid b$

$V \rightarrow aVc \mid W$

$W \rightarrow Wc \mid c$

CFGs for Languages

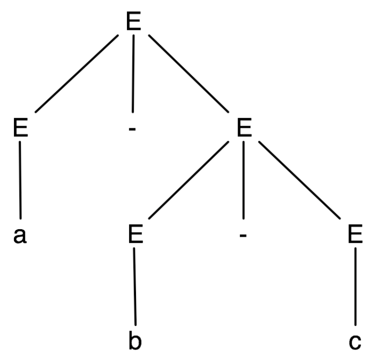
- Recall that our goal is to describe programming languages with CFGs
- We had the following example which describes limited arithmetic expressions

  $E \rightarrow a \mid b \mid c \mid E+E \mid E\text{-}E \mid E*E \mid (E)$

- What's wrong with using this grammar?
  - It's ambiguous!

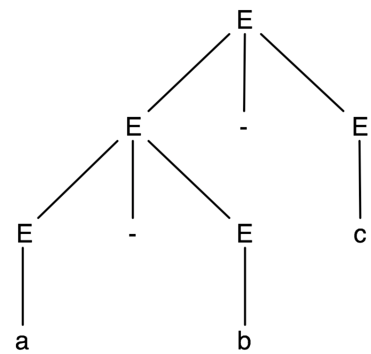
Example: a-b-c

E ⇒ E-E ⇒ a-E ⇒ a-E-E ⇒ a-b-E ⇒ a-b-c



Corresponds to a-(b-c)

E ⇒ E-E ⇒ E-E-E ⇒ a-E-E ⇒ a-b-E ⇒ a-b-c



Corresponds to (a-b)-c

The Issue: Associativity

- Ambiguity is bad here because if the compiler needs to generate code for this expression, it doesn't know what the programmer intended
- So what do we mean when we write a-b-c?
  - In mathematics, this only has one possible meaning
  - It's (a-b)-c, since subtraction is *left-associative*
  - a-(b-c) would be the meaning if subtraction was *right-associative*

Another Example: If-Then-Else

<stmt> ::= <assignment> | <if-stmt> | ...

<if-stmt> ::= if (<expr>) <stmt> |
    if (<expr>) <stmt> else <stmt>

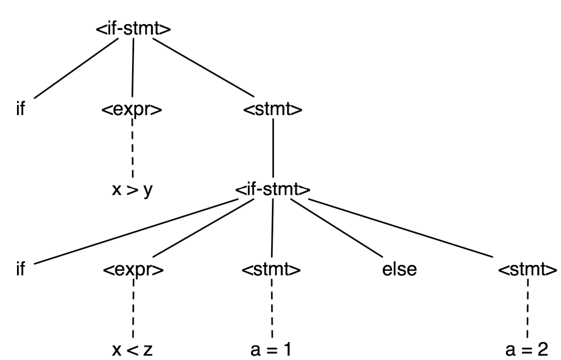
– (Here <>'s are used to denote nonterminals and ::= for productions)

- Consider the following program fragment:

```
if (x > y)
  if (x < z)
    a = 1;
  else a = 2;
```

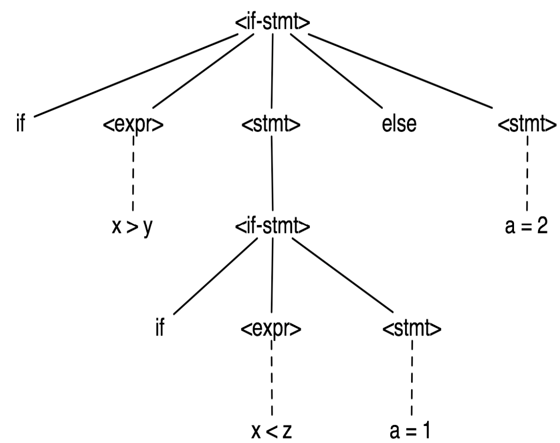
– Note: Ignore newlines

Parse Tree #1



- Else belongs to inner if

Parse Tree #2



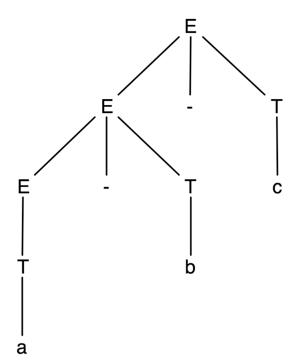
- Else belongs to outer if

Fixing the Expression Grammar

- Idea: Require that the right operand of all of the operators is not a bare expression

  E → E+T | E-T | E*T | T

  T → a | b | c | (E)



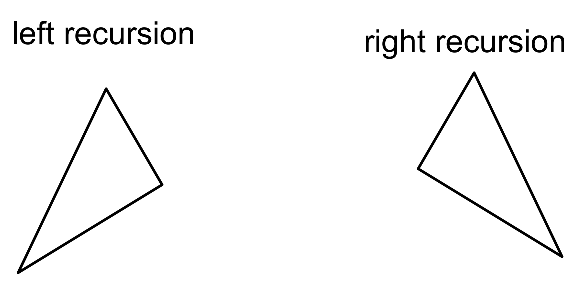
- Now there's only one parse tree for a-b-c
  - Exercise: Give a derivation for the string a-(b-c)

What if We Wanted Right-Associativity?

- Left-recursive productions are used for left-associative operators
- Right-recursive productions are used for right-associative operators
- Left:
  - $E \rightarrow E+T \mid E-T \mid E*T \mid T$
  - $T \rightarrow a \mid b \mid c \mid (E)$
- Right:
  - $E \rightarrow T+E \mid T-E \mid T*E \mid T$
  - $T \rightarrow a \mid b \mid c \mid (E)$

Parse Tree Shape

- The kind of recursion/associativity determines the shape of the parse tree

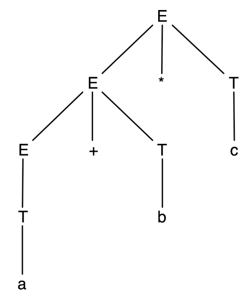


  - Exercise: draw a parse tree for a-b-c in the prior grammar in which subtraction is right-associative

A Different Problem

- How about the string a+b*c ?

  E → E+T | E-T | E*T | T
  T → a | b | c | (E)



- Doesn't have correct precedence for *
  - When a nonterminal has productions for several operators, they effectively have the same precedence
- How can we fix this?

Final Expression Grammar

E → E+T | E-T | T	lowest precedence operators
T → T*P | P	higher precedence
P → a | b | c | (E)	highest precedence (parentheses)

- Exercises:
  - Construct tree and left and and right derivations for
    - a+b*c a*(b+c) a*b+c a-b-c
  - See what happens if you change the last set of productions to P → a | b | c | E | (E)
  - See what happens if you change the first set of productions to E → E +T | E-T | T | P