Problem Set #3 Solutions

ENEE 426, Spring 2009

Complete the following problems:

1. (3 points) People arrive at an amusement park at a rate of 10 people per minute. The average customer spends 5 hours at the park. What is the average number of people in the amusement park?

   $\lambda = 10$, $T = 5 * 60 = 300$
   $N = \lambda * T = 10 * 300 = 3000$

2. (3 points) Customers arrive at a fast-food restaurant at a rate of five per minute and wait to receive their orders for an average of 5 minutes. Customers eat in the restaurant with probability 0.5 and carry out their order without eating with probability 0.5. A meal requires an average of 20 minutes. What is the average number of customers in the restaurant?

   Half eat in and half eat out, so let $\lambda 1$ reflect the arrival rate of those eating out, and $\lambda 2$ be those eating in.

   $\lambda 1 = 5 * 0.5 = 2.5 = \lambda 2$
   $N = \lambda 1 * T1 + \lambda 2 * T2 = 2.5 (5 + (5+20)) = 75$

3. (8 points) Consider a supermarket check-out line where arrivals and departures have a Poisson distribution. Customers get in line at a rate of 1 customer per 3 minute. It takes an average of 2 minutes to scan groceries scanned and for customers to pay.
   a. How long can a customer expect to wait in line?

      $\lambda = 1/3$, $\mu = 1/2$, $\rho = \lambda/\mu = 2/3$
      $W = \rho/(\mu-\lambda) = (2/3)/(1/6) = 4$

   b. How long can a customer expect to wait both in line, having their items scanned, and paying?

      $T = 1/(\mu-\lambda) = 1/(1/6) = 6$

   c. On average how long is the line?

      $NQ = \rho^2/(1-\rho) = (4/9)/(1/3) = 4/3$
      (Given ambiguity, also accept $N = \rho/(1-\rho) = (2/3)/(1/3) = 2$)

4. (6 points) People arrive at a taxi stand at a rate of 2 per minute. Taxis arrive at a rate of 1 per minute. The inter-arrival and inter-departure times are exponentially distributed. The taxi stand can hold at most 5 taxis.
    a. Compute the probability that at any given time, there are no taxis waiting.

       M/M/1/5 queueing system for the taxis

       $\lambda = 1$, $\mu = 2$, $\rho = 1/2$

       Pn = ρ^n (1-ρ) / (1-ρ^(m+1)) = 2^(5-n) / 63

       P0 = 32/63

    b. Compute the probability that at any given time exactly 5 taxis are waiting.

       P5 = 1/63

5. (10 points) A switch has 4 interfaces each running at 10Mbps, connected to 4 computers C1, C2, C3, and C4. Packet lengths are exponentially distributed with an average length of 1000 bytes. The network is currently supporting 7 traffic flows as follows. C1 transmitting to C2 at a rate of 4 Mbps. C1 is transmitting to C4 at a rate of 2 Mbps. C2 is transmitting to C3 at a rate of 6 Mbps. C3 is transmitting to C2 at a rate of 2 Mbps. C3 is transmitting to C4 at a rate of 1 Mbps. C4 is transmitting to C1 at a rate of 5 Mbps. C4 is transmitting to C2 at a rate of 1 Mbps.

   The switch has four queues, one on each output port of the switch. Packets arrive to an interface queue at the aggregate rate for all streams whose destination connected to the interface. Thus Q1 (connected to C1) receives 5 Mbps, Q2 receives 7 Mbps, Q3 receives 6 Mbps, and Q4 receives 3 Mbps. The associated packet arrival rates can be computed by dividing the flow rates by the average packet size. Thus $\lambda 1$=5/8 kpps, $\lambda 2$= 7/8 kpps, $\lambda 3$=6/8 kpps, and $\lambda 4$=3/8 kpps. The service rate μ=10/8 kpps, as each is a 10 Mbps output link.

    a. What is the average number of packets queued or being transmitted by the switch at any given time?

       N1 = $\lambda 1 / (\mu - \lambda 1) = (5/8) / (10/8 - 5/8) = 1$

       N2 = $\lambda 2 / (\mu - \lambda 2) = (7/8) / (10/8 - 7/8) = 7/3$

       N3 = $\lambda 3 / (\mu - \lambda 3) = (6/8) / (10/8 - 6/8) = 3/2$

       N4 = $\lambda 4 / (\mu - \lambda 4) = (3/8) / (10/8 - 3/8) = 3/7$

       N = N1+N2+N3+N4 = 5.27 packets

b. What is the average latency for each of the flows, assuming negligible processing and propagation delay?

Latency = Propagation + Transmission + Processing + Queueing
Processing = Propagation = 0
Latency = Transmission + Queueing
Transmission = Service Time
Latency = Queueing + Service = T

$T = (T1+T2+T3+T4) / 4$
$= (N1/\lambda 1 + N2/\lambda 2 + N3/\lambda 3 + N4/\lambda 4) / 4$
$= (1/(5/8) + (7/3)/(7/8) + (3/2)/(6/8) + (3/7)/(3/8)) / 4$
$= (8/5 + 8/3 + 8/4 + 8/7) /4$
$= 2 (1/5 + 1/3 + 1/4 + 1/7)$
$= 1.85$

Original units kpps = kp/sec, so 1/kpps = sec/kp = ms/packet, thus 1.85 ms