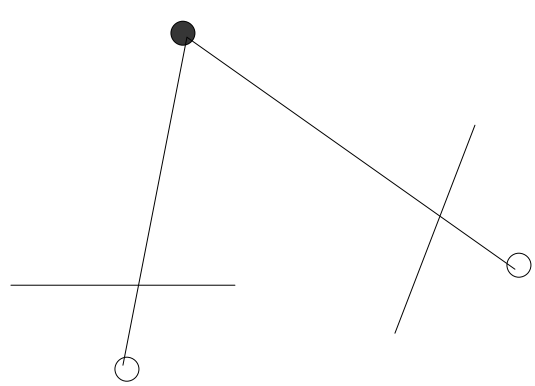
Motion Flow



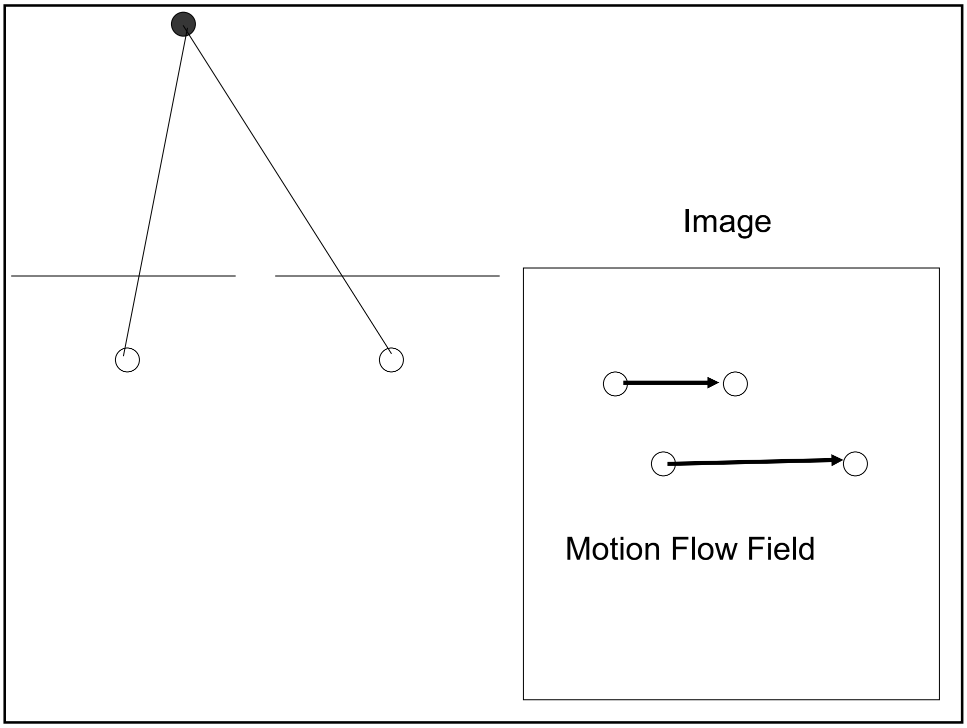
Motion flow: how does the image of a point move as the camera moves?

Motion

- We'll divide motion into two components.
  - Translation: focal point and image plane translate in same way.
  - Rotation: about the focal point, which doesn't move.
- Note that it is equivalent to move camera with scene fixed, or move scene with camera fixed.

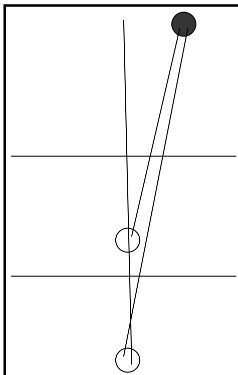
Example: Translation in +x direction

- We have already studied this in stereo.
- All points move horizontally
- Image motion is in opposite direction as camera (non-negative disparity).
- Motion is inversely proportional to depth.



Translation in the y direction

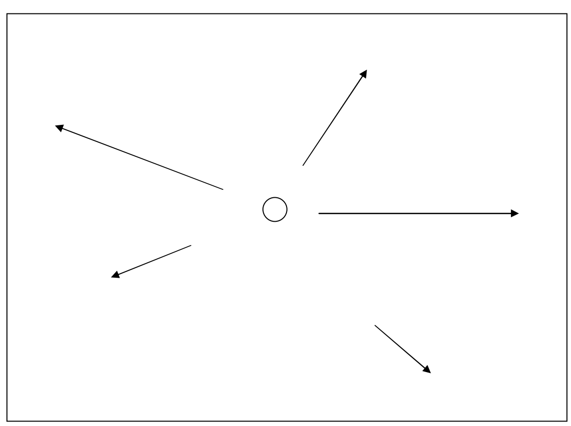
- By symmetry, this is the same as motion in the x direction, with flow in the vertical direction.



Translation in the Z direction

- The point in the center of the image doesn't move.
  - Consider the line of points that lie on the line connecting the two focal points. This line intersects the center of both images. So the image center point in image 1 must lie on this line, and any point on this line projects to the image center in image 2.
- Any other point moves in a line directly away from the image center.
  - Image is (u,v) = f(x,y)*1/Z, and Z decreases, so (u,v) is scaled.
  - Speed is greater as point is closer.

Fixed (center) point is called the focus of expansion (contraction, if we are moving away).



Rotation around optical (z) axis

- This doesn't move the image plane, just rotates it.
- So image is rotated.

Rotation about y axis

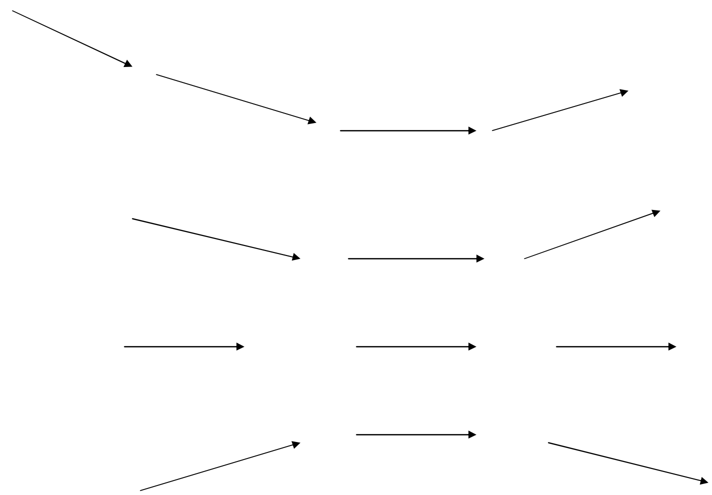
- Focal point doesn't move (with any rotation).
- For image point p, let L be the line through this point and the focal point (o).
  - Scene point P that produces p is on L also.
- o, p, and P will be collinear after rotation.
- This means that motion of p determined by its rotation.
  - Distance to P (depth) doesn't affect motion.
  - Rotation gives no information about depth.

Rotation about y axis

- Angle between L and y axis is preserved by rotaton.
  - As point rotates its longitude stays the same while latitude changes.
- Motion forms a curve that is at max height above optical center.

To see the effect of this rotation, let's consider an image point, p, and a unit vector v from the focal point to p. Rotating the scene around the y axis rotates v to Rv, and will move p to be the intersection of the image plane and a ray in the direction of Rv. Rotating v around the y axis doesn't effect its y coordinate, but rotates the x and z coordinates.

To find the location of p in the rotated image, we scale the Rv until it lies in the focal plane. That is, we scale it by f/z, where z is the z coordinate of Rv. So, if the z coordinate of Rv is less than the z coordinate of v, we'll have to scale v by more to get to the image plane, which will scale its y coordinate more. So, a rotation that decreases z will increase the y component of the image point. Note that v has the biggest z component when the point is directly above the image center. Moving to the image periphery, this z component of v decreases so the y component of the image point increases. This means the flow field looks like:



However, the real effect is much more subtle. This makes it very difficult to distinguish rotation about the y axis from translation in the x direction with little depth variation in the scene.

General Epipolar Constraint

Now consider general motion. The same reasoning used to find epipolar constraints for stereo continues to apply. Consider the family of planes that include the two focal points. Each plane intersects each image plane in a line. All the points on this line in one image are images of points on the plane, which must appear on the corresponding line in the second image. So we still have conjugate epipolar lines. If the camera geometry is known, these lines can be computed, and image matching is still a 1D problem.

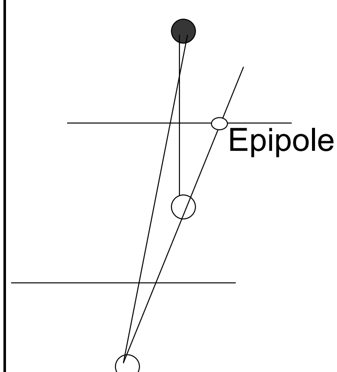
This makes sense, because we could always have rectified the two images to create a standard stereo configuration.

Epipole

However, there is a new wrinkle. All these planes include the line that connects the two focal points. Where this line intersects an image plane, that image point is called the epipole. Since every epipolar line is the intersection of the image plane with a plane that contains the epipole, every epipolar line contains the epipole. So the epipolar lines all intersect at the epipole.

Note that in the stereo setup, we had translation in the x direction. The line connecting the epipoles doesn't intersect either image. We think of this as having the epipole be at infinity, producing parallel epipolar lines.

General Translation



There is no motion at the epipole. All motion is along radial lines from the epipole. If camera motion is forward, flow is away from epipole. The closer points are the faster they move.

This focus of expansion can be used to determine the direction of motion.

When the scene is a plane, we can use trigonometry to determine time to collision, even without knowing distance or velocity.

Equations

Projection : $\mathbf{p} = f\dfrac{\mathbf{P}}{Z}$ with $\mathbf{p}$ the image point, $\mathbf{P}$ the scene point with depth Z.

3D Motion : $\mathbf{V} = -\mathbf{T} - \omega \times \mathbf{P}$ with $\mathbf{V}$ the 3D motion, $\mathbf{T}$ the translation, x the cross product and ω a vector in the direction of rotation, with magnitude that indicates the amount of rotation. The components of $\mathbf{V}$ are :

$$V_x = -T_x - \omega_y Z + \omega_z Y$$
$$V_y = -T_y - \omega_z X + \omega_x Z$$
$$V_z = -T_z - \omega_x Y + \omega_y X$$

When motion is small, we can compute image motion by taking the time derivative of both sides of the projection equation :

$$\mathbf{v} = f\frac{Z\mathbf{V} - V_z\mathbf{P}}{Z^2}$$

Looking at each component of image motion, with some substitutions we get :

$$v_x = \frac{T_z x - T_x f}{Z} - \omega_y f + \omega_z y + \frac{\omega_x xy}{f} - \frac{\omega_y x^2}{f}$$
$$v_y = \frac{T_z y - T_y f}{Z} + \omega_x f - \omega_z x - \frac{\omega_y xy}{f} + \frac{\omega_x y^2}{f}$$

From this equation we can more rigorously derive some previous statements. For example, it is clear that if motion is only rotation, flow doesn't depend on depth. When rotation is about y axis, omega_y is its only non-zero component. We can see that v_y will not be zero, so height of point changes, and we have an equation to describe it. Also, pure translation case becomes fairly simple.