Boundary Detection - Edges

• Edge Detection is a local (for now) decision about whether there is a boundary in an image.
• Boundaries of objects
  – Usually different materials/orientations, intensity changes.
We also get:
Boundaries of surfaces

Boundaries of materials
properties
Boundaries of lighting

Edge is Where Change Occurs

• Change is measured by derivative in 1D
• Biggest change, derivative has maximum magnitude
• Or 2\textsuperscript{nd} derivative is zero.
Noisy Step Edge

- Derivative is high everywhere.
- Must smooth before taking derivative.

Implementing 1D Edge Detection

1. Filter out noise: correlate with Gaussian
2. Take a derivative: correlate with [-0.5 0 0.5]
   - We can combine 1 and 2.
How do we take the first derivative with a convolution? Let $y = f(x)$. Recall that $y' = \lim_{dx \to 0} \frac{f(x+dx) - f(x)}{dx}$. In a discrete image, the smallest we can make $dx$ is 1 pixel, so we can take $f(x+1)-f(x)$, which is correlation with a filter of $[-1 1 0]$. This is asymmetric, we could just as easily use a filter like $[0 -1 1]$. And, it’s also reasonable to say: $y' = \lim_{dx \to 0} \frac{f(x+dx) - f(x-dx)}{2dx}$, which leads to a filter of $[-.5 0 .5]$. In the limit, these are the same, but before that they are different.

Implementing 1D Edge Detection

3. Find the peak: Two issues:
   - Should be a local maximum.
   - Should be sufficiently high.
Scale

- Smoothing more removes small scale structures.
- Varying smoothing varies the scale of edges we locate.
- *Matlab*

2D Edge Detection: Canny

1. Filter out noise
   - Use a 2D Gaussian Filter. \( J = G \circ I \)

2. Take a derivative

   \[
   \nabla J = (J_x, J_y) = \left( \frac{\partial J}{\partial x}, \frac{\partial J}{\partial y} \right)
   \]

   is the Gradient

   \[
   \| \nabla J \| = \sqrt{J_x^2 + J_y^2}
   \]

   tells how fast image changes

   \[
   \frac{\nabla J}{\| \nabla J \|}
   \]

   is the direction of fastest change.
Take a derivative - discretely

\[
\frac{\partial I}{\partial x} \approx \begin{bmatrix} -\frac{1}{2} & 0 & \frac{1}{2} \end{bmatrix} \otimes I \quad \text{(Just like in 1D)}
\]

\[
\frac{\partial I}{\partial y} \approx \begin{bmatrix} -\frac{1}{2} \\ 0 \\ \frac{1}{2} \end{bmatrix} \otimes I
\]

What is the gradient?

\[
\left( \frac{\partial I}{\partial x}, \frac{\partial I}{\partial y} \right) = (k, 0)
\]
What is the gradient?

\[ \left( \frac{\partial I}{\partial x}, \frac{\partial I}{\partial y} \right) = (0, k) \]

No Change

Gradient direction is perpendicular to edge.

Gradient Magnitude measures edge strength.

What is the gradient?

\[ \left( \frac{\partial I}{\partial x}, \frac{\partial I}{\partial y} \right) = (k_1, k_2) \]

Less Change

Much Change
Gradient – More formal

• Suppose we move $\Delta$ in direction $\theta$.
  - Motion: $v=(\Delta \cos \theta, \Delta \sin \theta)$
• Intensity change: $\langle v, \nabla I \rangle$
• Recall $\langle v, w \rangle = ||v|| \ ||w|| \cos \alpha$.
• So $\langle v, \nabla I \rangle$ is maximized when $v$ is in the direction of $\nabla I$.
• In that direction, rate of image change is: $||\nabla I||$.

Gradient and Convolution

• How do we compute gradients with convolution?
  - $\frac{\partial I}{\partial x}$ = derivative in $x$ direction
    - Correlation with $[-.5, 0, .5]$ for derivative in $x$ direction.
• Likewise, correlation with $[-.5; 0; .5]$ for derivative in $y$ direction.
Smoothing and Differentiation

- Need two derivatives, in x and y direction.
- We can use a derivative of Gaussian filter
  - because differentiation is convolution, and convolution is associative

Scale

Smoothing
- Eliminates noise edges.
- Makes edges smoother.
- Removes fine detail.
- *Matlab*

(Forsyth & Ponce)
Finding the Peak

1) The gradient magnitude is large along thick trail; how do we identify the significant points?
2) How do we link the relevant points up into curves?

We wish to mark points along the curve where the magnitude is biggest. We can do this by looking for a maximum along a slice normal to the curve (non-maximum suppression). These points should form a curve.

(Forsyth & Ponce)
Non-maximum suppression

At q, we have a maximum if the value is larger than those at both p and at r. Interpolate to get these values.

(Forsyth & Ponce)

Linear Interpolation

- Given a function defined at two points, f(0), f(1), we want to find values for intermediate points, eg., f(x), 0 < x < 1.
- Can take weighted average:
  \[ f(x) = (1-x)f(0) + xf(1) = f(0) + x(f(1)-f(0)) \]
- This is equation for line with slope f(1)-f(0).
Bilinear Interpolation – 4 points

- Given values at (0,0), (1,0), (0,1), (1,1) find value at (x,y).
- Linearly interpolate (x,0), (x,1), then interpolate (x,y).
- Or, find (0,y) and (1,y) and interpolate.
- These produce same results.

If we interpolate to get $f(x,0) = (1-x)f(0,0) + xf(1,0)$, $f(x,1) = (1-x)f(0,1) + xf(1,1)$. Then $f(x,y) = ((1-x)f(0,0) + xf(1,0))(1-y) + ((1-x)f(0,1) + xf(1,1))y$.

If we interpolate to get $f(0,y) = (1-y)f(0,0) + yf(0,1)$, $f(1,y) = (1-y)f(1,0) + yf(1,1)$. Then $f(x,y) = ((1-y)f(0,0) + yf(0,1))(1-x) + ((1-y)f(1,0) + yf(1,1))x$.

These are the same.
Hysteresis

• Check that maximum value of gradient value is sufficiently large
  – drop-outs? use **hysteresis**
    • use a high threshold to start edge curves and a low threshold to continue them.
Demo of Edge Detection
What is missing from Canny?

• Texture.
• Scale selection
• Learning for specific domains
• Edge classification