Formalizing the Interference Temperature Model

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***Abstract*— To combat recent spectral overcrowding in unlicensed bands, the FCC has been investigating new ways to manage RF resources. The idea is to let people use licensed frequencies, provided they can guarantee interference perceived by the primary license holders will be minimal. With advances in software and cognitive radio, practical ways of doing this are on the horizon. In 2003 the FCC released a memorandum seeking comment on the *interference temperature model* for controlling spectrum use. Formally defining and analyzing techniques for implementing this model are the primary goals of this paper.**

Two interpretations of the interference temperature model are developed, and for each we examine tradeoffs between power, bandwidth, and capacity. From these relationships, algorithms for computing RF transmission parameters are developed. These algorithms seek to maximize both capacity and spectral efficiency for a given RF environment. Additionally, we describe ways to choose a center frequency that will optimize future performance, subject to the constraints of the interference temperature model.

I. Introduction

The twenty-first century has seen an explosion in personal wireless devices. From mobile phones to WiFi, people want to be perpetually networked no matter where they are. Much of the wireless technological innovation is happening in the unlicensed bands, and as a result these small frequency ranges are becoming crowded, with many personal electronic devices interfering with each other.

To help alleviate the overcrowding, the FCC has begun considering other ways of managing spectrum. Rather than static allocations based on detailed site surveys, a more real-time, dynamic approach must be adopted. To that end, they have new policy [1] that allows cognitive radios to operate in licensed frequency ranges, provided they are smart enough to sense their RF environment and steer clear of frequencies where they detect licensed carriers. This offers many new frequency bands for communications (see Figure 1).

A motivating example is the current broadcast television frequency bands. Of the 68 channels, on average only 8 channels are used in any given TV market, or roughly 12%. Were unlicensed devices allowed to coexist with broadcast television, an additional 350 MHz of prime spectrum real estate would be available.

However, in the original FCC-proposed model [1] a channel is either occupied or it isn't. If it's occupied by a licensed signal, an unlicensed device may not use it. A logical extension to this binary model is one of true coexistence, where unlicensed transceivers can operate on the same frequencies as licensed signals, provided they can quantify and bound the additional interference. To that end, the FCC proposed the *interference temperature model* (ITM) [2], which provides a metric to measure interference experienced by licensed receivers.

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Fig. 1. FCC frequency allocations in the 300 MHz to 3 GHz microwave band

This paper seeks to concretely define the interference temperature model, and derive algorithms for computing RF transmission parameters that achieve a desired capacity. Additionally, we derive bounds on interference caused to primary licensees due to unlicensed transmissions.

Section 2 overviews related work. Section 3 introduces the interference temperature model. Section 4 describes techniques for measuring interference temperature subject to various constraints. Section 5 investigates ways of selecting a good center frequency. Section 6 concludes.

II. Related Work

Many approaches to dynamic spectrum allocation have been proposed. Here we overview some of the major efforts.

Much research was conducted as a part of the *Spectrum Efficient Uni and Multicast Services over Dynamic multi-Radio Networks in Vehicular Environments* (OverDRiVE) projects starting in 2000. Their main concern was video content delivery to vehicles [3], [4], [5]. The OverDRiVE architecture involves partitioning spectrum in space, frequency, and time. Each Radio Access Network (RAN) would be allocated certain blocks by a central authority in response to their predicted capacity needs.

Later, the CORVUS project at Berkeley used similar ideas [6]. They create a channelized spectrum pool from unused licensed spectrum, and have algorithms to allocate it efficiently.

These approaches are centralized, requiring someone to decide who should use which spectral resources at what time, while guaranteeing minimal interference to licensed devices. While achieving good results, current politics involved in frequency licensing would make adopting such an approach unlikely. The need for a central authority hampers feasible deployment.

More recently, research has begun on distributed techniques for dynamic spectrum allocation, where no central spectrum authority is required. Decentralized approaches may be less efficient, but require much less cooperation. Game theoretic aspects of this are studied in [7].

Of the decentralized approaches, most require control channel communication between devices. They make local, independent decisions about how to best communicate, and use these low-bandwidth side channels to negotiate communications parameters [8], [9]. An open question still exists though – what if your control channel is overcome by interference? How do you negotiate a transition to a new frequency?

The interference temperature model is an entirely new concept for dynamic spectrum access. Radio nodes treat licensed users, other unlicensed radio networks, other unlicensed nodes within the same network, interference, and noise all as *interference* affecting its signal-to-interference ratio (SIR). Higher interference yields lower SIR, which means lower capacity is achievable for a particular signal bandwidth. Radio nodes search for gaps in frequency and time where the measured interference is low enough to achieve communication at a target capacity, subject to overall interference constraints defined by the interference temperature model. A major distinction is that all other proposed schemes *avoid* licensed signals, while we try to *coexist* with them.

III. Interference Temperature

The concept of interference temperature is identical to that of noise temperature. It is a measure of the power and bandwidth occupied by interference. Interference temperature T_I is specified in Kelvin and is defined as

$$T_I(f_c, B) = \frac{P_I(f_c, B)}{kB}, \tag{1}$$

where $P_I(f_c, B)$ is the average interference power in Watts centered at f_c, covering bandwidth B measured in Hertz. Boltzmann's constant k is $1.38 \cdot 10^{-23}$ Joules per Kelvin degree.

The idea is that by taking a single measurement, a cognitive radio can completely characterize both interference and noise with a single number. Of course, it has been argued that interference and noise behave differently. Interference is typically more deterministic and independent of bandwidth, whereas noise is not.

For a given geographic area, the FCC would establish an *interference temperature limit*, T_L. This value would be a maximum amount of tolerable interference for a given frequency band in a particular location. Any unlicensed transmitter utilizing this band must guarantee that their transmissions added to the existing interference must not exceed the interference temperature limit at a licensed receiver.

While this may seem clear cut, there is ambiguity over which signals are considered interference, and which f_c and B to use. Should they reflect the unlicensed transceiver or the licensed receiver? Analyzing these ambiguities results in two possible interpretations, which we term the *ideal model* and the *generalized model*, which are illustrated in Figure 2.

A. Ideal Model

In the *ideal interference temperature model* we attempt to limit interference specifically to licensed signals. Assume the unlicensed transmitter is operating with average power P, at center frequency f_c, with bandwidth B. Assume also that this band $[f_c - B/2, f_c + B/2]$ overlaps n licensed signals, with respective frequencies and bandwidths of f_i and B_i. The goal is to then guarantee that

$$T_I(f_i, B_i) + \frac{M_i P}{kB_i} \le T_L(f_i) \quad \forall\ 1 \le i \le n, \tag{2}$$

where M_i will be defined shortly.

In other words, the constraint guarantees that the transmission does not violate the interference temperature limit at licensed receivers, as shown in Figure 2. Each signal overlapped by the unlicensed transmission adds a new power constraint, over which the minimum is taken. If the unlicensed signal does not overlap a licensed one, then the transmit power is unconstrained, though a regulatory maximum would likely be set.

In Figure 2, the dashed lines represent the interference power limit computed using the interference temperature and the bandwidth of the licensed signals. Notice that each licensed signal places a different constraint on the total allowable interference, and an unlicensed transmitter must guarantee that none of the individual interference constraints are violated.

Note the introduction of constants M_i in (2). This is a fractional value between 0 and 1, representing a multiplicative attenuation due to fading and path loss between the unlicensed transmitter and the licensed receiver. The idea is that the interference temperature model restricts interference at the licensed receiver, not the unlicensed transmitter, and therefore we must account for attenuation between these two devices. Since typically it's impossible to know the distance to all licensed receivers, assume that this value is fixed by a regulatory body to a single constant M.

There are two main challenges in implementing the ideal model. The first involves identifying licensed signals. One key question arises: how do you distinguish licensed signals from unlicensed ones? For specific cases, this can be relatively easy. In particular, consider the problems faced by IEEE 802.22 [10], currently under investigation by their spectrum sensing task group. They wish to coexist with digital broadcast television (DTV) signals, and can implement very specialized, matched filter sensors to look for DTV transmissions. If you know exactly with whom you are coexisting, then this problem becomes simpler.

The second problem involves measuring T_I in the presence of a licensed signal. The unlicensed transceiver must measure the interference floor *underneath* the licensed signal. Again, this can be relatively easy if it has knowledge of the licensed waveform's structure. For example, perhaps it can measure during an interval when the signal is not present, as often occurs in bursty, time-multiplexed signals. Also, if the radios have precise knowledge of the signal's bandwidth B and center frequency f_c, they can approximate the interference

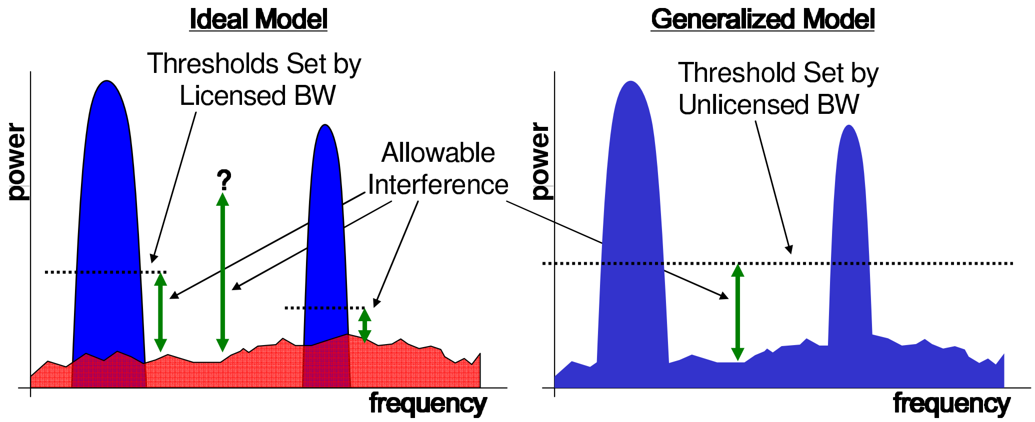

Fig. 2. Ideal and Generalized interpretations of the Interference Temperature Model

temperature as

$$T_I(f_c, B) \approx \frac{P(f_c - B/2 - \tau) + P(f_c + B/2 + \tau)}{2kB}, \quad (3)$$

where $P(f)$ is the sensed signal power at frequency f, and τ is a safety margin of a few kHz.

Assuming a specialized environment where cognitive radios can locate licensed signals and measure interference temperature, the next goal is to determine radio parameters f_c, B, and P that achieve a desired capacity C. This will be a piecewise-continuous optimization problem with constraints defined in (2).

B. Generalized Model

The generalized interference temperature model has a different interpretation to signals and bandwidths. The fundamental premise of the generalized model is that we have no apriori knowledge of the RF environment, and consequently have no way of distinguishing licensed signals from interference and noise, as shown in Figure 2.

Again, the dashed line in Figure 2 represents the interference power constraint. However, in this model it is computed using the interference temperature and the bandwidth of the transmitter, resulting in a single constraint rather than many as we saw in the ideal model.

Under these assumptions, we must apply the interference temperature model to the entire frequency range, and not just where licensed signals are detected. This translates into the following constraint:

$$T_I(f_c, B) + \frac{MP}{kB} \leq T_L(f_c). \quad (4)$$

Notice that the constraint is in terms of the *unlicensed transmitter's* parameters, since the parameters of the licensed receivers are unknown. One question that immediately comes to mind: under what conditions does the generalized model limit interference as well as the ideal model?

If both constraint equations are solved for P, the following results:

$$\begin{aligned} P^{id} &= B_i(T_L(f_c) - T_I^{id}(f_i, B_i)) \\ P^{gen} &= B(T_L(f_c) - T_I^{gen}(f_c, B)) \end{aligned} \quad (5)$$

To cause *less* interference in the generalized model, we are interested in the case where $P^{id} \geq P^{gen}$, thus:

$$B(T_L(f_c) - T_I^{gen}(f_c, B)) \leq B_i(T_L(f_c) - T_I^{id}(f_i, B_i)) \quad \forall\, 1 \leq i \leq n \quad (6)$$

Assuming each licensed signal has power P_i and otherwise the interference floor is defined by the thermal noise temperature T_N, we can transform (6) into the following:

$$kBT_L(f_c)(B - B_i) + kBT_N \sum_{j=1}^{n} B_j \leq \sum_{j=1}^{n} B_j P_j \quad \forall\, 1 \leq i \leq n \quad (7)$$

In general, provided B_i and P_i are sufficiently large, this condition can be easily met.

Considering only one licensed receiver simplifies the inequality to

$$\frac{kBT_L}{P_1 - kBT_N} \leq \frac{B_1}{B - B_1}. \quad (8)$$

Thus a small T_L, large B_1, or large P_1 will generally satisfy the constraint.

The next section describes challenges inherent in selecting transmission bandwidths necessary to meet a particular target capacity in each interference temperature model.

IV. Measuring Interference Temperature

In this section we describe techniques for selecting a power and bandwidth to meet a particular capacity requirement. Each model imposes different constraints on how interference temperature should be measured, and what those measurements signify.

A. Properties of Interference Temperature

One shortcoming in the design of the interference temperature model is its simplicity. The goal was to define a single metric that fully captures both the properties of interference and noise. In the end, a *temperature* approach was used rather than a *power* approach. This accurately models the noise portion of the metric, but not the interference portion.

The eventual goal is to determine the difference between the regulatory interference temperature limit and the measured interference temperature. This then defines the *transmission temperature* the cognitive radio can use, where for a given bandwidth the cognitive radios can compute the maximum allowed power.

Let's define things a little more concretely. Thus, the interference temperature T_I can be specified as a function of bandwidth B as

$$\begin{aligned} T_I(f_c, B) &= \frac{1}{Bk} P_I(f_c, B) \\ &= \frac{1}{Bk}\left(\frac{1}{B}\int_{f_c-B/2}^{f_c+B/2} S(f)\,df\right) \\ &= \frac{1}{B^2k}\int_{f_c-B/2}^{f_c+B/2} S(f)\,df \end{aligned} \tag{9}$$

where $S(f)$ represents power spectral density of the current RF environment.

Recall that in the ideal model, P_I reflects only the interference and noise, where in the generalized model, P_I reflects both the interference, noise, and any licensed signals. This same characterization extends to $S(f)$.

Next, consider how the transmission will affect the received interference temperature $\hat{T}_I(f_c, B)$. As described before, the end goal is to compute a transmit power P and bandwidth B that satisfy the constraints (2) and (4), depending on the model.

There are two basic cases to consider. First, B is known, and the goal is to compute a valid P. In the ideal model, this is fairly straightforward. Rewriting (2) we have

$$P \leq \frac{B_i k}{M}(T_L(f_i) - T_I(f_i, B_i)) \quad \forall\ 1 \leq i \leq n, \tag{10}$$

where the assumption is that for a selected B the unlicensed signal overlaps n licensed signals with parameters f_i and B_i. If $n = 0$ then $P \leq P_{\max}$, where $P_{\max}$ is the radio's maximum transmit power. For $n > 0$, to meet this constraint, we minimize i:

$$P \leq \min_{i\in[1..n]}\left(\frac{B_i k}{M}(T_L(f_i) - T_I(f_i, B_i))\right). \tag{11}$$

This gives us a way to compute P as a function of B.

In the generalized model, solve for P to obtain

$$P \leq \frac{Bk}{M}T_L(f_c) - \frac{1}{BM}\int_{f_c-B/2}^{f_c+B/2} S(f)\,df. \tag{12}$$

If the radios are trying to compute a valid B in terms of P, things get a little more complicated for both models. In the ideal model, the maximum bandwidth is going to depend on the interference at various licensed signals. Let there be n^* signals possibly overlappable by the radio for a given f_c and maximum transmit bandwidth $B_{\max}$. For each, licensed signal compute the maximum transmit power P_i in that band as

$$P_i = \frac{B_i k}{M}(T_L(f_i) - T_I(f_i, B_i)). \tag{13}$$

If $\forall\ 1 \leq i \leq n^*$, $P_i > P$, then the unlicensed transmitter will not cause any harmful interference regardless of the bandwidth, and therefore the constraint is

$$B \leq B_{\max}. \tag{14}$$

Otherwise, find the index i^* of the signal closest to f_c to which the unlicensed transmitter will cause harmful interference as

$$i^* = \arg\max_{i\in\{i:P_i<P\}} |f_c - f_i|. \tag{15}$$

From this compute the maximum bandwidth as

$$B \leq 2\left(|f_c - f_{i^*}| - B_{i^*}/2\right). \tag{16}$$

For the generalized model, there is no closed-form solution for a general $S(f)$. However, since $S(f)$ is a real, nondecreasing, continuous function of B, there is a solution $\forall S(f)$, even though it may be outside the radio's dynamic range of $(0, B_{\max}]$.

Since bandwidth and power are so interrelated, in the next sections we consider them jointly in terms of capacity.

B. Capacity in the Ideal Model

So far we've bounded bandwidth in terms of power, and vice versa. Let's change the formulation somewhat, and consider them jointly in terms of capacity. From the Shannon-Hartley Theorem[1] we can derive

$$C = B\log_2\left(1 + \frac{LP}{P_I + P_L}\right), \tag{17}$$

where B and P are as before, P_I represents interference power, and P_L represents the average power contributed by licensed signals.

Notice the addition of another constant, L. This value is similar to M, except it represents multiplicative path loss between the unlicensed transmitter and unlicensed receiver. We are measuring capacity at the *receiver*, and therefore need knowledge of the bandwidth and power at the receiver.

As before, let n be the number of licensed signals overlapped[2] by the unlicensed transmission, and let it be a function of f_c and B, such that $n(f_c, B)$ is the number of signals overlapped by the frequency range $[f_c - B/2, f_c + B/2]$.

For a particular bandwidth B where

$$\alpha = \min_{i\in[1..n(f_c,B)]}\left(\frac{B_i k}{M}(T_L(f_i) - T_I(f_i, B_i))\right) \tag{18}$$

the maximum transmit power is

$$P^*(f_c, B) = \begin{cases} P_{\max} & n(f_c, B) = 0 \\ \min(P_{\max}, \alpha) & n(f_c, B) > 0 \end{cases}. \tag{19}$$

Note that $P^*(f_c, B)$ is a non-increasing function of B. As the bandwidth is increased, more signals are overlapped, which could lower the maximum transmission power.

[1]The Shannon-Hartley Theorem requires an AWGN channel. Thus, the derived results will only be accurate if the underlying modulation scheme has a whitening effect on noise, like CDMA.

[2]For simplicity, we do not examine partially overlapping signals. The analysis could be extended to account for this, but the notation becomes particularly awkward. Capacity would then become a continuous function of B.

Looking at P_I and P_L, we can compute the interference to the unlicensed transmission as

$$P_I(f_c,B) = kBT_I(f_c,B)$$
$$P_L(f_c,B) = \frac{1}{B}\sum_{i=1}^{n(f_c,B)} P_i B_i. \tag{20}$$

Interference $P_I(f_c,B)$ is increasing with B, as the noise floor increases due to thermal noise. We cannot say anything about $P_L(f_c,B)$: it could either be increasing, decreasing, or both.

Thus the achievable capacity is

$$C^*_{id}(f_c,B) = B\log_2\left(1+\frac{LP^*(f_c,B)}{P_I(f_c,B)+P_L(f_c,B)}\right). \tag{21}$$

As long as $P^*/(P_I+P_L)$ is decreasing at a subexponential rate, increasing B will generally increase C. However, it will be highly dependent on the RF environment.

In a real radio, assuming the signal processing issues associated with the ideal model are solved, computing P and B subject to some C should be relatively simple. For a given f_c, simply characterize all n^* licensed signals and measure the interference temperature at each. From that data, a numeric version of $C^*_{id}(f_c,B)$ can be calculated, and solved for C.

C. Capacity in the Generalized Model

Interference temperature must always be measured at some bandwidth B, due to deterministic interference sources. To measure $T_I(f_c,B)$, down-sample the passband signal such that f_c is at $B/2$. Then quantize the spectrum at rate $2B$, and compute its PSD. This will yield a power spectrum for the frequency range $f_c - B/2$ to $f_c + B/2$, which is $\hat{S}_B(f)$. To compute the interference temperature, integrate as follows:

$$T_I(f_c,B) = \frac{1}{B^2 k}\int_0^B \hat{S}_B(f)\,df. \tag{22}$$

Thus, we can now compute the interference temperature as a function of B.

Let's say a minimum capacity of C is necessary for the target application running over our cognitive radio network. The next goal is to find a P and B that both meet regulatory requirements and achieve the capacity constraints. We showed that choosing a B and solving for a maximum P was relatively simple; however, considering $S(f)$ may have very steep slopes when f_c is close to powerful licensed signals, this may be problematic.

Thus, we must combine some of our concepts. Compute a capacity function $C^*_{gen}(f_c,B)$ in terms of B, and then solve $C^*_{gen}(f_c,B) = C$ for B. Let's assume a maximum transmit power is used for the bandwidth selection, or

$$P^*(f_c,B) = \frac{Bk}{M}(T_L(f_c) - T_I(f_c,B)). \tag{23}$$

This selects the maximum regulatory power allowable in the interference temperature model.

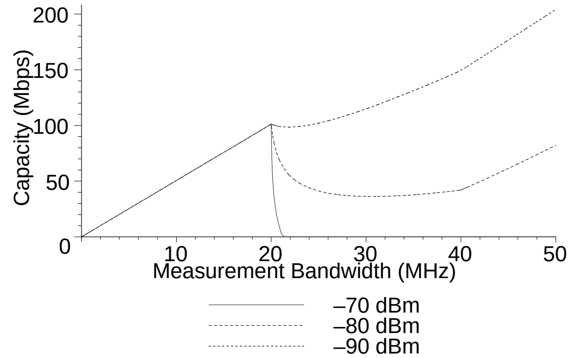

Fig. 3. Example capacities as a function of B for the generalized model, assuming a licensed signal of varying strengths located at $[f_c+10$ MHz, f_c+20 MHz$]$, with $T_L = 10000$ Kelvin and a noise temperature of 300 Kelvin. As the interference power increases, the capacity past 20 MHz falls off more.

Then define the capacity as

$$\begin{aligned} C^*_{gen}(f_c,B) &= B\log_2\left(1+\frac{LBk(T_L(f_c)-T_I(f_c,B))}{MBkT_I(f_c,B)}\right) \\ &= B\log_2\left(1+\frac{L(T_L(f_c)-T_I(f_c,B))}{MT_I(f_c,B)}\right) \end{aligned} \tag{24}$$

This result is similar to the capacity in the ideal model, as generally it is increasing with B, but could vary greatly from RF environment to RF environment.

D. Solving for Capacity

Solving $C^*(f_c,B) = C$ can be difficult for both models[3]. For a general interference environment, this must be done numerically. Figure 3 shows a simple example of a 10 MHz licensed signal with square power spectral density located 10 MHz from the carrier frequency. We can see that as long as the signal's power is relatively low, e.g. -90 dBm, the capacity function for the generalized model remains relatively linear. However, for -70 dBm, we can see that it will significantly hamper the capacity. The unlicensed transceiver can achieve maximum capacity if it avoids the signal all together.

As the previous example illustrates, the capacity function is not strictly increasing, and therefore there may be multiple bandwidths that give the same capacity. Certainly the best choice is to select the smallest bandwidth possible that will achieve your desired capacity in order to maximize spectral efficiency.

If the unlicensed device wishes to solve $C^*(f_c,B) = C$ for B, then it must employ numeric techniques. Using the above equations, for a particular B it can compute $T_I(f_c,B)$ and consequently $C^*(f_c,B)$. The next sections describe algorithms for accomplishing this numeric computation.

1) Hill Climbing Approach: We can frame the problem as a constrained optimization with objective function

$$\left|C^*(f_c,B) - C\right|. \tag{25}$$

[3]We use C^* to represent both C^*_{id} and C^*_{gen}, when distinguishing is unimportant.

One approach is to hill climb, trying to minimize the objective function with respect to B [11]. This function may have several global minimizers over the bandwidth range of the radio. The goal is to locate the one corresponding to the smallest bandwidth.

A good approach is to run our hill climbing algorithm several times with

$$B_0 = \left\{ \frac{iB_{\max}}{N} \right\}_{i=1..N}. \tag{26}$$

This will yield N, likely non-unique, solutions. Simply select the one with the smallest bandwidth.

A simple pricing scheme can also be used. To find global capacity maximizers, set $C = \infty$ and run the same algorithm. This will yield a set of points $(B_i, C_i)_{i=1..N}$. Let the capacity utility function be $U(C)$ and the pricing function be $P(B)$. We can then select the best point i^* as

$$i^* = \arg\max_{i=1..N} \left(U(C_i) - P(B_i) \right). \tag{27}$$

Lastly, we should address selection of N. The number of local minima will be proportional to the number of interfering signals. This could be computed by the radio by determining the number of local maxima n in $S(f)$ for $f_c - B/2 \le f \le f_c + B/2$. If solving for a specific C, let $N > 2n$, since there would likely be a solution on either side of the signal. If searching for global capacity maximizers, then $N > n$ should be sufficient. This operation could be done infrequently, and would provide a good estimate for N, assuming interfering signals are relatively uniformly spaced over the target spectrum band.

While we can use the hill climbing approach to both optimize $C^*(f_c, B)$ and solve it for a target capacity, we will see in the next section that fixed-point iteration is a more elegant way to solve $C^*(f_c, B)$ for a target capacity. Therefore, hill climbing is most appropriate when trying to maximize capacity.

2) Fixed-Point Iteration Approach: Many problems that you can solve with hill climbing can also be solved using fixed-point iteration. The basic idea is to take the original problem, $C^*(f_c, B) = C$ and rewrite it as $B_i = f(C^*(f_c, B_{i-1}), C)$, and hope that $\{B_i\}_{i=0...n}$ converges.

Due to the complexity of $C^*_{id}(f_c, B)$, in this section we will only consider application of fixed-point iteration to the generalized model. Next, consider a reformulation of the original problem.

Theorem 1: The sequence $\{B_i\}_{i=1..n}$ where

$$B_{i+1} = \frac{C}{\log_2\left(1 + \frac{L(T_L(f_c) - T_I(f_c, B_i))}{M T_I(f_c, B_i)}\right)} \tag{28}$$

converges linearly to a solution to

$$C = B \log_2\left(1 + \frac{L(T_L(f_c) - T_I(f_c, B_i))}{M T_I(f_c, B_i)}\right) \tag{29}$$

as long as

$$B_0 > \frac{2CT_I^*}{T_N} \log_2\left(1 + \frac{L(T_L(f_c) - T_I^*)}{M T_I^*}\right)^{-2} \tag{30}$$

where

$$T_I^* = \max_{B \in (0, B_{\max}]} T_I(f_c, B) \tag{31}$$

and a solution exists in $B \in (0, B_{\max}]$.

Proof: We're examining the problem in terms of fixed-point approximation. Let

$$g(B) = \frac{C}{\log_2\left(1 + \frac{L(T_L(f_c) - T_I(f_c, B))}{M T_I(f_c, B)}\right)}. \tag{32}$$

The theory of fixed-point iteration methods dictates that if $B = g(B)$ has at least one solution in some interval $[a, b]$, $g(B)$ is continuous, and $|g'(B)| < 1$ then any starting point in that interval will converge to a solution [12]. Intersect the interval $[a, b]$ with the feasible interval, $(0, B_{\max}]$. The result is a range for B_0

$$B_0 \in [a, \min\{b, B_{\max}\}]. \tag{33}$$

$T_I(f_c, B)$ is continuous, so consequently $g(B)$ is continuous. The derivative constraint can be expressed as follows:

$$\begin{aligned} &\frac{CLT_L(f_c)|T_I'(f_c, B)|}{T_I(f_c, B)(LT_L(f_c) + (M - L)T_I(f_c, B))} \\ &\qquad < \log_2\left(1 + \frac{L(T_L(f_c) - T_I(f_c, B))}{M T_I(f_c, B)}\right)^2 \end{aligned} \tag{34}$$

Obviously this constraint is not entirely useful, as it is in terms of B, which we do not yet know. In order to simply this further, we need to remove the dependence on B. First, use the definition of T_I^* provided in the theorem statement, and notice that

$$T_N \le T_I(f_c, B) \le T_I^*. \tag{35}$$

Next examine the derivative of the interference temperature:

$$T_I'(f_c, B) = \frac{\hat{S}(B)}{B^2 k} - \frac{2}{B} T_I(f_c, B) \tag{36}$$

Thus to maximize $|T_I'(f_c, B)|$, let $\hat{S}(B) = 0$ and $T_I(f_c, B) = T_I^*$. The result is

$$|T_I'(f_c, B)| \le 2T_I^*/B. \tag{37}$$

Substituting,

$$\begin{aligned} B_0 &> \frac{CLT_L(f_c)2T_I^*}{T_N(LT_L(f_c) + (M - L)T_N)} \log_2\left(1 + \frac{L(T_L(f_c) - T_I^*)}{M T_I^*}\right)^{-2} \\ &> \frac{CLT_L(f_c)2T_I^*}{T_N(LT_L(f_c))} \log_2\left(1 + \frac{L(T_L(f_c) - T_I^*)}{M T_I^*}\right)^{-2} \\ &> \frac{2CT_I^*}{T_N} \log_2\left(1 + \frac{L(T_L(f_c) - T_I^*)}{M T_I^*}\right)^{-2} \end{aligned} \tag{38}$$

Thus we have proved the theorem. ■

We now have a viable algorithm for computing the required bandwidth B in terms of desired capacity C. If $B_0 > B_{\max}$, this does not necessarily mean a solution does not exist, since we derived a *sufficient* condition, and not a *necessary* one. If divergence is detected, then the capacity C must be decreased in order to find a solution.

The key point is that fixed-point iteration can find a solution if one exists, but may not always succeed. As a result, it may be useful to implement a hybrid algorithm that first tries fixed-point iteration, and if divergence is detected, switches over to a hill climbing approach. Note that the algorithms can be executed on a PSD snapshot taken with bandwidth $B_{\max}$, and consequently radio sensing resources need not be tied up during algorithm execution.

V. Frequency Selection

In the previous sections we describe how to select a bandwidth given a center frequency f_c. However, one of the major uses for cognitive radio is to dynamically select your center frequency to exploit spectrum access opportunities.

There are two main schools of thought on dynamic center frequencies. In particular, the ability to change f_c in real time increases higher-layer protocol complexity, since the receiver must know that the transmitter has changed frequency. These competing ideas are related to how radios exchange radio parameters.

The first assumes there is a management or control channel through which radios can coordinate. Devices can indicate the center frequency, waveform, destination, and time of their next transmission. Thus, f_c is something to be optimized and changed in real time.

However, others consider the management channel an unrealistic assumption. In a dense, busy packet network environment, management of the management channel becomes a problem. Also, how can we guarantee the management channel is not causing harmful interference?

Here, we look at how to select f_c for optimal performance, and ignore protocol issues for coordination. We simply address how to select the best f_c at a particular time. The approach is a simple extension of the ideas in the last section.

We defined the capacity functions for each model in the previous sections, and described techniques to solving

$$C^*(f_c, B) = C \tag{39}$$

for B. However, if we assume f_c is no longer fixed, how does that change things?

We advocate selecting an f_c at the beginning to maximize your eventual per-packet capacity, and leaving it fixed unless communication at that frequency becomes impossible. Thus, the optimal center frequency is

$$f_c^* = \max_{f \in [f_{\min}, f_{\max}]} \left(\max_{B \in (0, B_{\max}]} C^*(f, B) \right). \tag{40}$$

Maximizing over B can be done using the hill climbing approach. Assuming the space of frequencies is channelized, then $[f_{\min}..f_{\max}]$ is a discrete set, and the hill climbing can be executed for each f.

Alternatively, we can look at the structure of $C^*(f_c, B)$ in more detail. In particular, in the presence of uniform interference, both capacity functions are maximized when licensed signals are completely avoided. Assume n licensed signals are detected within the radio's overall candidate frequency band. Let each be located at center frequency f_i and have bandwidth B_i. Assume $\{f_i\}_{i=1}^n$ is an ordered set, where

$$f_1 \leq f_2 \leq \cdots \leq f_n. \tag{41}$$

The best frequency is going to be half way between the two signals with furthest distance between them. In particular, if

$$i^* = \arg\max_{i=1..n-1} \left(f_{i+1} - \frac{B_{i+1}}{2} \right) - \left(f_i + \frac{B_i}{2} \right) \tag{42}$$

then

$$f_c^* = \frac{1}{2} \left(\left(f_{i^*+1} - \frac{B_{i^*+1}}{2} \right) + \left(f_{i^*} + \frac{B_{i^*}}{2} \right) \right). \tag{43}$$

Recall, however, that this assumes the interference is uniform. If interference varies some, but not a significant amount, adapt the previous optimization somewhat. In particular, if

$$\left| \frac{d}{df} T_I^{id}(f, B) \right| < \epsilon \qquad \forall\ f \in [f_{\min}, f_{\max}] \tag{44}$$

then define the channelization $\{c_i\}_{i=1}^{n-1}$ as

$$c_i = \frac{1}{2} \left(\left(f_{i+1} - \frac{B_{i+1}}{2} \right) + \left(f_i + \frac{B_i}{2} \right) \right) \tag{45}$$

and then maximize over the channels to compute

$$f_c^* = \arg\max_{f = c_1..c_{n-1}} \left(\max_{B \in (0, B_{\max}]} C^*(f, B) \right). \tag{46}$$

As discussed, center frequency should be selected to promote a radio environment that will maximize the potential capacity. Typically, this involves steering clear of licensed signals, so we use this fact to pick a set of candidate center frequencies. By computing the maximum capacity at each, we can decide which is optimal.

VI. Conclusion

This paper has considered how to use both interference temperature and the regulatory interference temperature limit to select an optimal radio bandwidth and power for a particular interference environment. We've discussed two interpretations of the interference temperature model, and showed the conditions under which the general model yields interference at least as small as the ideal model.

Two main techniques for solving for the desired bandwidth in both the ideal and generalized model were presented. The first uses hill climbing and is best suited for scenarios where we wish to maximize capacity for the radio's dynamic bandwidth range. The second uses fixed-point iteration and is designed to find a bandwidth for a specific target capacity. Future work involves a detailed analysis of how pricing functions can be applied to to bandwidth selection, and how this affects network-wide spectral efficiency.

These two techniques have been implemented within an interference temperature simulator and have been successfully used for dynamic spectrum access. In a noisy RF environment with several licensed carriers and many unlicensed tranceivers, unlicensed devices can successfully locate and utilize spectrum. A key result of the simulator is, however, that when forced to overlap a licensed signal, achievable capacity is low.

Consequently, research is underway to investigate techniques for shaping unlicensed waveforms to fit *around* licensed signals rather than overlap them with full power [13].

Overall, the interference temperature model offers an exciting new paradigm for dynamic spectrum access.

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