Problem Set 1 Math Review for Machine Learning CMSC 422, Fall 2017 Assigned, Wednesday, August 30. Due Wednesday, Sept. 6.

Please show your work for all problems.

- 1. Calculus: Consider the function $f(x) = (x-7)^2$.
 - a. Using intuition, determine x_{min} , the value of \boldsymbol{x} that minimizes this function.
 - b. What is the derivative of f?
 - c. What is the value of the derivative at $x = x_{min}$, which you came up with for (a).
 - d. What important fact from calculus explains the connection between the derivative of f and its minimum value?
- 2. Partial Derivatives: Consider the function $f(x,y) = (x-7)^2 + (y-3)^2$.
 - a. Using your intuition, determine the point (x_{min}, y_{min}) at which this function has a minimum value.
 - b. What is the partial derivative of f with respect to x?
 - c. What is the partial derivative of f with respect to y?
 - d. What is the gradient of f at the point (x_{\min}, y_{\min}) ? Use the values of (x_{\min}, y_{\min}) that you gave as your answer to (a).
 - e. Explain in about one sentence what you think will be the general connection between the gradient of a function and its minimum value.
- 3. Vectors and Inner Products
 - a. Scale the vector (3,4) so that it is a unit vector. The resulting vector should point in the same direction as (3,4) but have a Euclidean norm of 1.
 - b. Suppose you are beginning at the point (0,0). You will travel a distance, *d*, in the direction of the vector (3,4). Then you will turn 90 degrees to the right, and travel in a straight line, to get to the point (10, 5). What is *d*? (See figure at bottom).
- 4. Probability
 - a. Consider the following joint probability table over variables y and z, where y takes a value from the set $y \in \{a,b,c\}$ and $z \in \{0,1\}$.

	y = a	y = b	y = c
z = 1	0.2	0.1	0.2
z = 0	0.05	0.15	0.3

What is the value of p(z=1|y=b)?

- b. True or false: P(A or B) = P(A) + P(B) P(A and B). Note P(A or B) indicates the probability that either A is true, B is true, or both are true. P(A and B) indicates the probability that both are true.
- c. True or false: P(A|B) = P(B|A)?

