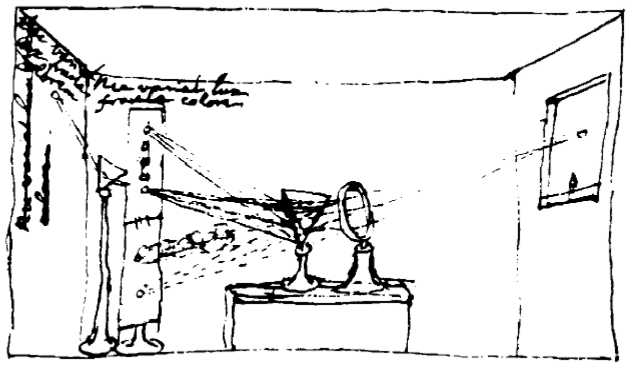
What is color for?

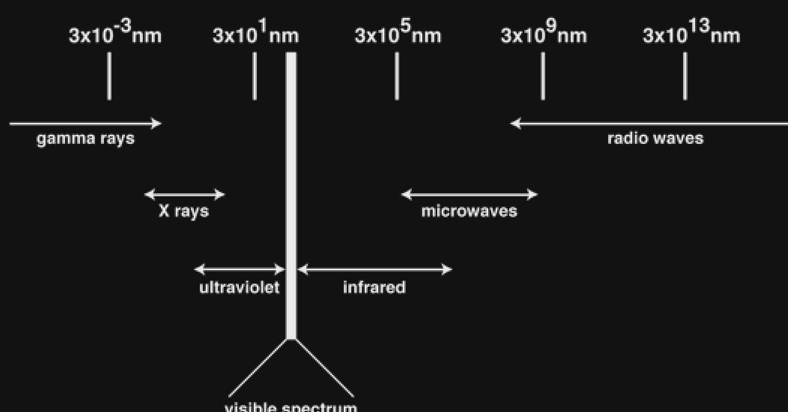
Color

- Physics
  - Light is E-M radiation of different frequencies.
  - Superposition principle
- Perception
  - 3 cones -> 3D color space. (Metamers).
  - Convex subset of 3D linear space.
- Color Spaces
  - RGB – standard representation, Monitors, OpenGL
  - HSV – More intuitive
- More Perception
  - Constancy
- Refs: H&B Chapter 12; “The Foundations of Color Measurement and Color Perception”, by Brian Wandell:

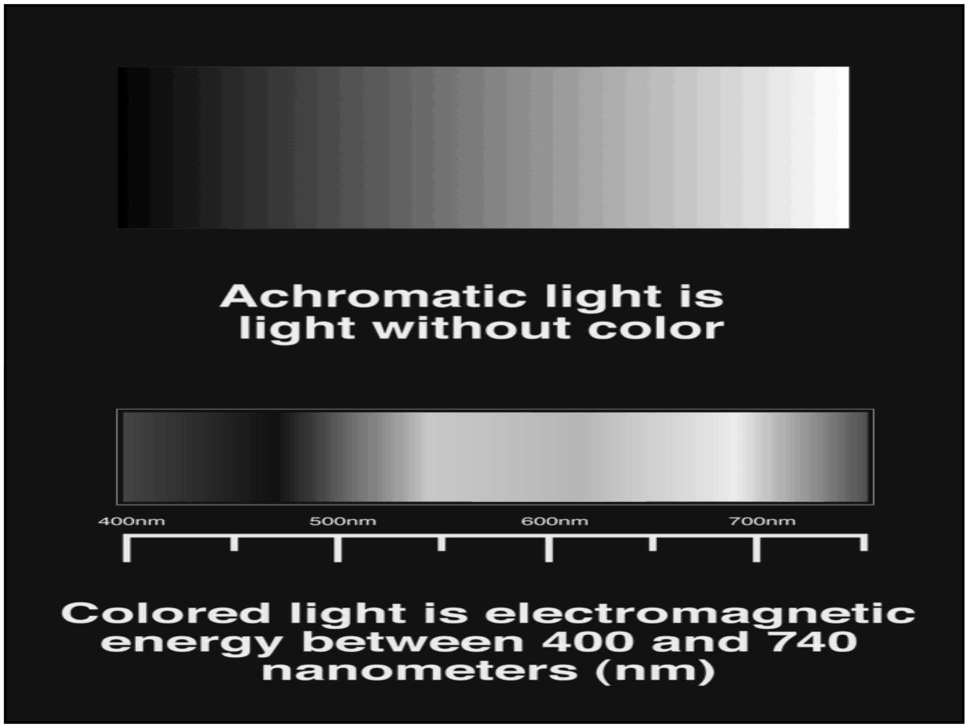
Newton’s drawing:



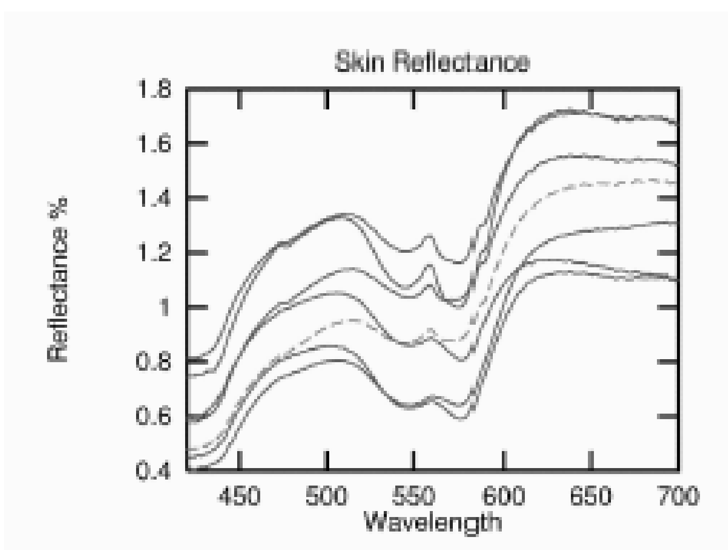
(Wandell)



(Varshney)



Color is a function



(Angelopoulou)

Superposition

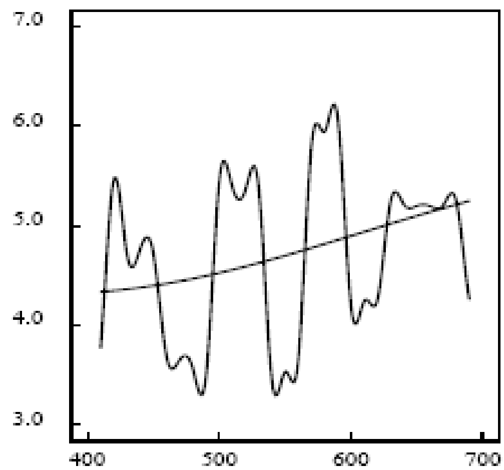
- Light is linear.
- Light from source A + light from source B = Light from sources A & B.
  - Any color is a combination of pure colors.
- Doubling intensity of source doubles amount of light reaching us.

Human Color Perception

- *Cones* allow color perception.
- 3 types of cones sensitive to different frequencies
- Perceptual color depends on how these are stimulated.

tristimulus theory

Metamers



(Wandell)

Perceptual color space

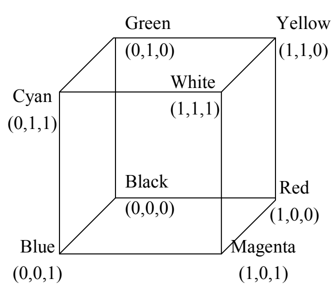
- 3D
- Convex subspace
  - Cones don’t have negative response
- In general, any three colors projected onto this space span it.
  - But not with non-negative coefficients.
  - So not all colors can be produced by (positive) combinations of RGB

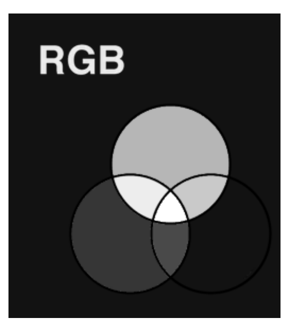
Grassman's Additivity Law

- Color matching follows superposition
- If we know how to produce all pure colors, we can produce any color.

Additive Color Model RGB

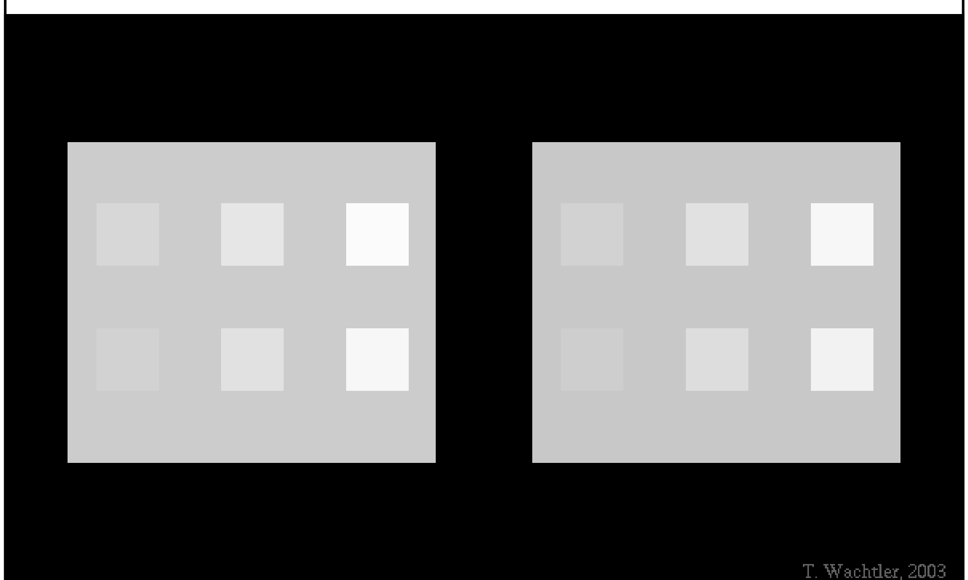
- Mix Red, Green, Blue primaries to get colors
- Cartesian Coordinate System with origin as black.
- Used in display devices: CRTs, LCDs.





Color Vocabulary

- Hue: Distinguishes among colors
  - red, yellow, blue
- Saturation: Color *Purity* (difference from white)
  - blue and sky blue
- Value: overall intensity of light.
- Lightness: Perceived intensity of reflected light
  - blue and darker shades of blue
- Brightness: Perceived intensity of self-luminous objects
- Artists:
  - Tint: Add white (decrease saturation)
  - Shade: Add black (decrease lightness)



http://neuro.physik.uni-marburg.de/~wachtler/cc.html

Color Constancy

Our color vision is based on signals of the three photoreceptor types which respond to light of three different wavelength regions. But if we look at an object, its color that we perceive is not only determined by the spectral composition of the light coming from the object. Object colors depend on the context in which the object is seen.

Look at the image below. On the left are six color fields on a grey field, representing six objects on a background. On the right, esentially the same arrrangement is shown, but all colors have a slightly bluish tint. It is as if we see the same scene under a bluish illumination. Incidentally, the spectral composition of the light coming from the three fields in **the upper row on the right side** are **exactly the same** as those of **the lower row on the left side**. Furthermore, the colors on the right that match most closely those on the left are the ones in the corresponding positions of the scenes, not those with the same physical spectrum:

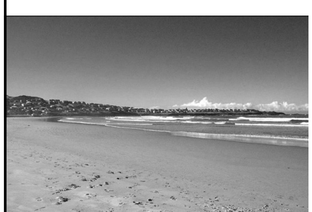
Color Constancy

- A red object will produce a wide variety of RGB values, depending on the light.
- Separate color of materials from color of light.
  - Possible if we assume some distribution of colors in the scene.
- Interesting algorithms exist
  - Mostly for somewhat controlled/idealized conditions
  - Useful in applications, but not so much in natural images
  - This makes it hard to use color in recognition.
  - Segmentation can be ok as long as lighting is locally constant.

Color Quantization

Compression by using a small set of colors.

- Represent each color with one of these.
- So we need to pick a small number of colors so that all the colors in the image are “close” to them in some way.
- Cost function of k-means is natural.
  - Spatial position is irrelevant.



Full Color

64

16

8

4

K-means clustering

- Brute force difficult because many spheres, many pixels.
- Assume all spheres same radius; just need sphere centers.
- Iterative method.
  - If we knew centers, it would be easy to assign pixels to clusters.
  - If we knew which pixels in each cluster, it would be easy to find centers.
  - So guess centers, assign pixels to clusters, pick centers for clusters, assign pixels to clusters, ….

Why is this better?

- With a greedy algorithm, once we make a decision we cannot undo it.
- With an iterative algorithm, we can make changes.

K-means Algorithm

1. Initialize – Pick k random cluster centers
   – Pick centers *near* data. Heuristics: uniform distribution in range of data; randomly select data points.
2. Assign each point to nearest center.
3. Make each center average of pts assigned to it.
4. Go to step 2.

Let's consider a simple example. Suppose we want to cluster black and white intensities, and we have the intensities: 1 3 8 11. Suppose we start with centers c1 = 7 and c2=10. We assign 1, 3, 8 to c1, 11 to c2. Then we update c1 = (1+3+8)/3 = 4, c2 = 11. Then we assign 1,3 to c1 and 8 and 11 to c2. Then we update c1 = 2, c2 = 9 ½. Then the algorithm has converged. No assignments change, so the centers don't change.

K-means Properties

- We can think of this as trying to find the optimal solution to:
  - Given points p1… pn, find centers c1…ck
  - and find mapping f:{p1…pn}->{c1…ck}
  - that minimizes C = (p1-f(p1))^2 + …+ (pn-f(pn))^2.
- Every step reduces C.
  - The mean is the pt that minimizes sum of squared distance to a set of points. So changing the center to be the mean reduces this distance.
  - When we reassign a point to a closer center, we reduce its distance to its cluster center.
- Convergence: since there are only a finite set of possible assignments.

Local Minima

- However, algorithm might not find the best possible assignments and centers.
- Consider points 0, 20, 32.
  - K-means can converge to centers at 10, 32.
  - Or to centers at 0, 26.
- Heuristic solutions
  - Start with many random starting points and pick the best solution.

Histogram Tesselation

- We can think of clustering as another way to divide a histogram into bins.
- Each cluster is a bin.
- Bins are adapted to the data
  - The width of bins is as small as possible.

E-M

- Like K-means with soft assignment.
  - Assign point partly to all clusters based on probability it belongs to each.
  - Compute weighted averages (c_j) and variance (σ).

$$f_j(p_i) = \frac{e^{-\|p_i - c_j\|^2 / \sigma^2}}{\sum_j e^{-\|p_i - c_j\|^2 / \sigma^2}}$$

Cluster centers are c_j.

Also Useful for Image Segmentations

Two clusters in color space.