

# USING LOCALIZED RANDOM WALKS TO MODEL DELAY-TOLERANT NETWORKS

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**Abstract**—Mobile Wireless Delay-Tolerant Networks (DTNs) are wireless networks that suffer from intermittent connectivity, but enjoy the benefit of mobile nodes that can store, carry, and forward packets or messages, bringing them closer to their destinations through a selective forwarding policy. The evaluation of DTN routing protocols has primarily relied on simulation because most theoretical mobility models are unable to represent the mobility patterns that such protocols seek to take advantage of.

In this paper we present and analyze a mobility model that we call Localized Random Walk. This model is simple enough that it can be incorporated into mathematical models, but is spatially localized, which unlike other common mobility models, will make it possible to showcase the properties of heuristic-based DTN routing protocols. We derive the stationary spatial distribution of the mobility model, approximate what we call its spatial cross section, approximate the properties of its interaction with nodes following other mobility models, and use it to model some relatively simple DTN scenarios.

## I. INTRODUCTION

The key difference between Delay-Tolerant Networks (DTNs) and Mobile Ad-Hoc Networks (MANETs) is that DTNs may lack end-to-end connectivity [7]. In mobile wireless DTNs the network must rely on intermediate mobile nodes to store, carry, and forward each message as the opportunity arises, eventually delivering the message to its intended destination. The mobility patterns of the nodes in such a network will be crucial to the performance of the network.

In many applications DTN nodes will be carried by humans, animals, vehicles, or planets, which exhibit patterns in their movements. Many DTN routing protocols aim to take advantage of these patterns [15], [10], [16], [13], [6]. To evaluate these routing protocols, researchers often rely on simulations with mobility models that exhibit some type of patterned movement.

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The traditional mobility models such as uniform, random walk, and random direction [3] will generally fail to show any benefit under intelligent DTN protocols because past encounters will be poor predictors of future encounters, and all nodes obeying any particular one of these mobility models will have the same uniform spatial distribution in the long term. From a DTN standpoint the essential problem with all of these models is that once two nodes with the same mobility model coincide on a particular grid point, both are equally likely to encounter any other node. DTN routing protocols seek to take advantage of *differences* in the encounter probabilities. We have found that a heuristic DTN protocol run with random walk mobility can lead to a huge increase in latency, but little or no increase in efficiency.

On the other hand, several complex and more realistic mobility models have been developed [14], [17] based on intuition and experience with human and animal mobility patterns. Other specially tuned mobility models have been built based on real user traces [12], and several researchers have begun evaluating DTN protocols based on simulations driven by the encounter traces themselves [6], [18]. All of these models are useful for simulating a mobile network, but they are too complex to model mathematically.

In this paper we present, analyze, and use the Localized Random Walk (LRAW) mobility model. LRAW geographically biases node movement, making it possible to study the benefits of DTN routing protocols, yet is simple enough that it can be modeled mathematically. Nodes following the LRAW model have a *home cell* which they tend to remain close to. The home cell could represent, for example, the home or den of a human or animal, or the home base of a military unit.

### A. Prior Work

Localized mobility centered around a “home location” has been observed in wireless traces [1]. The need for a mobility model with non-uniform spatial distribution is noted in [8], though the authors do not propose a specific mobility model. Centrally biased random walks

have long been studied in other contexts, for example [9]. Also the LRAW mobility model is used for simulations, but not mathematically analyzed, in [20].

## II. SPECIFICATION OF THE MODEL

### A. Sparse Network Model

We view a DTN as a sparse disconnected collection of nodes that may occasionally meet. We model this situation using discrete space and discrete time. Each node occupies a cell in the grid, and it can only communicate with other nodes in the same cell. A model like this should be a reasonable approximation for very sparse networks such as the Saami Network or Zebra Net [13], [11], or in a denser urban environment where nodes may have short range and are surrounded by obstructions.

### B. Localized Random Walk Mobility Model

The mobility model operates as follows:

- Each node is assigned a fixed *home cell* in the physical space of the experiment.
- At each time step the node makes a list of all of its neighboring cells.
- For each cell  $i$  in the neighbor list let  $p_i = e^{-d_i/2\tau}$  where  $d_i$  is the (taxicab) distance from cell  $i$  to the home cell and  $\tau$  is the “tightness parameter”.
- the node moves from its current cell with a certain fixed probability  $s \in (0, 1]$  (this will not affect the stationary distribution).
- If the node decides to move, it selects its next cell from its list of neighboring cells with probability  $\text{Prob}(\text{choosing cell } i) = p_i / \sum_i p_i$

To be clear, by taxicab distance we mean the metric given by  $d((x_1, y_1), (x_2, y_2)) = |x_1 - x_2| + |y_1 - y_2|$ .

### C. Justification

It is common sense that humans and many animals have homes or dens that they return to periodically. If we are modeling foot or bicycle traffic, it will take more time and energy to travel further from home. At any time a traveler is more likely to turn back home than to head further away. Other scenarios also have this property in the short-term. For example a tourist may walk around a city and then return to their hotel, or a commuter may take a train or bus to a station, move locally in the vicinity of the station, and then have to return to the station for the trip back. This argument applies to automobile traffic to some extent also. Since traveling further from home usually requires more time and fuel, travelers will be more likely to limit themselves to nearby destinations.

## III. ANALYSIS OF LRAW

### A. Spatial Distribution

We represent the LRAW process as a discrete time Markov model [2] with home cell corresponding to state 0 (see fig 1). Let  $s \in (0, 1]$  be the Bernoulli probability of moving on any given step. The model is symmetric, so the probabilities of going left or right out of state 0 are both  $\frac{s}{2}$ . Let  $s\beta$  be the probability of moving one step further away from state 0, so then  $s(1 - \beta)$  is the probability of moving one step closer to state 0, for  $\beta \in [0, \frac{1}{2})$ .

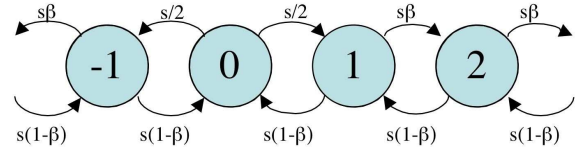


Fig. 1. Part of the Markov model for a 1D localized random walk. The transitions from the states to themselves have probability  $(1 - s)$  and have been omitted from the drawing.

This Markov model is represented by the matrix

$$A = \begin{pmatrix} \ddots & & & & & & \\ & 1-s & s\beta & 0 & 0 & 0 & \\ & s(1-\beta) & 1-s & \frac{s}{2} & 0 & 0 & \\ \dots & 0 & s(1-\beta) & 1-s & s(1-\beta) & 0 & \dots \\ & 0 & 0 & \frac{s}{2} & 1-s & s(1-\beta) & \\ & 0 & 0 & 0 & s\beta & 1-s & \\ & & & & & & \ddots \end{pmatrix} \quad (1)$$

This model is ergodic and will have a steady state for  $1 > s > 0$  and  $0 \leq \beta < \frac{1}{2}$ . The reader can check that the following is a solution to  $Ax = x$ .

$$x_0 = \frac{1 - 2\beta}{2 - 2\beta} \quad (2)$$

$$x_{i \neq 0} = \left( \frac{\beta}{1 - \beta} \right)^{|i|} \left( \frac{1 - 2\beta}{4\beta(1 - \beta)} \right) \quad (3)$$

Note that  $s$  does not appear in the solution. This is very close to a double exponential distribution. For  $i \neq 0$ :

$$\ln(x_i) = -|i| \ln\left(\frac{1 - \beta}{\beta}\right) + \ln\left(\frac{1 - 2\beta}{4\beta(1 - \beta)}\right) \quad (4)$$

which has the same form as

$$\ln\left(\frac{1}{2\tau} e^{-|x|/\tau}\right) = -|x| \frac{1}{\tau} + \ln\left(\frac{1}{2\tau}\right) \quad (5)$$

if we let  $\tau = 1 / \ln\left(\frac{1 - \beta}{\beta}\right)$ .

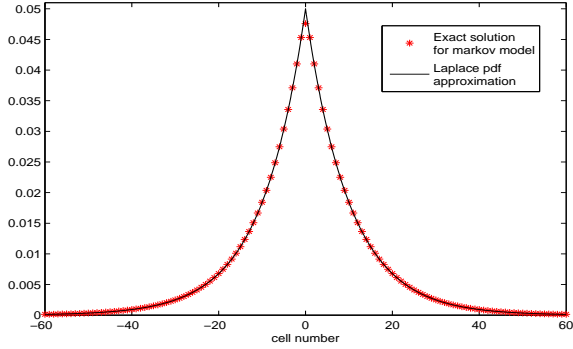


Fig. 2. The continuous double exponential (Laplace) pdf and the exact solution to the 1D LRAW Markov model for tightness parameter  $\tau = 10$ .

The fact that  $x_0$  does not quite conform to the double-exponential throws off the normalization of all the terms slightly. For  $\beta > 0.25$  the discrepancy very small, and we can approximate the spatial distribution of the 1D LRAW mobility model as a double-exponential.

$$x_i := \text{Prob}(\text{node in cell } i) \approx \frac{1}{2\tau} e^{-|i|/\tau} \quad (6)$$

1) *Generalizing to 2D:* As long as we use the taxicab distance in the mobility model, the 2D cell-occupancy probabilities are just the products of 1D cell-occupancy probabilities.

$$\text{Prob}(\text{node in cell } i, j) = x_i x_j \quad (7)$$

$$= \frac{1}{4\tau^2} e^{-(|i|+|j|)/\tau} \quad (8)$$

### B. LRAW-Uniform interactions

When several nodes obeying the LRAW mobility model share the same home cell and tightness parameter we will refer to them as an “**LRAW cloud**”. An LRAW cloud could be used to model a group of humans or animals that live in the same area, or a platoon of soldiers operating in the vicinity of some home base. By **uniform mobility** we mean the mobility model where a node may appear at any location with uniform probability, independent of where it was last seen. This could be seen as an approximation to other mobility models with uniform spatial distributions such as random direction or random waypoint on a torus.

Picture the scenario in which we have a cloud of LRAW nodes, all with the same home cell and tightness parameter, and some collection of more highly mobile **carrier nodes** with uniform mobility. In this scenario some fraction,  $\alpha \in (0, 1]$ , of the nodes in the cloud

carry a copy of a message destined to a node in some other cloud (distant enough that the probability of direct delivery is effectively zero). The question is: given  $N$  nodes in the LRAW cloud,  $N\alpha$  of which have a copy of the message, and  $M$  carrier nodes, at what rate can we expect carriers to pick up copies from the source cloud?

In this scenario the probability of an encounter with a carrier does not increase linearly with the number of nodes in the LRAW cloud. To quantify this non-linearity we compute what we call the **spatial cross-section**,  $\sigma(N)$ , of the LRAW cloud. This quantity is the expected number of cells occupied by a cloud of  $N$  nodes. We start the analysis in one dimension.

For a cloud of  $N$  nodes with home cell at  $i = 0$ , let  $p(N, i) = \text{Prob}(\text{cell } i \text{ is occupied})$ . Then

$$p(N, i) = 1 - \text{Prob}(i \text{ is unoccupied}) \quad (9)$$

$$= 1 - (1 - p(1, i))^N \quad (10)$$

$$= 1 - \left(1 - \frac{1}{2\tau} e^{-|i|/\tau}\right)^N \quad (11)$$

since  $p(1, i) = \frac{1}{2\tau} e^{-|i|/\tau}$

Given an LRAW cloud that contains  $N$  nodes, we can now view the probability that that cell,  $i$ , is non-empty as a Bernoulli random variable,  $X_i$ , with success probability  $p(N, i)$ . We are interested in the total number of non-empty cells, so we define a new random variable  $Y = \sum X_i$ . The spatial cross section is the expectation of  $Y$ .

$$\sigma(N) := E[Y] \quad (12)$$

$$= E\left[\sum_{i=-\infty}^{\infty} X_i\right] \quad (13)$$

$$= \sum_{i=-\infty}^{\infty} E[X_i] \quad (14)$$

$$= \sum_{i=-\infty}^{\infty} 1 - \left(1 - \frac{1}{2\tau} e^{-|i|/\tau}\right)^N \quad (15)$$

We approximate this sum with an integral.

$$\sigma(N) \approx 2 \int_0^{\infty} 1 - \left(1 - \frac{1}{2\tau} e^{-x/\tau}\right)^N dx \quad (16)$$

$$= 2\tau \left(\sum_{i=1}^N \frac{1}{i} - \sum_{i=1}^N \frac{1}{i} k^i\right) \quad (17)$$

where  $k = (1 - \frac{1}{2\tau})$ . This give us our best expression for the spatial cross section of an  $N$ -node cloud.

$$\sigma(N) \approx 2\tau \sum_{i=1}^N \frac{1}{i} (1 - k^i) \quad (18)$$

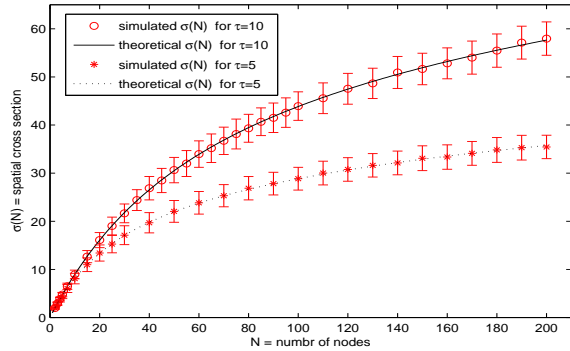


Fig. 3. Theoretical and simulation cross-section data for  $\tau = 5$  and  $\tau = 10$  in 1D. Experimental data points are averaged over runs of 100,000 steps. Error bars are one standard deviation.

Note that if  $k < 1$  then  $\sum_{i=1}^N k^i/i$  is bounded by a constant and therefore  $\sigma(N) = 2\tau(H_N - O(1))$  where  $H_N$  is the  $N$ th harmonic number. This means that  $\sigma(N) = O(\ln(N))$  [5].

1) *Generalizing to 2D*: The same basic u-substitution used in 1D can be applied twice to derive

$$\sigma_{2D}(N) \approx 4\tau^2 \sum_{i=1}^N \sum_{j=1}^i \frac{1 - k^j}{ij} \quad (19)$$

where now  $k = (1 - \frac{1}{4\tau^2})$ .

### C. LRAW-LRAW interactions

Suppose we have two LRAW nodes with home cells  $c_1 = (x_1, y_1)$  and  $c_2 = (x_2, y_2)$ ,  $x_1 \leq x_2$ ,  $y_1 \leq y_2$ , and common tightness parameter  $\tau$ . We would like to approximate their encounter rate.

We will approximate the meeting probability by integrating the joint 2D pdf of the two nodes.

$$f_{joint}(x, y|c_1, c_2, \tau) = f(x, y|c_1, \tau)f(x, y|c_2, \tau) \quad (20)$$

$$= \frac{1}{16\tau^4} e^{-(d_{c_1}(x, y) + d_{c_2}(x, y))/\tau} \quad (21)$$

where  $d_{c_i}(x, y) = |x - x_i| + |y - y_i|$ .

There are three types of regions to integrate.

- The box region  $(x, y) \in [x_1, x_2] \times [y_1, y_2]$  where the joint pdf has constant value  $\frac{1}{16\tau^4} e^{-(|x_2 - x_1| + |y_2 - y_1|)/\tau}$
- The fringes along the edges of the box region  $(x, y) \in ((-\infty, x_1] \cup [x_2, \infty)) \times [y_1, y_2]$  and  $(x, y) \in [x_1, x_2] \times ((-\infty, y_1] \cup [y_2, \infty))$
- The corners  $(x, y) \in ((-\infty, x_1] \cup [x_2, \infty)) \times ((-\infty, y_1] \cup [y_2, \infty))$

The final integral gives the probability of the two nodes meeting at any given time:

$$Prob(meeting|d_x, d_y, \tau) = \quad (22)$$

$$\frac{1}{16\tau^2} \left( \frac{d_x d_y}{\tau^2} + \frac{d_x + d_y}{\tau} + 1 \right) e^{-(d_x + d_y)/\tau} \quad (23)$$

where  $d_x = |x_2 - x_1|$  and  $d_y = |y_2 - y_1|$ .

For two LRAW nodes with tightness parameter  $\tau$  and home cells  $c_1$  and  $c_2$  that are taxicab distance  $d_x, d_y$  apart, we will call the value we have just computed the **pair-encounter rate**.

$$R_{1,2} = R_{2,1} = Prob(meeting|d_x, d_y, \tau) \quad (24)$$

## IV. EXAMPLES AND RESULTS

In this section we model some simple DTN scenarios and compare to simulation results. The metric we focus on is message latency. In each case we look at the properties under epidemic flooding [19], [21].

### A. Transfer within a single LRAW cloud

Based on the analysis in section III-C we can estimate the encounter rate of nodes in the same cloud by setting  $d_x = d_y = 0$ . We omit any subscripts on the pair-encounter rate since we have only one LRAW cloud.

$$R = \frac{1}{16\tau^2} \quad (25)$$

As a first approximation we can assume that the encounter pattern is uniform. That is, at any time each node is equally likely to encounter any other node. Note that this is not quite the same as assuming a uniform mobility model (where the probability of appearing in any location at any time is uniform) since the encounter rate is based on the LRAW mobility model.

Suppose a single node generates a message at time  $t = 0$ . Let  $c(t)$  denote the number of copies in circulation at time  $t$ . Since  $c$  is limited by the number of nodes,  $N$ , this process is modeled by the differential equation

$$\frac{dc}{dt} = NRc \left( 1 - \frac{c}{N} \right) \quad (26)$$

Here the factor  $(1 - c/N)$  can be thought of as the probability that a message carrier encounters a non-carrier. This is solved by a logistic function, given  $c(0) = 1$ .

$$c(t) = \frac{Ne^{NRt}}{N + (e^{NRt} - 1)} \quad (27)$$

1) *Better modeling with a modified logistic:* In reality LRAW (and random walks in general) produce a non-uniform encounter pattern. At any given time a message carrier is more likely to be near other message carriers, and therefore less likely to encounter a non-carrier than the factor  $(1 - c/N)$  in equation 26 would predict. We need some way of approximating this nonlinearity.

Our approach is based largely on empirical observations. We have observed that the  $c(t)$  curve produced in a single LRAW cloud looks like a standard logistic function with the  $t$ -axis rescaled in a continuous way. If this is the case then  $c(t)$  can be written as

$$c(t) = \frac{NAe^{NRm(t)}}{N + A(e^{NRm(t)} - 1)} \quad (28)$$

for some continuous function  $m(t)$ . Note that this is the solution to the differential equation

$$\frac{dc}{dt} = NRc(t)m'(t) \left(1 - \frac{c(t)}{N}\right) \quad (29)$$

for initial condition  $c(0) = A$ , and that it allows us to solve for  $m(t)$  as a function of  $c(t)$ .

$$m(t) = \frac{1}{NR} \ln \left( \frac{c(N-A)}{A(N-c)} \right) \quad (30)$$

This last equation expresses  $m(t)$  as a function of  $c(t)$ . We determined that using  $m(t) = 2 \ln(NRt + 1)/NR$  provides a good fit for a wide range of  $N$  and  $\tau$  values. This amounts to replacing the factor  $(1 - c/N)$  with  $2(1 - c/N)/(NRt + 1)$  in eqn 26. The resulting differential equation can be solved, and the solution inverted to get  $t$  as a function of  $c$ . Figure 4 illustrates the ability of the modified logistic to model the epidemic spread of messages in an LRAW cloud.

$$\frac{dc}{dt} = \frac{2NRc}{NRt + 1} (1 - c/N) \quad (31)$$

$$c(t) = N \frac{(NRt + 1)^2}{N + (NRt + 1)^2 - 1} \quad (32)$$

$$t(c) = \frac{1}{NR} \left( \sqrt{\frac{c(N-1)}{N-c}} - 1 \right) \quad (33)$$

We can use this improved  $c(t)$  function to estimate the mean and median latency. Note that at any time,  $t$ ,  $c(t)/N$  is exactly the probability that any particular node is a message carrier. Therefore, supposing the source and destination reside in an LRAW cloud of  $N$  nodes, we can view  $c(t)/N$  as the cumulative distribution function (CDF) of the message delivery latency.

$$Prob(latency \leq t) = c(t)/N \quad (34)$$

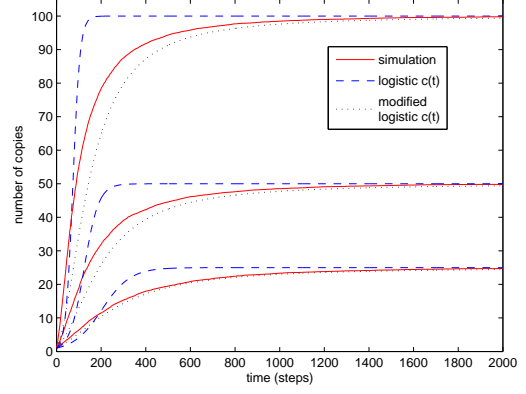


Fig. 4. The plain and modified logistic  $c(t)$  compared to simulation data. The three sets of curves are for  $N = 100$ ,  $N = 50$ , and  $N = 25$ , and  $\tau = 10.0$ .

Therefore the median latency will be approximated by

$$median\ latency = c^{-1} \left( \frac{N}{2} \right) = \frac{\sqrt{N-1} - 1}{NR} \quad (35)$$

Mean latency will generally be larger than the median latency because of cases where the source or the destination is out on the fringes of the cloud. Since we are viewing  $c(t)/N$  as the CDF of the latency distribution, the mean will be computed as

$$expected\ latency = \int_0^\infty t \frac{c'(t)}{N} dt \quad (36)$$

This mean does exist for the  $c(t)$  in equation 32 but it is very complex. If we make an approximation to  $c(t)$

$$c(t) \approx N \frac{(NRt)^2}{N + (NRt)^2} \quad (37)$$

we obtain a much more useful approximation to mean latency.

$$mean\ latency \approx \frac{\pi}{2R\sqrt{N}} \quad (38)$$

Compare this to the median (and mean) latency of a plain logistic:

$$c_{logistic}^{-1} \left( \frac{N}{2} \right) = \frac{1}{NR} \ln(N-1) \quad (39)$$

Figure 5 shows that our estimate of mean latency is reasonable for a range of values for  $N$ , though it becomes a poorer for small numbers of nodes.

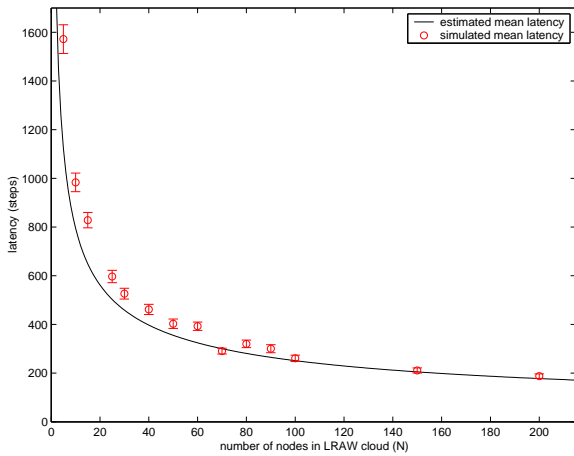


Fig. 5. Simulated mean latency data compared to mean latency predicted by eqn 38 for  $\tau = 10.0$  and varying values of  $N$ . Data points are averaged over 500 runs.

### B. Clouds and carriers

We will use results from section III-B to model the scenario where a message is transferred from one LRAW cloud to another via highly mobile *carrier nodes*. This could be modeling a scenario where a platoon of soldiers is trying to send data to another group, and the data is relayed by arial vehicles or unreliable satellite uplinks.

We suppose that there are two LRAW clouds, each with  $N$  nodes and tightness parameter  $\tau$ , centered about home cells far enough apart that the probability of direct delivery is effectively zero.

We also assume there are  $M$  carrier nodes in the  $L \times L$  simulation grid, and that  $M \ll L^2$ . We model the carrier node mobility with the uniform model.

We model this scenario as the concatenation of three phases:

- **1** The saturation of the source cloud with copies of the message.
- **2** The pickup/dropoff process. For an individual carrier node this process consists of picking up a copy of the message from the source cloud, then delivering a copy to the destination cloud.
- **3** The delivery process in the destination cloud.

The separation of the problem into consecutive phases is justified under our assumptions. A carrier node is much less likely to pick up a message copy when only a few nodes in the source cloud have copies. Therefore phase 2 will rarely commence until phase 1 is well underway. Similarly once a message copy is dropped off in the destination cloud it will spread rapidly through the cloud. Any additional drop-offs will have little effect on the time it takes for the message to reach its destination.

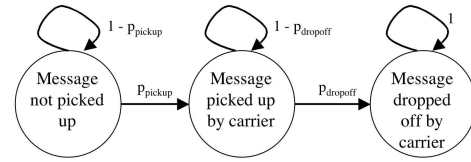


Fig. 6. Markov model for the Clouds & Carriers process.

The latency for the clouds and carriers scenario is a random variable,  $T_{c\&c}$ , which we estimate as the sum of the latencies of the three phases:

$$T_{c\&c} = T_1 + T_2 + T_3 \quad (40)$$

Since  $M \ll L^2$  we may assume that  $T_1$  and  $T_3$  are independent of the number of carriers. Recall that the median delivery latency in an LRAW cloud is  $c^{-1}(N/2)$ . We estimate the time for phase one (effective saturation of the source cloud) as twice the median latency, and phase three as the median delivery latency for a single LRAW cloud.

$$T_1 + T_3 \approx 3 \frac{1}{NR} \left( \sqrt{N-1} - 1 \right) \quad (41)$$

where  $R$  is the expected intra-cloud pair encounter rate used in section III-C.

The slowest part of this process is the pickup/dropoff phase,  $T_2$ , which we examine in more detail. For a single carrier node, the pickup/dropoff process can be modeled by the three-state Markov model in figure 6. The first state corresponds to the carrier node not having a copy of the message. Once the source cloud is more or less completely saturated with copies, the probability of the carrier node picking up a copy on any step is given by  $p_{pickup} = \sigma(N)/L^2$ , where  $\sigma(N)$  is the spatial cross section of the source cloud, derived in section III-B.

The second state corresponds to the carrier node having a copy of the message, but not having delivered it to the destination cloud. The probability of the message-laden carrier encountering the destination cloud on any step is the same as the pickup probability,  $p_{dropoff} = p_{pickup} = \sigma(N)/L^2$ . The last state corresponds to the message having been delivered to the destination cloud, and the end of phase 2.

This is a truncated pure birth process [2], and we can use transient state analysis to derive the time-dependent state probability,  $\pi_i(t)$ , for each state  $i$ . Let  $\lambda = p_{pickup} = p_{dropoff}$

$$\pi_1(t) = e^{-\lambda t} \quad (42)$$

$$\pi_2(t) = \lambda t e^{-\lambda t} \quad (43)$$

$$\pi_3(t) = 1 - e^{-\lambda t} (1 + \lambda t) \quad (44)$$

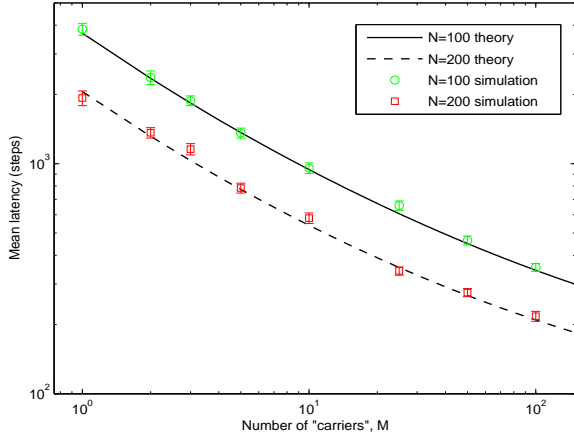


Fig. 7. Predicted vs experimental latency for the Clouds and Carriers scenario. The simulation results are averaged over 100 independent runs.

$\pi_3(t)$  is the CDF of the random variable  $T_2$ . We can find its distribution by differentiating w/r/t  $t$ :

$$\begin{aligned} \text{Prob}(T_2 = t|\lambda, M = 1) &= \frac{d}{dt}(1 - e^{-\lambda t}(1 + \lambda t)) \quad (45) \\ &= \lambda^2 t e^{-\lambda t} \quad (46) \end{aligned}$$

This analysis was for a single carrier node,  $M = 1$ . In the case where  $M > 1$  we are interested in the time of the first delivery. Since we have the pdf of  $T_{2,M=1}$  we can use the order statistic transform [4] to compute the general pdf.

$$\text{Prob}(T_2 = t|\lambda, M) = M\lambda^2 t(1 + \lambda t)^{M-1} e^{-M\lambda t} \quad (47)$$

We evaluate the expectation numerically and compare to simulation results. Figure 7 shows  $E[T_{C\&C}]$  for varying cloud sizes,  $N$ , and numbers of carriers,  $M$ . The simulation was done with our **GridSim** simulator on a toroidal 400x400 grid, with LRAW clouds with tightness parameter  $\tau = 5.0$  centered on coordinates (100,100) and (300,300).

## V. CONCLUSIONS AND FUTURE WORK

In this paper we have introduced the Localized Random Walk mobility model, derived some of its properties, used it to construct models of some simple DTN scenarios, and checked the predictions against simulation results. We believe that developing mathematical models for Delay-Tolerant Networks will help us understand the fundamental properties governing their behavior. This, in turn, will lead to the development of more robust DTN protocols that are applicable to a wider range of scenarios.

Some other extensions of this work that we hope to explore include:

- Other localized models.
- Comparison to real-world mobility data.
- Independent home cells.

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