

# Image-based Rendering and Interpolation

## New Images From Old

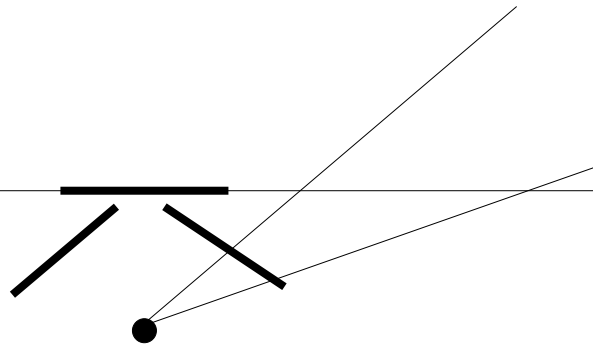
- Transfer Rays of Light from One Image to Another
  - Rectification and mosaicing
  - Light Fields
- Interpolate missing pixels
  - Interpolate in the image
  - Interpolate light rays

## Light-Field Rendering

- Sample the set of light rays in the world.
- Then generate an image by selecting the right rays.
- Mosaicing: simpler, just sample rays through one focal point.
- If one has all rays then camera can also move.

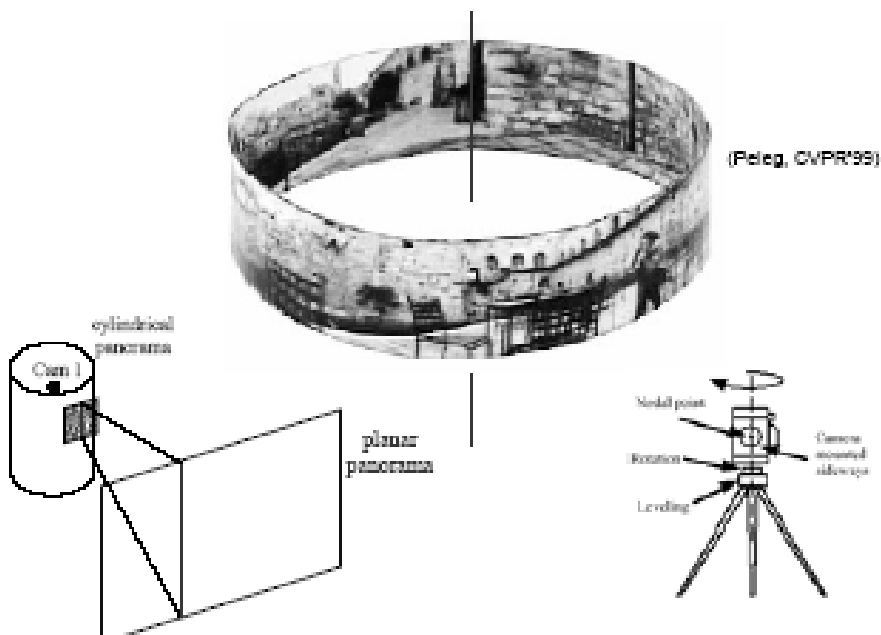
## Mosaics

- Take multiple images and construct one big image.
  - Represented as image, cylinder or sphere.
- Allows panning and zooming.
  - Simplest kind of motion.



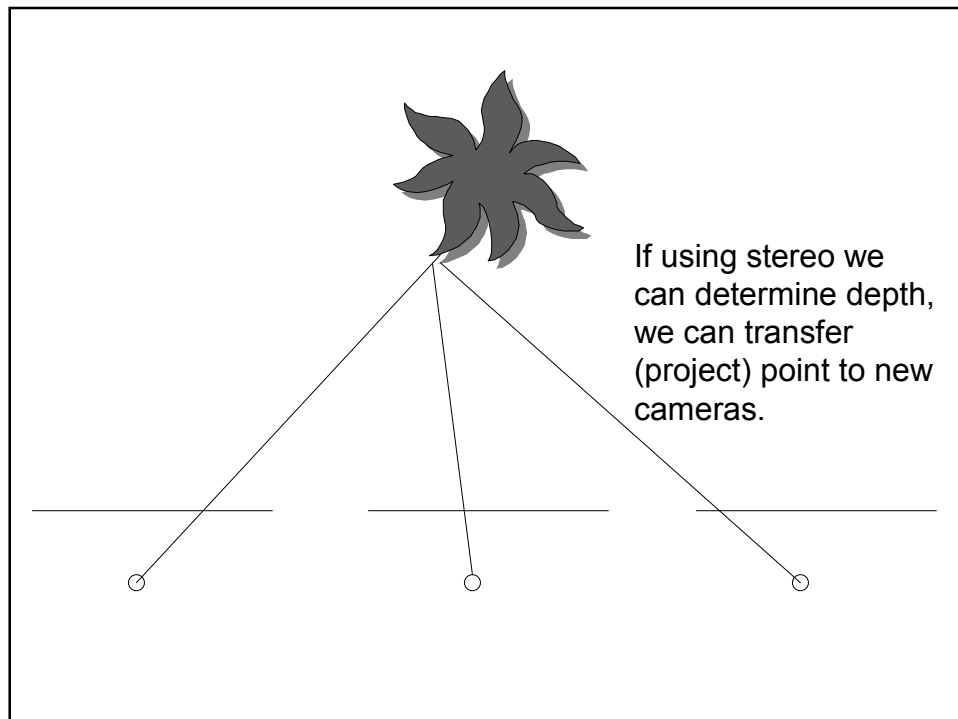
- Fixed focal point.
- Correspondence needed to align images.
- Image rectification

## Cylindrical Panoramas for 360° Viewing



## Cameras with different focal points

- Stereo Transfer



## Stereo Transfer

- Note that we can just interpolate disparity to determine where a point will appear in a new image, providing focal points are all collinear.
  - Given  $Z$  and  $f$ ,  $d$  and  $T$  are linearly related.

$$Z = f \frac{T}{d}$$

## Linear Interpolation

- Given a function defined at two points,  $f(0)$ ,  $f(1)$ , we want to find values for intermediate points, eg.,  $f(x)$ ,  $0 < x < 1$ .
- Can take weighted average:  
$$f(x) = (1-x)*f(0) + x*f(1) = f(0) + x(f(1)-f(0))$$
- This is equation for line with slope  $f(1)-f(0)$ .

## Bilinear Interpolation – 4 points

- Given values at  $(0,0)$ ,  $(1,0)$ ,  $(0,1)$ ,  $(1,1)$  find value at  $(x,y)$ .
- Linearly interpolate  $(x,0)$ ,  $(x,1)$ , then interpolate  $(x,y)$ .
- Or, find  $(0,y)$  and  $(1,y)$  and interpolate.
- These produce same results.

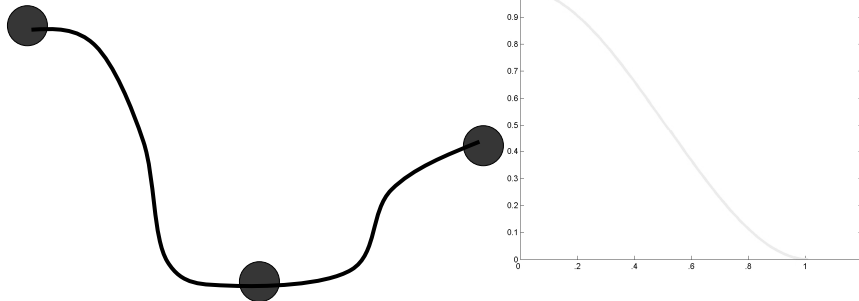
If we interpolate to get  $f(x,0) = (1-x)f(0,0) + xf(1,0)$ ,  $f(x,1) = (1-x)f(0,1) + xf(1,1)$ . Then  $f(x,y) = ((1-x)f(0,0) + xf(1,0))(1-y) + (f(x,1) = (1-x)f(0,1) + xf(1,1))y$ .

If we interpolate to get  $f(0,y) = (1-y)f(0,0) + yf(0,1)$ ,  $f(1,y) = (1-y)f(1,0) + yf(1,1)$ . Then

$$f(x,y) = ((1-y)f(0,0) + yf(0,1))(1-x) + ((1-y)f(1,0) + yf(1,1))x$$

These are the same.

# Cubic Interpolation



Instead of weighting by distance,  $d$ , weight by:

$$1 - 3d^2 + 2|d|^3$$

- Smooth
- Symmetric

Suppose  $0 \leq x \leq 1$ , and a function  $f$  is defined on  $f(0)$ ,  $f(1)$ . We want to define it for  $f(x)$  so that  $f(x)$  is smooth.

If we do this by averaging neighbors, we have:

$f(x) = g(x)f(0) + g(1-x)f(1)$ . Then we want a function  $g$  that is smooth, and in which  $g(0) = 1$  and  $g(1) = 0$ , and in which  $g$  is symmetric so that  $g(x) + g(1-x) = 1$ .

With linear interpolation  $g(x) = 1-x$ . This fits the second two criteria, but this  $g$  is not smooth. There is a discontinuity at  $f(0)$ , since we suddenly switch between averaging  $f(0)$  and  $f(1)$  and averaging  $f(0)$  and  $f(-1)$

So instead, we want  $f(x)$  near  $f(0)$  to be based mostly on the value of  $f(0)$ , and only to gradually average in  $f(1)$  as we get closer to it.

A nice function that does this is  $1 - 3d^2 + 2|d|^3$

Note that  $g(1-x) = 1 - 3(1-x)^2 + 2(1-x)^3$

$$= 1 - 3 + 6x - 3x^2 + 2 - 6x + 6x^2 - 2x^3 = 3x^2 - 2x^3$$

$$= 1 - (1 - 3x^2 + 2x^3)$$

Also, we can see that when  $x \rightarrow 0$ ,  $g(x) \rightarrow 1 - 3x^2 + 2x^3 \rightarrow 1$ , and that  $g(1-x)$  similarly goes to 0. This means that  $(g(x)f(0) + g(1-x)f(1)) - f(0) / x \rightarrow 0$ , which shows that the tangent at  $f(0)$  on the right side of the curve is 0. Similarly, the tangent on the other side is also zero, so two interpolating curves meet at  $x=0$  with the same tangent, ie., smoothly.

## Application: Image Resizing

- When we enlarge an image, we need values for the new pixels.
- Common methods:
  - Nearest neighbor
  - Bilinear interpolation
  - Bicubic interpolation





