Problem Set 2
CMSC 426
Due February 18, 2010

Pencil and Paper Assignments

1. **10 points.** Consider the following 1D image:

36	A	21	B	C	18	24

Suppose we filter this using the averaging filter:

1/3	1/3	1/3

and the resulting image is:

34	29	23	21	20	22	22

What were the values of the pixels labeled A, B, and C?

2. Consider the following 1D image:

63	59	63	55	55	43	39

   a. **5 points.** Show the 1D image that results from filtering this with the three element filter:

1/4	½	¼

   b. **5 points.** The filter in problem 2 will smooth an image, because applying it is equivalent to repeated application of a smoothing filter. Show that this is true, explaining which filter is being applied, how many times it is being applied, and why this is equivalent to the single filter in (a). Hint: recall that convolution is associative.

3. In class, we defined correlation with the following equation:

$$F \circ I(x) = \sum_{i=-N}^{N} F(i) I(x+i)$$

We can define a continuous version of correlation, when we have an image and filter that are continuous 1D functions of location, as:

$$F \circ I(x) = \int_{-\infty}^{\infty} F(t) I(x+t) dt$$

   a. **4 points.** Define a box filter, F, of width *2w* as, so that it performs the same continuous operation as a discrete box filter. That is, filtering with F

should replace $I(x)$ with the average of all values of I that are between $I(x-w)$ and $I(x+w)$. Give a mathematical expression that defines F.

b. **4 points.** Suppose we have an image in which the intensities vary from 0 to 2 according to a cosine function. That is:

$$I(x) = 1 + \cos(x)$$

What is the result of correlating I with the box filter, F, you defined in problem 4? Give the result as an explicit mathematical function.

c. **2 points.** How does this relate to the convolution theorem?

4. Suppose we have an image whose intensities follow the pattern $I(x) = x^2$. So the discrete image might look like:

0	1	4	9	16	25	36

a. **5 points.** There are three reasonable choices for a filter that computes a derivative. List these filters and show the result of correlating the above image with each filter.

b. **5 points.** Use calculus to compute the exact value for the derivative of $I(x) = x^2$ at the points $x = 1$, 3 and 5. How much error is there in the numerical values you computed in (a), compared to these exact values? Which method of computing the derivative is most accurate?

5. **10 points.** Construct a 1D filter that will compute the second derivative of a 1D image. Explain how you built it.

6. Consider a 2D image with intensities $I(x,y) = x^2 + y$.

a. **5 points.** Show what the discrete image looks like for x and y between 0 and 3. Then use derivative filters to compute the value of the gradient at $(x,y) = (2,1)$. What is the gradient? What is the magnitude of the gradient? What is the direction of the gradient?

b. **5 points.** Use calculus to compute the exact value of the gradient at $(2,1)$. (if you haven't done multivariable calculus, note that to take the derivative in the x direction, treat y as if it were a constant. The same idea applies to take the derivative in the y direction). How does this compare to the value you obtained in part (a)?

7. **Up to 10 points.** Challenge problem: Consider a 1D box kernel of width $2m$, and height 1. Call this kernel, k. k must lie entirely in the interval 0 to 2π, so assume $m < \pi$ Write k as a linear combination of the basis elements:

$$1, \frac{\cos\theta}{\|\cos\theta\|}, \frac{\sin\theta}{\|\sin\theta\|}, \frac{\cos 2\theta}{\|\cos 2\theta\|}, \frac{\sin 2\theta}{\|\sin 2\theta\|}, \ldots$$

Here, 1 refers to a function that has the constant value of 1, from 0 to 2π. Hint: you are free to place *k* anywhere in the interval 0 to 2π. Put it somewhere that will make your life easier.