Problem Set 2

Due: Tuesday, March 4, 2003, 11:00, at the start of class. Email all Matlab code to the TA before 11:00 also.

Readings:
Forsyth & Ponce: Chapters 7 and 8.

1. **10 points** Prove that convolving a 1-D signal twice with a Gaussian kernel is equivalent to convolving the signal with a single Gaussian kernel with a larger standard deviation. (Hint: Make use of the identity:

$$\int_{-\infty}^{\infty} e^{-\frac{1}{2}Ax^2+Zx}\,dx = \sqrt{\frac{2\pi}{A}}\,e^{\frac{Z^2}{2A}}$$

with A>0.)

2) 1D Edge Detection

a) **10 points** Write a function to produce a Gaussian kernel, in which the standard deviation, sigma, is a parameter. Make sure the kernel is big enough to capture 99% of the area of the Gaussian. Plot it for sigma=2. Turn in the plot.

b) **10 points** Write a function to convolve a signal with a kernel in 1D. Test it by convolving a Gaussian with itself 2 and 3 times. Plot both. Turn in the plots.

c) **20 points** Write a 1D edge detector. This should take 2 parameters, the amount of smoothing and a threshold on the strength of the edge. Test it by finding the edges in the following 1D image.

I = [zeros(1,50), .9*ones(1,10), zeros(1,10), .6*ones(1,40), zeros(1,50)].

d) **20 points** Use a very low threshold, which should work since there's no noise. For sigma = .5, 1, 1.5, 2, 2.5, 3, … 25 determine the location of the edges in image I (from part 2c). Plot them in a 2D picture, where each row shows the edges detected for a different sigma. Turn in the plot.

e) **20 points** Form and turn in the same plot, this time adding Gaussian noise to the image with a standard deviation of 1 (see function **randn**).

3) **10 points** Download two images from the internet, or take two images yourself if you have a digital camera. The first image should be one you think will be easy to find edges in, the second image should be one that seems hard. Choose a black and white image, or convert it with rgb2gray. For each image: Run the canny edge detectors, with the default parameters: edge(I,'canny',[.02,.1],1). Then play with the parameters until you get the best results you can. On this final, best, version, circle and label two places where you think the edge detector made a mistake, if it did. Explain why you think it got the wrong answer.

For each image, turn in the image, the edges found using default parameters, and edges with the best hand-chosen parameters with points labeled. List the parameters you used. Also turn in your explanation for why any failures occurred.

4) **20 points** Challenge problem (this is optional for extra credit): Consider a 1D box kernel of width 2m, and height 1. Call this kernel, *k*. *k* must lie entirely in the interval 0 to 2pi, so assume m<pi Write *k* as a linear combination of the basis elements:

$$\frac{\cos\theta}{\|\cos\theta\|}, \frac{\sin\theta}{\|\sin\theta\|}, \frac{\cos 2\theta}{\|\cos 2\theta\|}, \frac{\sin 2\theta}{\|\sin 2\theta\|}, \ldots$$

Hint: you are free to place *k* anywhere in the interval 0 to 2pi. Put it somewhere that will make your life easier.