Problem Set 3
CMSC 426, Spring 2014
Due Feb. 26

1. **Histogram Equalization (10 points)**

   a. Perform histogram equalization on the following image:

0	0	1	4
0	1	2	5
0	2	6	8
1	2	3	9

   Assume that all pixel intensities vary between 0 and 9. There was a little confusion about this in class, so be sure to review the description of histogram equalization in the current version of the class notes. As intermediate steps, show the histogram and the CDF of the image.

   b. Perform histogram equalization on the same image, but this time suppose that the intensities could vary between 0 and 15.

2. **Image Gradients (20 points)**

   a. Suppose an image can be described by the function, $I(x,y) = 2x^2+7y+9$.
      i. What is the gradient of the image at the point (5,6)?
      ii. What is the magnitude of the gradient?
      iii. What is the direction of the gradient?

   b. Consider a different image. Suppose at some point of this new image, the gradient is (3,1). What is the partial derivative of the image in the direction (1,1) at that point? (That is, if you take a tiny step of length delta in the direction (1,1), what is the change in intensity divided by delta)?

   c. Suppose at some point the gradient of an image is (1,0). What is the partial derivative of the image in the direction (0,1)?

   d. Suppose you have a 2D image described by the function:

$$I(x,y) = \begin{matrix} 0 & \text{for} & x < 10 \\ 50 & \text{for} & x \geq 10 \end{matrix}$$

   That is, the image is just a step function that goes from black to gray. Suppose we now convolve this image with an unknown filter, to produce a new image J. Is the following statement true or false: When the image gradient is not (0,0), the direction of the image gradient of J is always in the x direction? Either give an example to show that this is false, or prove that it is true. Imagine that the image is infinite, so that there are no boundary effects in which part of the filter lies outside the image.

3. **Creating filters (20 points)**. You may use Matlab and numerical techniques in answering these questions. The fspecial command may be particularly helpful for (a), but note that you will have to do more than just call that function to get a correct answer. Explain your reasoning in creating these filters. Especially, explain how you decided on the size of the filter.

    a. Create the smallest possible 1D Gaussian filter with sigma = 1.2, that captures 99% of the values of a full Gaussian.
    b. Build a 1D filter of $1/(1+x^4)$. This should also be the smallest possible filter that captures 99% of the values of the full filter. Note that you do not have to normalize this filter so that all the values add up to 1. You do not have to determine the filter size analytically; you can use Matlab to calculate its size.
    c. Construct a 1D filter that computes a second derivative.
        i. Show that you can construct this by convolving together two filters that each compute a first derivative.
        ii. What happens if you correlate two first derivative filters? How is the result different? Does this show that correlation is not associative?
    d. Create a 2D filter that computes the derivative of an image in the diagonal direction (ie., in the direction of the vector (1,1)). That is, this filter estimates the amount the image would change if you move one pixel in a diagonal direction. Apply this filter to the "image"

1	2	3	4	5	6	7	8
2	4	6	8	10	12	14	16
3	6	8	12	15	18	21	24

Challenge Problem (10 points): Consider an ideal case of a continuous, smooth image. Prove that performing histogram equalization on such an image does not alter the direction of the image gradient anywhere in the image.