Dynamic Spectrum Access:
The Capacity Trade-off

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Abstract

Since 2000, cognitive radio has slowly been moving from theory into reality. One major driving force for that is tangible commercial applications for cognitive radio, the most major of which is dynamic spectrum access (DSA). A wide variety of DSA techniques have been proposed, in an effort to take advantage of every ounce of unused spectrum.

This paper analyzes the three main approaches: frequency coexistence, time coexistence, and interference coexistence. An ideal radio would employ all three to fully exercise all the available spectrum, however each in many scenarios it's often not worthwhile to do much more than basic dynamic frequency selection. We analyze the different scenarios and provide guidance on when different approaches are practical.

1 Introduction

Over the past several years, research in dynamic spectrum access has accelerated, as it allows cognitive algorithms to answer very real problems facing our current radio-frequency spectrum usage. Endless techniques have been designed that either adapt many current communications algorithms or invent entirely new ones that emphasize non-cooperative sharing between primary and secondary users of the spectrum [1, 2, 3, 4, 5].

Three basic classes of coexistence currently exist. The first is coexistence in frequency, commonly called dynamic frequency selection (DFS). Here algorithms attempt to find unused frequencies for secondary users. The second is coexistence in time where secondary users attempt to determine when a primary user is idle, and then use the same frequency. The last is coexistence in space or power. Here secondary users transmit simultaneous to primary users, but try to constrain interference by bounding distances between devices or the power with which they can transmit. This third technique is exemplified by the Interference Temperature Model [6].

In an ideal cognitive radio environment, a secondary radio could take advantage of all three of these coexistence techniques. Each one offers additional capacity to secondary users. Unfortunately, the algorithms necessary to ensure minimum interference to primary users become increasingly more complex as you start looking at coexistence in time and space, and increase the risk of causing interference.

This paper takes a step back and asks, "is it worth it?" Given complexity constraints derived from either financial, power, interference, or speed constraints on your radio hardware, there are diminishing capacity returns for an escalating cost of hardware and increased risk of harmful interference to primary users.

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Assuming utility and price functions gauges the relative benefit of capacity versus interference, we derive conditions on which type of dynamic spectrum access technologies make sense give then their respective complexities. The major result is that in many commercial cases, simple DFS is sufficient.

The remainder of this paper is organized as follows. Section 2 describes the capacity models for the dynamic access to spectrum by secondary users. Section 3 evaluates these models subject to a variety of real-world examples. Section 4 concludes.

2 Capacity Model

This section defines Shannon capacity for our basic channel access model. Section 2.1 defines the notation we use to refer to primary users. Section 2.2 through 2.4 define the individual coexistence methods. Section 2.5 combines the approaches into a single hybrid approach. Throughout we assume Shannon capacity, which can be viewed as a theoretical upper bound, due to the AWGN assumptions. To make these assumptions more plausible, we can implement direct-sequence spread spectrum (DSSS) secondary waveforms that have an inherently whitening effect on the channel [7].

2.1 Primary User Model

We assume an intelligent radio can classify signals of N primary users in the frequency range from $f_{\min}$ to $f_{\max}$, and determine when they are or are not transmitting. Each primary user $S_i \in \mathcal{S}$ can be represented by the tuple

$$S_i(t) = (f_i, B_i, T_i, \alpha_i(t)) \tag{1}$$

where f_i is the center frequency for signal i, $B_i(t)$ is the occupied bandwidth, T_i is the transmission temperature, and $\alpha_i(t)$ is time varying and either 0 or 1, indicating whether the user is idle or transmitting, respectively. Note that we assume the values B_i and $\alpha_i(t)$ include suitable guard bands in both frequency and time to prevent cochannel interference, thus if signal i was a 5 MHz signal requiring 250 kHz guard bands on either side, the occupied bandwidth $B_i = 5.5$ MHz.

Additionally, we shall assume that the elements of $\mathcal{S}$ are sorted by center frequency. In particular, $f_i \leq f_{i+1}$. Lastly, let us add to our set two additional values to simplify notation later on:

$$\begin{aligned} S_0 &= (f_0 = f_{\min}, B_0 = 0, \alpha_0(t) = 0) \\ S_{N+1} &= (f_{N+1} = f_{\max}, B_{N+1} = 0, \alpha_{N+1}(0) = 0) \end{aligned} \tag{2}$$

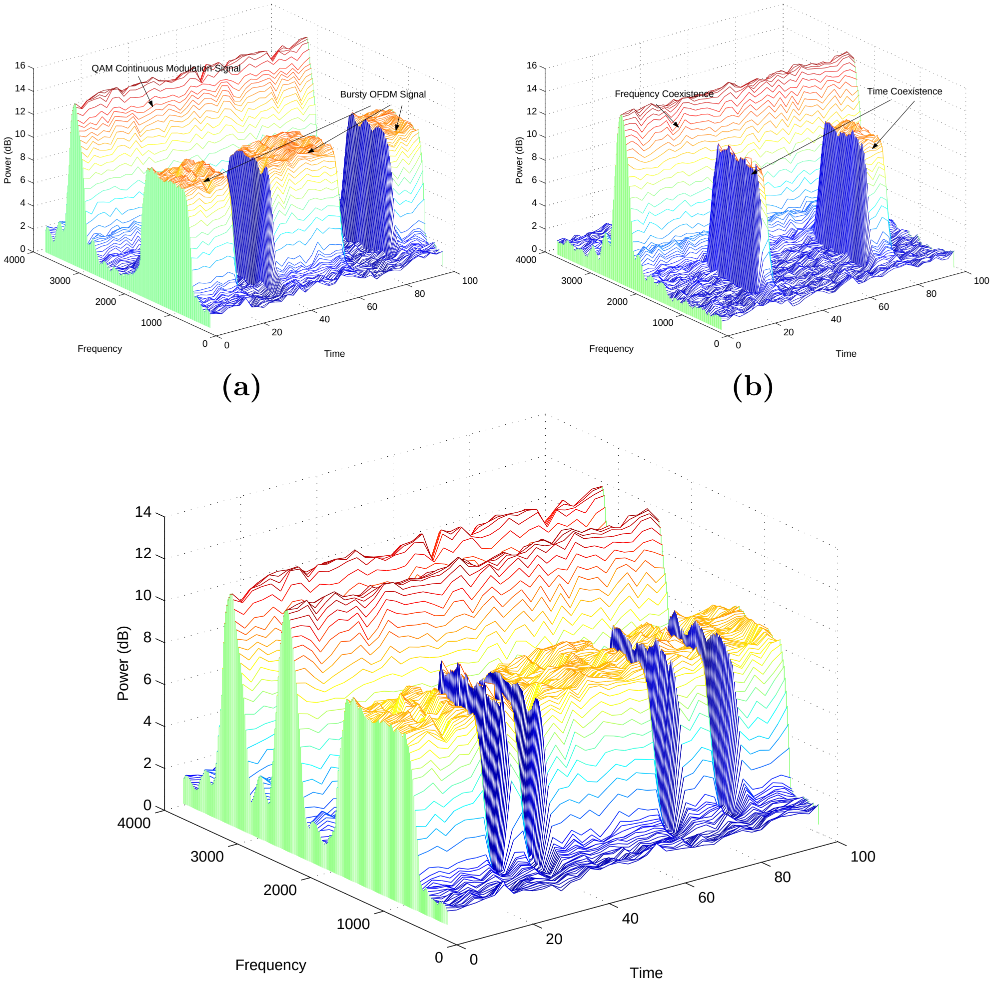
As an example, see Figure 1(a), where two licensed signals are visible. One is continuously modulated such that $\forall t \; \alpha_1(t) = 1$, and the second is bursty where $\alpha_2(t)$ would be 1 when the signal is active.

2.2 Frequency-Domain Coexistence

In frequency-domain coexistence, we try to find frequencies that are unoccupied for some large amount of time, and occupy those frequencies until they are no longer idle. Once a primary user becomes active on that frequency, we seek out a new frequency to use for communications. In Figure 1(b) the continuously-modulated secondary user has selected a vacant frequency based on the observed and identified signals from Figure 1(a).

For frequency-domain coexistence, there are two types of radios to consider. The first involves simple radios whose waveforms can only occupy a continuous span of unoccupied bandwidth. The second involves pooling radios who can create waveforms to pool non-continuous spans of unoccupied bandwidth up to some maximum bandwidth W [8]. Assuming a radio can compute $\mathcal{S}$, frequency-domain coexistence is fairly straight forward. Also assume an interference temperature T_I and a maximum transmit temperature of T_S, and consequently our transmit SNR is T_S/T_I. For simple radios, our maximum occupied bandwidth B_S is

$$\begin{aligned} B_S &= \max_{0 \leq i \leq N} f_{i+1} - \frac{B_{i+1}}{2} - \left(f_i + \frac{B_{i+1}}{2} \right) \\ &= \max_{0 \leq i \leq N} f_{i+1} - f_i - \frac{B_i + B_{i+1}}{2} \end{aligned} \tag{3}$$



(a) **(b)**

(c)

Figure 1: Power Spectrograms (power as a function of both frequency and time) of **(a)** two primary users, one continuous-modulation QAM and the other bursty OFDM; **(b)** two different secondary user coexistence strategies, one multiplexing in frequency and the other in time; and **(c)** the two power spectra together showing noninterference.

While for the pooling radio, our maximum occupied bandwidth B_P is

$$B_P = \max_{0 \le i \le N} \left(W - \sum_{j=i+1}^{m(i,W)} \min(B_j, \beta(j)) \right)$$

where

$$\begin{aligned} \beta(i) &= F_{\max} - f_i - B_j/2 \\ m(i,W) &= \arg\max_{j=i+1..N} (j \cdot I(f_j - B_j/2 < W)) \end{aligned} \tag{4}$$

and $I(C)$ evaluates to 1 if and only if the condition C evaluates to true.

Given we can compute our occupied bandwidths B_S and B_P for the two scenarios, the achievable capacities C_F^S and C_F^P are

$$C_F^{\{S,P\}} = B_{\{S,P\}} \log_2(1 + P \cdot T_S/T_I) \tag{5}$$

where P is the multiplicative path loss between the transmitter and receiver.

Assuming the accuracy of $\mathcal{S}$, the ratio of time I_F we are causing interference is $I_F = 0$.

2.3 Time-Domain Coexistence

In time-domain coexistence, secondary users look for channels that are only partially occupied by bursty primary users, and transmit when the channel is idle [9]. It is often implemented as a CSMA-like scheme where the primary user gets ultimate priority in channel access. This scheme is distinguished from frequency-domain coexistence because in time-domain coexistence we do not move to a new center frequency when the channel becomes active, as the overhead of doing so would be greater than the additional capacity achieved by simply waiting for the channel to once again become free. This type of coexistence is depicted in Figure 1(b), where the orthogonal frequency-division multiplexing (OFDM) secondary user is transmitting in the gaps left by the bursty primary user.

Consider we have elected to multiplex in the time domain with a bursty primary signal S_i, matching its bandwidth B_i. Ideally, our secondary signal should operate using the following tuple: $S_{sec} = (f_i, B_i, T_S, 1 - \alpha_i(t))$. Consequently, or capacity $C_T(i)$ can be computed as

$$C_T^{(\mathrm{ideal})}(i) = B_i \Delta_i \log_2(1 + P \cdot T_S/T_I) \tag{6}$$

where idle cycle Δ_i is

$$\Delta_i = 1 - \lim_{j \to \infty} \frac{1}{j} \int_0^j \alpha_i(t)\, dt \tag{7}$$

However, in realistic systems there is a reaction time associated with our ability to detect changes in $\alpha_i(t)$. There is some time τ_1 to realize a channel is idle and begin transmitting, and some time τ_2 to realize a primary user has resumed transmitting.

Thus for every idle period of length t, our achieved capacity must be discounted by the fraction $(t - \tau_1)/t$, and for every busy period we are interfering for τ_2 units of time. Time-domain coexistence is less efficient when sharing spectrum with a burstier primary users.

If $\alpha_i(t)$ is modeled by a random process with mean arrival rate λ_i, then our capacity discount is $(1 - \lambda_i \tau_1)$ Note that we assume $\tau_1 < 1/\lambda_i$ and $\tau_2 < 1/\lambda_i$. Were this not the case, we would be constantly causing interference and never achieve any useful communications.

Taking this into account, our capacity is more accurately represented as

$$C_T(i) = (\Delta_i - \lambda_i \tau_1) B_i \log_2(1 + P \cdot T_S/T_I) \tag{8}$$

and the fraction of time we are causing interference is

$$I_T(i) = \lambda_i \tau_2 \tag{9}$$

Thus, we can compute our expected capacity $C_T(i)$ and interference $I_T(i)$ in terms of our signal bandwidth, idle cycle, and burstiness.

2.4 Interference-Domain Coexistence

In interference-domain coexistence we seek to exploit the fact that most primary users already expect and account for a small amount of interference in their link budgets. Consequently, secondary users should be able to transmit at precisely the same time and frequency as primary users, provided they can control their power levels and consequently constrain the amount of interference caused.

This is the main goal of the Interference Temperature Model [6], where T_S is constrained such that for path loss M between the a secondary transmitter and the closest primary receiver,

$$M \cdot T_S \leq T_L \tag{10}$$

where T_L is the *interference temperature limit*.

For a secondary network consisting of mesh nodes, the achievable capacity and resulting interference can be very difficult to analyze [10], therefore here we shall only consider a simple point-to-point case. Generally, interference-domain coexistence would be attempted only when frequency-domain and time-domain coexistence fail due to a very occupied spectrum.

Thus, we shall assume we are occupying the same bandwidth as a primary signal S_i, and transmitting when that signal is active, using transmission power $T_S = T_L/M$. If S_i is a continuously-modulated signal, the resulting capacity is

$$C_I(i) = B_i \log_2 \left(1 + \frac{PT_L}{MT_P}\right) \tag{11}$$

where T_P is the received temperature of the primary user at the secondary user. Assuming the value of M is correct, the probability of being located such that the increase in the ambient interference decreases the receive SNR for a primary receiver below its receive sensitivity [10] is

$$I_I = 1 - (T_I/T_L) \tag{12}$$

For a bursty primary signal, we would likely employ some sort of interference and time coexistence simultaneously. Thus, when the primary signal is not transmitting, our power is unconstrained, and when it *is*, our power *is* constrained. Figure 2 depicts this type of spectrum sharing. Note the use of τ_1 and τ_2 to illustrate our reaction time.

For every primary-user burst, there are four segments to consider, depending on the power level of the primary and secondary transmitters. When the primary user starts transmitting, for time τ_2 we achieve capacity

$$C_{I,T}^{(a)} = B_i \log_2 (1 + P \cdot T_S/T_P) \tag{13}$$

After we realize the channel is occupied, we reduce our power level for $t_{on} - \tau_2$ units of time and achieve capacity

$$C_{I,T}^{(b)} = B_i \log_2 (1 + (P/M) \cdot (T_L/T_P)) \tag{14}$$

When the primary user stops transmitting, there is τ_1 units of time before we realize it, and consequently achieve capacity

$$C_{I,T}^{(c)} = B_i \log_2 (1 + (P/M) \cdot (T_L/T_I)) \tag{15}$$

And finally, we realize the channel is vacant and begin transmitting at full power for $t_{off} - \tau_1$ time units, and achieve capacity

$$C_{I,T}^{(d)} = B_i \log_2 (1 + P \cdot (T_S/T_I)) \tag{16}$$

However, in reality, it would be impossible for our radio to change its modulation and coding rates to adapt on the same time scale as our sensing time. For example, $C_{I,T}^{(a)} = 0$, since our modulation and coding for those τ_2 time units assumed a noise temperature of T_I, not T_P. Additionally, $C_{I,T}^{(c)} = C_{I,T}^{(b)}$, since we don't know to use a faster modulation/coding scheme.

We can define our t_{on} and t_{off} in terms of our already-defined statistics:

$$\begin{aligned} t_{off} &= \Delta_i/\lambda_i \\ t_{on} &= (1 - \Delta_i)/\lambda_i \end{aligned} \tag{17}$$

Then combining everything, we have:

$$\begin{aligned} C_{I,T}(i) =& B_i \lambda_i \log_2 \left(\left(1 + \frac{PT_L}{MT_P}\right)^{t_{on} - \tau_2 + \tau_1} \right. \\ & \left. \cdot \left(1 + \frac{PT_S}{T_I}\right)^{t_{off} - \tau_1} \right) \end{aligned} \tag{18}$$

The secondary users' interference can then be computed as

$$\begin{aligned} I_{I,T}(i) &= I_T(i) + (1 - I_T(i)) \cdot I_I \\ &= \lambda_i \tau_2 + (1 - \lambda_i \tau_2) \left(1 - \frac{T_I}{T_L}\right) \end{aligned} \tag{19}$$

This is because we will interfere with everyone while transmitting at a high power for τ_2 time units each burst, and I_I fraction of users by transmitting at our reduced power level.

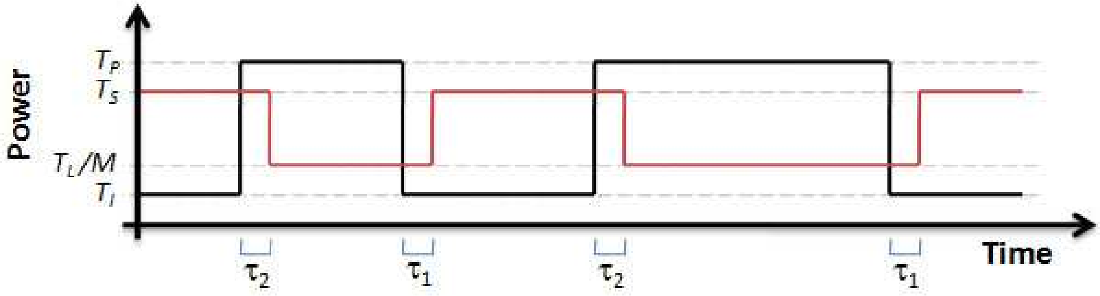


Figure 2: Power level for a primary (black) and secondary (red) signal occupying the same frequency, showing constrained power level when the primary signal is transmitting, and unconstrained power level when the primary signal is idle.

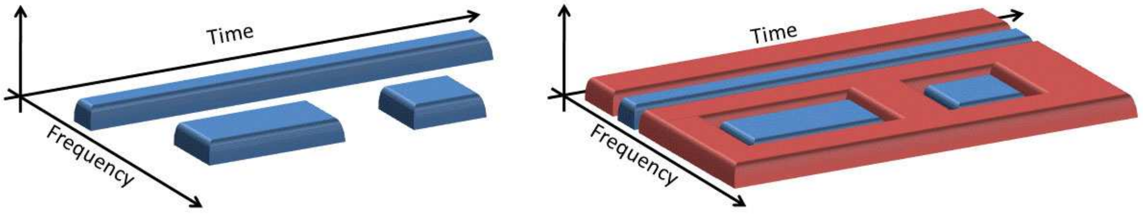


Figure 3: Dynamic spectrum access strategy combining both frequency-based and time-based coexistence: **(a)** primary user channel occupancy (blue) and **(b)** secondary user access (red)

2.5 Hybrid Coexistence

It is certainly possible to use multiple coexistence techniques simultaneously. We've already seen time-domain and interference-domain coexistence. For example, for the primary user access depicted in Figure 3(a), secondary users could engineer a waveform such as the one in Figure 3(b) to occupy all the vacant area surrounding the primary signal. This would involve both frequency and time-domain coexistence.

Combining all three coexistence techniques into a single approach, we can adjust our power levels in both frequency and in time to create a waveform that occupies all available spectrum. This could be an adaptive, power-controlled OFDM or shaped DSSS medium-access control (MAC) scheme [11].

Given we have a pooling radio overlapping N signals $S_i \in \mathcal{S}$. Our total capacity is

$$C_H = C_F^P + \sum_{i=1}^{N} C_{I,T}(i) \tag{20}$$

with total interference

$$I_H = \sum_{i=1}^{N} \frac{B_i}{W} I_{I,T}(i) \tag{21}$$

3 Analysis

In this section, we apply utility and price function to our channel capacity. First we define our different strategies, and present some numeric results for capacity and interference. We then define functions to penalize devices for causing interference, and show how this affects the selected strategies.

3.1 Strategies

There are three different strategies that can be used by a secondary user. The first, R_1, is DFS. The second, R_2, is DFS with time-domain multiplexing. The third, R_3, is to use all three, or hybrid coexistence. Let $R \in \{R_1, R_2, R_3\}$ be the selected strategy.

The achieved capacity C is then a function of the secondary users' maximum radio bandwidth and transmit power, the primary signals in the vicinity, and the selected strategy.

Table 1: WiMAX Signal Properties

Signal Bandwidth	$B_1 = 3.5$ MHz
Arrival Rate	$\lambda_1 = 50$ frames/sec
Idle Cycle	$\Delta_1 = 0.3$

Table 2: Secondary Radio's Properties

Frequency	$f_c = 3.5$ GHz
Bandwidth	$W = 20$ MHz
Sensing Time	$\tau_1 = \tau_2 = 1$ ms
Transmit Temp	$T_S = 2.3e15$ K (+28 dBm @ 20 MHz)
Rec from Primary	$T_P = 3.6e5$ K (-70 dBm @ 20 MHz)
Interference Temp	$T_I = 300$ K
IT Limit	$T_L = 500$ K
Dist to Primary	500 meters
Dist to Secondary	200 meters
Path Loss Exponent	3

$$C(R) = C_F + \begin{cases} R = R_1 & 0 \\ R = R_2 & \sum_{i=1}^{N} C_T(i) \\ R = R_3 & \sum_{i=1}^{N} C_{I,T}(i) \end{cases} \tag{22}$$

The total interference I is

$$I(R) = I_F + \begin{cases} R = R_1 & 0 \\ R = R_2 & \sum_{i=1}^{N} \frac{B_i}{W} I_T(i) \\ R = R_3 & \sum_{i=1}^{N} \frac{B_i}{W} I_{I,T}(i) \end{cases} \tag{23}$$

3.2 Examples

For the first example, let us consider coexisting with a single IEEE 802.16-2004 (fixed WiMAX) signal. Its properties are outlined in Table 1. The secondary radio's properties are outlined in Table 2.

Evaluating the resulting capacities and interference subject to our strategies, we obtain Table 3. We see a base capacity of 6.3 MHz for DFS, which increases to 6.7 MHz as we add additional coexistence techniques. We offer two different interference statistics. $I(1)$ is the interference

Table 3: Capacity and Interference Results for single WiMAX radio

Strategy	Capacity	$I(1)$	I
R_1	6.3 Mbps	0	0
R_2	6.6 Mbps	5%	0.9%
R_3	6.7 Mbps	43%	7.5%

Table 4: Capacity and Interference Results for multiple, unloaded WiMAX radios

Strategy	Capacity	$I(1..4)$	I
R_1	2.3 Mbps	0	0
R_2	4.0 Mbps	5%	3.5%
R_3	4.1 Mbps	43%	30%

specifically to signal S_1. I is overall average interference, and is equal to $I(1) \cdot \frac{3.5}{20}$, which is $I(1)$ multiplied by the ratio of signal S_1's bandwidth to the overall secondary bandwidth W. We can see that time-domain coexistence adds some interference, but interference-domain coexistence adds *significant* interference.

Next, let us consider another case where we are sharing the spectrum with 4 WiMAX radios as specified in Table 1, but for each the idle cycle $\Delta_1 = 0.8$. Table 4 shows the evaluation results. The individual interference is the same, but the overall interference I has increased due to the increase in the number of primary users. However, we see a significant improvement in using time-domain coexistence over just frequency-domain coexistence for this spectrally-crowded but relatively unloaded environment.

Finally, consider the case where there is both spectral crowding and heavy usage. Let's shrink $W = 18$, have 5 WiMAX primary users, with idle cycles $\Delta_i = 0.1$. The results are in Table 5. Again interference increases, but we can see a relatively significant advantage in using R_3 from a capacity standpoint.

Table 5: Capacity and Interference Results for multiple, loaded WiMAX radios

Strategy	Capacity	$I(1..5)$	I
R_1	190 kbps	0	0
R_2	520 kbps	5%	4.8%
R_3	1 Mbps	43%	41%

3.3 Utility Functions

Next, let's assume a utility function $U(C(R))$ which defines utility as a function of capacity, and a pricing function $P(I(R))$ which defines a price as a function of interference. Our goal is to then select a strategy R to maximize the difference:

$$R_{\text{selected}} = \underset{R \in \{R_1, R_2, R_3\}}{\arg\max} \; U(C(R)) - P(I(R)) \tag{24}$$

In most cases, $U(\cdot)$ would be selected by the secondary user while $P(\cdot)$ would be provided by either the primary user or a regulator. In most cases, $U(\cdot)$ will be logarithmic in C (diminishing return on additional capacity) and $P(\cdot)$ will be exponential in I (to help restrict the amount of interference). Thus

$$\begin{aligned} U(C(R)) &= u_1 \log(1 + u_2 C(R)) \\ P(I(R)) &= p_1 \mathbf{e}^{p_2 I(R)} \end{aligned} \tag{25}$$

In our experimentation, $u_1 = u_2 = p_1 = 1$ and $p_2 = 10$ provides for good decision making.

4 Conclusion

In this paper, we analyzed a variety of coexistence techniques, including frequency-domain, time-domain, and interference-domain. Each provided access to a different portion of unused spectrum.

For most current, commercial application, the main result of this paper is that dynamic frequency selection is sufficient. Currently IEEE 802.22 [5] is seeking to utilize UHF broadcast television spectrum. DTV signals are continuously-modulated, and therefore time-domain coexistence provides no additional capacity. Additionally, the interference constraints prevent the use of any interference-domain coexistence. Consequently, considering an average spectral occupacy of 12%, dynamic frequency selection is sufficient.

In frequency bands where most channels are active but only intermittently, time-domain techniques will be vital to allow coexistence; however, no such bands are currently under consideration for cognitive radio licensing. That's not to say it they won't be in the future.

In most cases, interference-domain coexistence, and consequently interference temperature in general, is ineffective. The benefit is insignificant to the incurred cost, in most cases. Certainly with better and faster spectrum sensing techniques, we can reduce incurred interference in all cases.

Overall, frequency-domain coexistence will take us from 10% spectral occupancy to 80%, which was the goal set by the DARPA XG project [4]. Going from 80% to 98% will require time-domain coexistence. It's unclear whether interference temperature is a viable approach to go the final 2%.

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