Evolutionary Game Framework for Behavior Dynamics in Cooperative Spectrum Sensing

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Abstract— **Cooperative spectrum sensing has been shown to greatly improve the sensing performance in cognitive radio networks. However, if the cognitive users belong to different service providers, they tend to contribute less in sensing in order to achieve a higher throughput. In this paper, we propose an evolutionary game framework to study the interactions between selfish users in cooperative sensing. We derive the behavior dynamics and the stationary strategy of the secondary users, and further propose a distributed learning algorithm that helps the secondary users approach the Nash equilibrium with only local payoff observation. Simulation results show that the average throughput achieved in the cooperative sensing game with more than two secondary users is higher than that when the secondary users sense the primary user individually without cooperation.**

I. Introduction

With the emergence of new wireless applications and devices, the last decade has witnessed a dramatic increase in the demand for radio spectrum, which has forced government regulatory bodies, such as the Federal Communications Commission (FCC), to review their policies. Since the allocated frequency bands to some licensed spectrum holders experience very low utilization [1], the FCC has been considering opening the under-utilized licensed bands to secondary users on an opportunistic basis with the aid of cognitive radio technology [2]-[4]. When the licensed spectrum holders (primary users) are sensed as inactive, the secondary users can operate in the licensed spectrum, if they do not conflict/interfere with the primary user.

In order to protect the primary users from interference due to secondary users' operation, spectrum sensing has become an essential function of cognitive radio devices. The authors of [5] investigated the fundamental limits and practical challenges in spectrum sensing. Recently, cooperative spectrum sensing with relay nodes' help and multi-user collaborative sensing has been shown to greatly improve the sensing performance [6]-[12]. In [6], the authors proposed collaborative spectrum sensing as a means to combat shadowing or fading effects. The work in [7] proposed light-weight cooperation in sensing based on hard decisions to reduce the sensitivity requirements on individual cognitive radios. The authors of [8] showed that by allowing the cognitive users operating in the same band to cooperate in sensing, the detection time of the primary user can be reduced, and thus the overall agility can be increased. It was shown in [9] that cooperating a group of secondary users with the highest primary user's signal to noise ratios (SNR) can optimize the cooperative sensing performance, while cooperating all secondary users may not. The authors of [10] studied the design of sensing slot duration to maximize the achievable throughput for the secondary users under the constraint that the primary users were sufficiently protected. Two energy-based cooperative detection methods using weighted combining for dynamic spectrum access were analyzed in [11]. Spatial diversity in multiuser networks to improve the spectrum sensing capabilities of centralized cognitive radio networks were exploited in [12].

In most of the existing cooperative spectrum sensing schemes [6]-[12], it is generally assumed that all secondary users belong to the same authority. They will voluntarily fuse their sensing outcomes to a centralized controller (e.g., the secondary base station), which makes a final decision on whether the primary user is present or not. However, with the emerging applications of mobile ad hoc networks envisioned in civilian usage, the secondary users may be selfish and do not serve a common goal. Sensing a licensed frequency band also consumes a certain amount of energy and time which may alternatively be diverted to data transmissions. If multiple secondary users occupy different sub-bands of one primary user and can overhear the other users' sensing outcomes, they tend to take advantage of the others and wait for the others to sense the primary user so as to reserve more time for their own data transmission.

In order to study the interactions between the selfish users and their stationary strategy in the long run, in this paper we propose to model the cooperative spectrum sensing as an evolutionary game. If some secondary users agree to cooperate in sensing, the cost can be equally shared among them, while the users who do not take part in cooperative sensing can enjoy a free ride. However, if no user senses the primary user, then all of them will be punished by a very low payoff. By using replicator dynamics [16], we obtain the equations that govern the users' behavior dynamics, and further derive the equilibrium strategy when all secondary users are assumed homogeneous in their individual data rates and the received SNRs of the primary user (e.g., the secondary users are located far away from the primary base station and clustering together). Moreover, we develop a distributed learning algorithm that can help the secondary users approach their optimal strategy with only their own payoff history. Simulation results show that as the number of secondary users and the cost of sensing increases, the users tend to have less incentive to contribute to the cooperative sensing. However, they can still achieve a higher average throughput in the spectrum sensing game than that of the single-user sensing, if there are more than two secondary users in the cognitive radio network.

The rest of this paper is organized as follows. The system model is presented in Section II. In Section III, we formulate the cooperative spectrum sensing as an evolutionary game, analyze the behavior dynamics of the secondary users, and develop a distributed learning algorithm that approaches the optimal equilibrium strategy. Simulation results are shown in Section IV. Finally, Section V concludes the paper.

II. System Model

A. Hypothesis of Channel Sensing

When a secondary user is sensing the licensed spectrum channel in a cognitive radio network, the received signal $r(t)$ from the detection has two hypotheses when the primary user is present or absent, denoted by H_1 and H_0, respectively. Then, $r(t)$ can be written as

$$r(t) = \begin{cases} hs(t) + w(t), & \text{if } H_1; \\ w(t), & \text{if } H_0. \end{cases} \tag{1}$$

In (1), h is the gain of the channel from the primary user's transmitter to the secondary user's receiver; $s(t)$ is the signal of the primary user, which is assumed to be an i.i.d. random process with mean zero and variance σ_s^2; $w(t)$ is an additive white Gaussian noise (AWGN) with mean zero and variance σ_w^2. $s(t)$ and $w(t)$ are assumed to be mutually independent.

Assume we use an energy detector to sense the licensed spectrum, then the test statistics $T(r)$ is defined as

$$T(r) = \frac{1}{N}\sum_{t=1}^{N} |r(t)|^2, \tag{2}$$

where N is the number of collected samples.

The performance of licensed spectrum sensing is characterized by two probabilities. The probability of detection, P_D, represents the probability of detecting the presence of primary user under hypothesis H_1. The probability of false alarm, P_F, represents the probability of detecting the primary user's presence under hypothesis H_0. The higher the P_D, the better protection the primary user will receive; the lower the P_F, the more spectrum access the secondary user will obtain.

If the noise term $w(t)$ is assumed to be circularly symmetric complex Gaussian (CSCG), using central limit theorem the probability density function (PDF) of the test statistics $T(r)$ under H_0 can be approximated by a Gaussian distribution $\mathcal{N}(\sigma_w^2, \frac{1}{N}\sigma_w^4)$. Then, the probability of false alarm P_F is given by [14]

$$P_F(\lambda) = \mathcal{Q}\left(\left(\frac{\lambda}{\sigma_w^2} - 1\right)\sqrt{N}\right), \tag{3}$$

where λ is the threshold of the energy detector, and $\mathcal{Q}(\cdot)$ denotes the complementary distribution function of the standard Gaussian, i.e.,

$$\mathcal{Q}(x) = \frac{1}{\sqrt{2\pi}}\int_x^{\infty} \exp\left(-\frac{t^2}{2}\right) dt.$$

Similarly, if we assume the primary signal is a complex PSK signal, then under hypothesis H_1, the PDF of $T(r)$ can be approximated by a Gaussian distribution $\mathcal{N}((\gamma+1)\sigma_w^2, \frac{1}{N}(2\gamma+1)\sigma_w^4)$, where $\gamma = \frac{|h|^2\sigma_s^2}{\sigma_w^2}$ denotes the received signal-to-noise ratio (SNR) of the primary user under H_1. Then, the probability of detection P_D can be approximated by [14]

$$P_D(\lambda) = \mathcal{Q}\left(\left(\frac{\lambda}{\sigma_w^2} - \gamma - 1\right)\sqrt{\frac{N}{2\gamma+1}}\right). \tag{4}$$

Given a target detection probability $\bar{P}_D$, the threshold λ can be derived, and the probability of false alarm P_F can be further rewritten as

$$P_F(\bar{P}_D, N, \gamma) \triangleq \mathcal{Q}\left(\sqrt{2\gamma+1}\,\mathcal{Q}^{-1}(\bar{P}_D) + \sqrt{N}\gamma\right), \tag{5}$$

where $\mathcal{Q}^{-1}(\cdot)$ denotes the inverse function of $\mathcal{Q}(\cdot)$.

B. Throughput of a Secondary User

When sensing the primary user's activity, the secondary users cannot perform data transmission at the same time. If we denote the sampling frequency by f_s and the frame duration by T, then the time duration for data transmission is given by $T - \delta(N)$, where $\delta(N) = \frac{N}{f_s}$ represents the time spent in sensing.

When the primary user is absent and no false alarm is generated, the average throughput of the secondary user is

$$R_{H_0}(N) = \frac{T - \delta(N)}{T}(1 - P_F)C_{H_0}, \tag{6}$$

where C_{H_0} represents the data rate of the secondary user under H_0. When the primary user is present while not detected by the secondary user, the average throughput of the secondary user is

$$R_{H_1}(N) = \frac{T - \delta(N)}{T}(1 - P_D)C_{H_1}, \tag{7}$$

where C_{H_1} represents the data rate of the secondary user under H_1.

If we denote P_{H_0} as the probability that the primary user is absent, then the total throughput of the secondary user is

$$R(N) = P_{H_0}R_{H_0}(N) + (1 - P_{H_0})R_{H_1}(N). \tag{8}$$

Then, from the secondary user's perspective, he/she wants to maximize his/her total throughput (8), given that $P_D \geq \bar{P}_D$. As mentioned in [10], in practice the target detection probability $\bar{P}_D$ are required by the primary user to be close to 1; moreover, we usually have P_{H_0} close to 1 and $C_{H_1} < C_{H_0}$ (due to the interference from the primary user to the secondary user). Therefore, (8) can be approximated by

$$\tilde{R}(N) = P_{H_0}R_{H_0}(N) = P_{H_0}\frac{T - \delta(N)}{T}(1 - P_F)C_{H_0}. \tag{9}$$

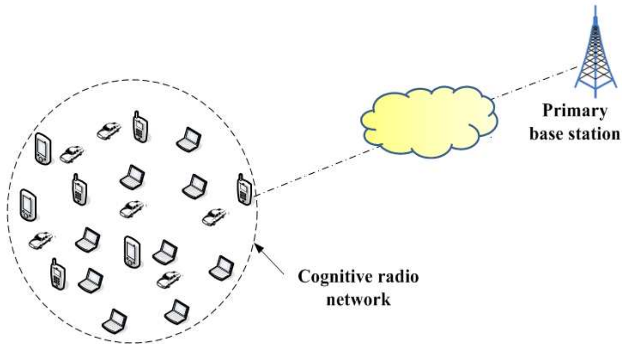

Fig. 1: System model

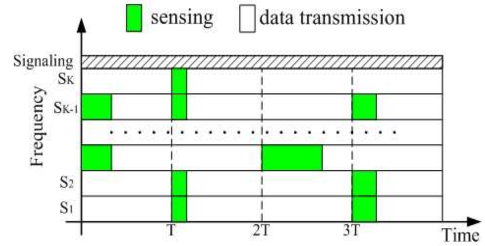

Fig. 2: Cooperative spectrum sensing

III. Spectrum Sensing Game

Before we model the spectrum sensing game, let's first analyze the secondary user's throughput $\tilde{R}(N)$ with respect to N, the number of collected signals from the primary user detection. From (5) we know that given a target detection probability $\bar{P}_D$, P_F is a decreasing function of N. As the secondary user reduces N (or $\delta(N)$) in the hope of having more time for data transmission, P_F will increase. This indicates a tradeoff for the secondary user to choose an optimal N that maximizes the throughput $\tilde{R}(N)$. In order to reduce both P_F and N, i.e., keep low false alarm P_F with a smaller N, a good choice for the secondary user is by cooperative spectrum sensing with the other secondary users in the same licensed band.

A. Problem Formulation

A snapshot of a cognitive radio network where multiple secondary users are allowed to access one licensed spectrum band is shown in Fig. 1, where the secondary users are clustering together, but far away from the primary base station. The cooperative spectrum sensing is shown in Fig. 2. We assume that the entire licensed band is divided into K sub-bands, and each secondary user operates exclusively in one of the K sub-bands when the primary user is absent. The transmission time is slotted into intervals of length T. Before each data transmission, the secondary users need to sense the primary user's activity. Since the primary user will operate in all the sub-bands once becoming active, the secondary users can jointly sense the primary user's presence, and exchange their sensing results via a narrow-band signalling channel, as shown in Fig 2. In this way, each of them can spend less time detecting while enjoying a low false alarm probability P_F via some decision fusion rule [13], and the spectrum sensing cost (N, or $\delta(N)$) can be shared by whoever is willing to contribute (C).

However, the secondary users may not serve for a common authority, and they will act selfishly by pursuing as high a throughput as possible. Once a secondary user is able to overhear the detection results from the other users, he/she tends to take advantage of that by denying (D) to take part in spectrum sensing. In this scenario, each secondary user in the cognitive radio network still achieves the same false alarm probability P_F, while the users who deny to join in cooperative sensing will have more time for their own data transmission. The secondary users are punished by a very low throughput if no one cooperates in sensing, in the hope that someone else will sense the spectrum.

Therefore, we can model the spectrum sensing as a non-cooperative game. The players of the game are the secondary users, denoted by $\mathcal{S} = \{s_1, \cdots, s_K\}$. Each player s_k has the same action/strategy space, denoted by $\mathcal{A} = \{\text{C,D}\}$. The payoff function is defined as the throughput of the secondary user. Assume that the secondary users contributing in cooperative sensing forms a set, denoted by $\mathcal{S}_c = \{s_1, \cdots, s_J\}$. Denote the false alarm probability of the cooperative sensing among set $\mathcal{S}_c$ with fusion rule "RULE" and a target detection probability $\bar{P}_D$ by $P_F^{\mathcal{S}_c} \triangleq P_F(\bar{P}_D, N, \{\gamma_i\}_{i\in\mathcal{S}_c}, \text{RULE})$. Then the payoff for a contributor $s_j \in \mathcal{S}_c$, can be defined as

$$\tilde{U}_{C,s_j} = P_{H_0}\left(1 - \frac{\delta(N)}{|\mathcal{S}_c|T}\right)(1 - P_F^{\mathcal{S}_c})C_{s_j}, \quad \text{if } |\mathcal{S}_c| \in [1, K], \tag{10}$$

where $|\mathcal{S}_c|$, i.e., the cardinality of set $\mathcal{S}_c$, represents the number of contributors, and C_{s_j} is the data rate for user s_j under hypothesis H_0. The payoff for a denier $s_i \notin \mathcal{S}_c$, who selects strategy D, is then given by

$$\tilde{U}_{D,s_i} = P_{H_0}(1 - P_F^{\mathcal{S}_c})C_{s_i}, \quad \text{if } \ |\mathcal{S}_c| \in [1, K-1], \tag{11}$$

since s_i will not spend time in sensing. If no secondary user contributes to spectrum sensing and waits for the others to sense, i.e., $|\mathcal{S}_c| = 0$, from (5), we know that $\lim_{N\to 0} P_F = 1$, especially for the low received SNR regime and high $\bar{P}_D$ requirement. In this case, the payoff for a denier becomes

$$\tilde{U}_{D,s_i} = 0, \quad \text{if } \ |\mathcal{S}_c| = 0. \tag{12}$$

Note that the contributors can further exchange information about their received SNR values and optimally distribute the total sensing time $\delta(N)$ among them. But this will increase computation complexity, and results in extra communication overhead and fairness issues. Therefore, in this paper we assume that the cost of sensing, $\delta(N)$, is equally shared by all contributors as in (10). N is a large number agreed by the group of contributors to guarantee a low P_F. For instance, as shown in Fig. 1, the secondary users are located far away from the primary base station and mutually nearby each other. Then, the received SNR values for the secondary users are similar. So N can be chosen as a proper multiple of the value that maximizes one secondary user's throughput (9) assuming single-user sensing. The decision fusion rule can be selected from the logical-OR rule, logical-AND rule, and majority rule [10]. In this paper, we mainly focus on the logical-OR rule to derive the $P_F^{\mathcal{S}_c}$, but the other fusion rules could be similarly analyzed. Denote the detection and false alarm probability for a contributor $s_j \in \mathcal{S}_c$ by P_{D,s_j} and P_{F,s_j}, respectively. Then, under OR rule we have the following

$$P_D = 1 - \prod_{s_j \in \mathcal{S}_c} (1 - P_{D,s_j}), \tag{13}$$

and

$$P_F = 1 - \prod_{s_j \in \mathcal{S}_c} (1 - P_{F,s_j}). \tag{14}$$

Hence, given a $\bar{P}_D$ for set $\mathcal{S}_c$, each individual user's target detection probability can be expressed as

$$\bar{P}_{D,s_j} = 1 - (1 - \bar{P}_D)^{(1/|\mathcal{S}_c|)}. \tag{15}$$

Then, from (5) we can write P_{F,s_j} as

$$P_{F,s_j} = \mathcal{Q}\left(\sqrt{2\gamma_{s_j}+1}\mathcal{Q}^{-1}(\bar{P}_{D,s_j}) + \sqrt{N/|\mathcal{S}_c|}\gamma_{s_j}\right), \tag{16}$$

and can further obtain $P_F^{\mathcal{S}_c}$ by substituting (16) in (14).

B. Analysis of the Game

Since the data transmission for each secondary user is continuous, the spectrum sensing game is played repeatedly and will evolve over time. Therefore, we can use evolutionary game theory [17] to analyze the evolutionary dynamics of the players and further derive the their equilibrium strategies [16].

*1) **Evolution Dynamics of the Sensing Game**:* The development of evolutionary game theory is a major contribution of biology to competitive decision making. The key concept of the evolutionary game is *replicator dynamics*, which describes the evolution of strategies in time. Specifically, consider a large population of homogeneous individuals who are programmed to the same set of pure strategies $\mathcal{A}$ in a symmetric game with payoff function U. At time t, let $p_{a_i}(t) \geq 0$ be the *number* of individuals who are currently programmed to pure strategy $a_i \in \mathcal{A}$, and let $p(t) = \sum_{a_i \in \mathcal{A}} p_{a_i}(t) > 0$ be the total population. Then the associated *population state* is defined as the vector $x(t) = \{x_{a_1}(t), \cdots, x_{|\mathcal{A}|}(t)\}$, where $x_{a_i}(t)$ is defined as the population share $x_{a_i}(t) = p_{a_i}(t)/p(t)$. By replicator dynamics, the evolution dynamics of $x_{a_i}(t)$ is given by the following differential equation

$$\dot{x}_{a_i} = \epsilon[\bar{U}(a_i, x_{-a_i}) - \bar{U}(x)]x_{a_i}, \tag{17}$$

where $\bar{U}(a_i, x_{-a_i})$ is the instantaneous average payoff of the individuals using a_i, $\bar{U}(x)$ is the instantaneous average payoff of the whole population, and ϵ is some positive number representing the time scale. The intuition behind (17) is as follows: if strategy a_i results in a higher payoff than the average level, the population share using a_i will grow, and the *growth rate* $\dot{x}_{a_i}/x_{a_i}$ is proportional to the difference between strategy a_i's current payoff and the current average payoff in the entire population.

By analogy, we can view $x_{a_i}(t)$ as the probability that one player in a symmetric game adopts pure strategy a_i, and $x(t)$ can be equivalently viewed as a mixed strategy for that player. Then, we can generalize (17) to the spectrum sensing game with heterogeneous players, as C_{s_i} may vary among different users.

Denote the probability that user s_j adopts strategy $h \in \mathcal{A}$ at time t by $x_{h,s_j}(t)$, then the time evolution of $x_{h,s_j}(t)$ is governed by the following differential equation:

$$\dot{x}_{h,s_j} = \frac{1}{\bar{U}_{s_j}(x)}\left[\bar{U}_{s_j}(h, x_{-s_j}) - \bar{U}_{s_j}(x)\right]x_{h,s_j}, \tag{18}$$

where $\bar{U}_{s_j}(h, x_{-s_j})$ is the average payoff for player s_j using pure strategy h, and $\bar{U}_{s_j}(x)$ is s_j's average payoff using mixed strategy x_{s_j}.

*2) **Equilibrium Analysis**:* If each user s_j maximizes his/her total payoff by choosing the optimal probability of being a contributor (or a denier), x_{h,s_j}, where h = C (or D), the outcome of the game can be characterized by the Nash Equilibrium [18]. In Nash equilibria (NE), no player can gain a higher payoff value by unilaterally deviating from the equilibrium strategy, given that the other players adopt their equilibrium strategies. If the probability x_{h,s_j} is required to be either 0 or 1, for all $s_j \in \mathcal{S}$, the equilibrium is called a pure strategy NE; if $x_{h,s_j} \in (0,1)$, we call the equilibrium a mixed strategy NE.

The stationary state solution to (18) given any initial condition is defined as the evolutionary stable strategy (ESS). It is shown that the ESS lies in the NE set [17]. In order to solve equation (18) and obtain the equilibrium of the game, we need to know all the $\bar{U}_{s_j}(h, x_{-s_j})$'s, which is of complexity $O(2^{|\mathcal{S}|})$. Moreover, according to the definition of (16) and (14), for different $\mathcal{S}_c$'s, the false alarm probability is different, even if they have the same cardinality. Thus, it is generally difficult to represent $\bar{U}_{s_j}(h, x_{-s_j})$ in a very compact form. Therefore, in this section, we first analyze a special case of the sensing game to get some insight, and will next develop a distributed learning algorithm for the players to achieve the NE in the long run.

As shown in Fig. 1, assume all the secondary users are located far away from the primary base station and clustering together, so all the received γ_{s_j}'s are very low and similar to each other. In order to guarantee low P_F given a target $\bar{P}_D$, the number of sampled signals N should be large. Under these assumptions, we can approximately view $P_F^{\mathcal{S}_c}$ as the same for different $\mathcal{S}_c$'s, denoted by $\hat{P}_F$. Further assume that all users have the same data rate, i.e. $C_{s_i} = C$, for all $s_i \in \mathcal{S}$. Then, the payoff functions defined in (10)-(12) become

$$U_C(J) = U_0\left(1 - \frac{\tau}{J}\right), \quad \text{if } J \in [1, K], \tag{19}$$

and

$$U_D(J) = \begin{cases} U_0, & \text{if } J \in [1, K-1]; \\ 0, & \text{if } J = 0, \end{cases} \tag{20}$$

where $U_0 = P_{H_0}(1 - \hat{P}_F)C$, $J = |\mathcal{S}_c|$, and $\tau = \frac{\delta(N)}{T}$.

According to the symmetric setting, (17) can be applied to the special case as all players have the same evolution dynamics and equilibrium strategy. Denote x as the probability that one secondary user contributes to spectrum sensing, then the average payoff for pure strategy C can be obtained as

$$\bar{U}_C = \sum_{j=0}^{K-1} \binom{K-1}{j} x^j (1-x)^{K-1-j} U_C(j+1), \tag{21}$$

where $\binom{K-1}{j} x^j (1-x)^{K-1-j}$ is the probability that $J+1$ users contributes to cooperative sensing. Similarly, the average payoff for pure strategy D is given by

$$\bar{U}_D = \sum_{j=0}^{K-1} \binom{K-1}{j} x^j (1-x)^{K-1-j} U_D(j). \tag{22}$$

Since the average payoff $\bar{U} = x\bar{U}_C + (1-x)\bar{U}_D$, then (17) becomes

$$\dot{x} = \epsilon x(1-x)(\bar{U}_C - \bar{U}_D). \tag{23}$$

In equilibrium x^*, any player will not deviate from the optimal strategy, indicating $\dot{x}^* = 0$, or $\bar{U}_C^* = \bar{U}_D^*$. Then, by equating (21) and (22), we can have the following K^{th}-order equation

$$\tau(1-x^*)^K + Kx^*(1-x^*)^{K-1} - \tau = 0, \tag{24}$$

and further solve the equilibrium.

*3) **Learning Algorithm for Nash Equilibrium**:* We have shown above that the equilibrium for a special symmetric sensing game is solvable. However, it is very difficult to solve the optimal strategy in a general asymmetric game with multiple ($K > 2$) players. In addition, solving the equilibrium requires the exchange of instantaneous γ_{s_j}'s and strategies adopted by the other users, which brings a lot of communication overhead. Therefore, a distributed learning algorithm that gradually converges to the NE is preferred.

From (18), we can derive the strategy adjustment for the secondary user as follows. Denote the pure strategy taken by user s_j at time t by $A_{s_j}(t)$. Define an indicator function $\mathbf{1}_{s_j}^h(t)$ as

$$\mathbf{1}_{s_j}^h(t) = \begin{cases} 1, & \text{if } A_{s_j}(t) = h; \\ 0, & \text{if } A_{s_j}(t) \neq h. \end{cases} \tag{25}$$

At some interval mT, we can approximate $\bar{U}_{s_j}(h, x_{-s_j})$ by

$$\bar{U}_{s_j}(h, x_{-s_j}) \doteq \frac{\sum_{t \le mT} \tilde{U}_{s_j}(A_{s_j}(t), A_{-s_j}(t)) \mathbf{1}_{s_j}^h(t)}{\sum_{t \le mT} \mathbf{1}_{s_j}^h(t)}, \tag{26}$$

where $\tilde{U}_{s_j}(A_{s_j}(t), A_{-s_j}(t))$ is the payoff value for s_j as determined by (10)-(12). Similarly, $\bar{U}_{s_j}(x)$ is approximated by

$$\bar{U}_{s_j}(x) \doteq \frac{1}{m} \sum_{t \le mT} \tilde{U}_{s_j}(A_{s_j}(t), A_{-s_j}(t)). \tag{27}$$

Then, the derivative $\dot{x}_{h,s_j}(mT)$ can be approximated by substituting (26) and (27) into (18). Therefore, the probability of user s_j taking action h can be adjusted by

$$x_{h,s_j}((m+1)T) = x_{h,s_j}(mT) + \eta_{s_j} \dot{x}_{h,s_j}(mT), \tag{28}$$

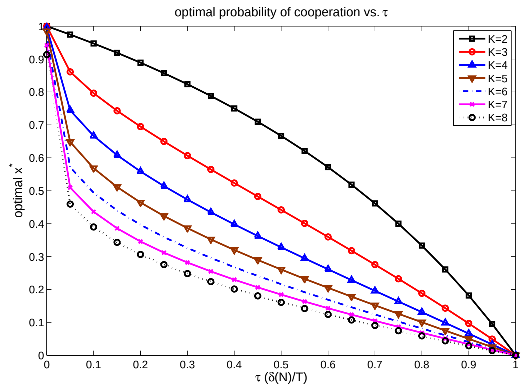

Fig. 3: Probability of being a contributor vs. τ

with η_{s_j} being the step size of adjustment chosen by s_j. Using the above distributed learning algorithm, each user can adapt a strategy with only his/her own payoff history, and the set of strategies gradually converges to the NE. We will demonstrate the convergence of the learning algorithm in the next section.

IV. Simulation Results and Analysis

To evaluate the performance of the proposed scheme, we performed simulations for a multi-player spectrum sensing game. The parameters used in the simulation are as follows. We assume that the primary signal is a baseband QPSK modulated signal, the sampling frequency is $f_s = 4$MHz, and the frame duration is $T = 5$ ms. The probability that the primary user is inactive is set as $P_{H_0} = 0.9$, and the required target detection probability $\bar{P}_D$ is 0.95. The noise is assumed to be a zero-mean CSCG process. The distance between the cognitive radio network and the primary base station is very large, so the received γ_{s_j}'s are in the low SNR regime, with an average value of -12 dB.

We first illustrate the optimal equilibrium strategy for the secondary users assuming a homogeneous setting as in Section III-B.2, where the data rate is $C = 1$ Mbps. In Fig. 3, we show the optimal probability of being a contributor x^* for a network with different number of secondary users. The x-axis represents $\tau = \frac{\delta(N)}{T}$, the ratio of sensing time duration over the frame duration. From Fig. 3, we can see that x^* decreases as τ increases. For the same τ, x^* decreases as the number of secondary users increases. This indicates that the incentive of contributing to the cooperative sensing drops as the cost of sensing increases and more users exist in the network. This is because the players tend to wait for someone else to sense the spectrum and can then enjoy a free ride, when they are faced with a high sensing cost and more counterpart players. In Fig. 4, we show the average throughput per user when all users adopt the equilibrium strategy. Note that when we calculate the optimal x^*, we approximately view $P_F^{\mathcal{S}^c}$ is identical for all different $\mathcal{S}^c$ sets. However, when τ is small, this approximation may not be precise. Therefore, in the simulations, we compute the actual value of $P_F^{\mathcal{S}^c}$ for each specific $\mathcal{S}^c$. From Fig. 4, we see that there is a tradeoff between the cost of sensing and the throughput for arbitrary number of

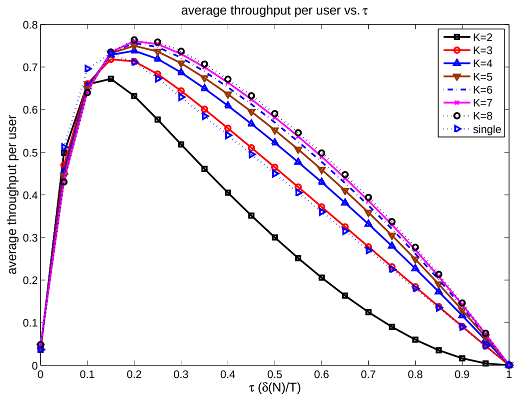

Fig. 4: Average throughput per user vs. τ

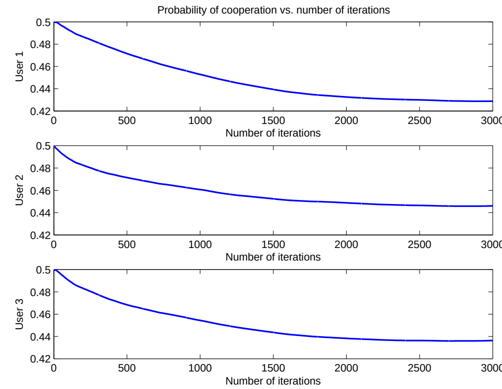

Fig. 5: Learning curve for a 3-user sensing game

users. The optimal value of τ is around 0.15, and will slightly increase as the number of user increases. This is because the false alarm probability P_F in (14) tends to increase as the number of user increases. In order to have a low P_F, the users need to collect more samples for better detection. Although the cost of sensing increases, as more users share the sensing cost, the optimal average throughput per user still increases. We also plot the optimal throughput for the single-user sensing (dotted line "single") for comparison. It is interesting that the average throughput values for games with more than 2 users are all higher than that of the single-user sensing, while the throughput for the 2-user game is not. The reason is that when there are more than 2 users in the game, the chance that no user contributes to sensing is smaller; it is more likely that neither user senses the spectrum in the 2-user game.

We finally show the learning curve for the probability of being a contributor in a 3-player game in Fig. 5, with $\tau = 0.5$, the step-size of learning $\eta_{s_j} = 0.002$, $\gamma_1 = -13$ dB, $\gamma_2 = -12$ dB, and $\gamma_1 = -11$ dB. We see that in the long run, all three users can gradually reach the equilibrium strategy, which is about 0.44 as shown in Fig. 3. By carefully adjusting the step size η_{s_j}, we can expect a higher convergence speed. It's worth noting that although the three users have different SNR values, the difference between the approximation of a symmetric game (Fig. 3) is very small, compared to the actual learning results.

V. Conclusion

In this paper, we propose an evolutionary game-theoretical framework for distributed cooperative sensing over cognitive radio networks. By employing the theory of replicator dynamics, we study the behavior dynamics of secondary users, and further propose a distributed learning algorithm that gradually converges to the Nash equilibrium. From the simulation results, the average throughput per user in a K-user sensing game ($K > 2$) is still higher than that in the single-user sensing.

References

[1] Federal Communications Commission, "Spectrum policy task force report," *FCC Document ET Docket No. 02-155*, Nov. 2002.

[2] Federal Communications Commission, "Facilitating opportunities for flexible, efficient and reliable spectrum use employing cognitive radio technologies: notice of proposed rule making and order," *FCC Document ET Docket No. 03-108*, Dec. 2003.

[3] J. Mitola III, "Cognitive radio: an integrated agent architecture for software defined radio," Ph.D. Thesis, KTH, Stockholm, Sweden, 2000.

[4] S. Haykin, "Cogntive radio: Brain-empowered wireless communications," *IEEE J. Selected Areas in Communications*, vol. 23, no. 2, pp. 201-220, Feb. 2005.

[5] A. Sahai and D. Cabric, "A tutorial on spectrum sensing: fundamental limits and practical challenges," *IEEE DySPAN 2005*, Baltimore, MD, Nov. 2005.

[6] A. Ghasemi and E. S. Sousa, "Collaborative spectrum sensing in cognitive radio networks," *IEEE DySPAN 2005*, pp. 131-136, Baltimore, MD, Nov. 2005.

[7] S. M. Mishra, A. Sahai, and R. W. Brodensen, "Cooperative sensing among cognitive radios," *IEEE ICC 2006*, pp. 1658-1663, Istanbul, Turkey, June 2006.

[8] G. Ganesan and Y. Li, "Cooperative spectrum sensing in cognitive radio," *IEEE Trans. Wireless Commun.*, vol. 6, no. 6, pp. 2204-2222, June 2007.

[9] E. Peh and Y.-C. Liang, "Optimization for cooperative sensing in cognitive radio networks," *IEEE WCNC 2007*, pp.27-32, Hongkong, Mar. 2007.

[10] Y.-C. Liang, Y. Zeng, E. Peh, and A. T. Hoang, "Sensing-throughput tradeoff for cognitive radio networks," *IEEE ICC 2007*, pp. 5330-5335, Glasgow, Scotland, June 2007.

[11] F. E. Visser, G. J. Janssen, and P. Pawelczak, "Multinode spectrum sensing based on energy detection for dynamic spectrum access," *IEEE VTC 2008-Spring*, Singapore, May 2008.

[12] G. Ganesan, Y. Li, B. Bing, and S. Li, "Spatiotemporal sensing in cognitive radio networks," *IEEE J. Selected Areas in Communications*, vol. 26, no. 1, pp. 5-12, Jan. 2008.

[13] Z. Chair and P. K. Varshney, "Optimal data fusion in multiple sensor detection systems," *IEEE Trans. on Aerospace and Elect. Syst.*, vol. 22, pp. 98-101, Jan. 1986.

[14] H. V. Poor, *An introduction to signal detection and estimation*, 2nd ed. Springer-Verlag, New York, 1994.

[15] C.-H. Chan, et al., "Evolution of cooperation in well-mixed N-person snowdrift games," *Physica A (2008)*, doi:10.1016/j.physa.2008.01.035.

[16] D. Fudenberg and D. K. Levine, The theory of learning in games, MIT Press, 1998.

[17] J. W. Weibull, Evolutionary game theory, MIT Press, 1995.

[18] D. Fudenberg and D. K. Levine, Game theory, MIT Press, 1991.