

Problem Set 5

CMSC 426

Assigned Tuesday, November 1, Due Tuesday, November 8

1. Projection

- a. **5 points:** Suppose we have a camera with focal point $(0,0,0)$ and image plane $z = 1$. We observe a point in the scene at the location $(18, 6, 3)$. Where does this point appear in the image?

We divide the x and y coordinates by the z coordinate, to obtain $(6,2,1)$. Note that we would multiply this by the focal length, but this is 1.

- b. **10 points:** Suppose we have a camera with focal point $(3,2,3)$ and image plane $x + y + z = 11$. We are looking at a scene point at the location $(13,7,8)$. Where does this appear in the image plane? That is, give the coordinates of a 3D point that is in the image plane.

The point is projected along the line joining the focal point and the scene point, which is: $(3,2,3) + t(10,5,5)$. We need to find the point where this intersects the image plane. This is where $3+10t + 2 + 5t + 3 + 5t = 11$, which implies $20t = 3$, $t = 3/20$. So the point is at $(4.5, 2.75, 3.75)$.

- c. **10 points:** Suppose we have a camera with a focal point at $(0,0,0)$, and an image plane at $z = 1$. We are looking at the line described by the equations $x = 1$, $y = z$. What is the vanishing point of this line?

A point on this line has coordinates $(1, z, z)$. The image of this point is $(1/z, 1)$. As the point gets very far away, that is, as z goes to infinity, this point converges to $(0, 1)$.

- d. **10 points:** Suppose we have a camera with a focal point at $(0,0,0)$, and an image plane at $z = 1$. We are looking at the plane $x = 1$. What is the horizon of this plane?

The horizon is a line. Any point on this plane will have coordinates $(1, y, z)$. For any point that is far away, the image of this point, $(1/z, y/z)$ will approach $(0, y/z)$. y/z could be anything, depending on what y is. So, all points on the horizon will be on the line, $x = 0$, (in the image plane, $z = 1$).

- e. **Challenge problem, 10 points:** Suppose we have a camera with a focal point at $(0,0,0)$, and an image plane at $z = 1$. We are looking at a circle in the plane $x = 1$. It has its center at the location $(1, 0, 10)$, and a radius of 1. What shape does the circle produce in the image? Describe this with an equation.

2. Reconstruction

- a. **5 points:** Suppose we have a camera with a focal point at $(0,0,0)$, and an image plane at $z = 1$. We see an image point at the location $(5,-2,1)$. Give a line in 3D that this point must lie on.

The line goes from the focal point to the image point, $(x,y,z) = (0,0,0) + t(5,-2,1)$.

- b. **10 points:** Suppose the point is known to be on a ramp, which is described by the equation $3y-z = -140$. What are the 3D coordinates of the scene point we are looking at?

We have to find a point at the intersection of $3y-z = -140$ and $(x,y,z) = (0,0,0) + t(5, -2, 1)$. Combining these equations we have $3(-2t)-t = -140$. So $t = 20$, and $(x,y,z) = (100, -40, 20)$.

3. Stereo

- a. **5 points:** Suppose we have two cameras. One has a focal point at $(0,0,0)$, the second has a focal point at $(10,0,0)$. Both have the same image plane, $z = 1$. Suppose we see a scene point at $(2, 5, 1)$ in the first image, and at $(10,5,1)$ in the second image. Where is the scene point?

We know $z = T/d$. In this setup, $T = 10$, $d = 2$, so $z = 5$. From the first image, we know that the scene point is on the line $(0,0,0) + t(2,5,1)$. This means $t = 5$, and the scene point is at $(10, 25, 5)$. We can check that this will project to the image point $(10,5,1)$ in the second image.

- b. **10 points:** Suppose the second camera has an image plane at $z = 2$. If we see a point in the first image at $(0,1,1)$, what is the corresponding epipolar line in the second image?

The focal points are $(0,0,0)$ and $(10,0,0)$, and we see a point at: $(0,1,1)$. First, let's find an equation for a plane that goes through these points. In general, we have $Ax + By + Cz = D$. From the first point, we know $D = 0$. From the second point, we know $A = 0$. So we have $B + C = 0$ from the third point. We can choose any values that satisfy this equation, so let's say the plane has the equation: $y - z = 0$. We can check that this fits all three points. Next, we need to intersect this with the image plane for the second image, which is $z = 2$. This gives us $z = 2$, $y = 2$. This describes a line, in which x can take on any value.

- c. **10 points:** Suppose the second camera has an image plane at $x = 9$. What is the epipole in the second image?

We just need to find a line through $(0,0,0)$ and $(10,0,0)$, and intersect it with the plane $x = 9$, which will occur at the point $(9,0,0)$.

- d. **10 points:** Suppose the second camera has an image plane at $x = 9$, and we see a scene point at $(1, 2, 1)$ in the first image. What is the epipolar line in the second image?

Let's find a plane that goes through the two focal points and $(1, 2, 1)$. From the last answer, we know that any plane through the focal points has an equation like $By + Cz = 0$. Plugging in $(1, 2, 1)$ we have: $2B + C = 0$. Let's take $B = 1$ and $C = -2$. Then the epipolar plane has the equation: $y - 2z = 0$. Now we intersect this with the plane $x = 9$. We can describe the epipolar line with these two equations.

4. Rectification

- a. **5 points:** Suppose we take an image with a camera that has a focal point at $(0, 0, 0)$ and an image plane of $x + z = 1$. We observe a point at the location $(-4, 3, 4)$. Where would this point have appeared if we had taken the picture with a camera with the same focal point, but an image plane of $z = 1$?

The line from the scene point to the focal point passes through $(-4, 3, 4)$, so it must have the equation: $(x, y, z) = (0, 0, 0) + t(-4, 3, 4) = t(-4, 3, 4)$. We need to intersect this line with $z = 1$. This happens when $t = 1/4$. So the intersection point is $(-1, 3/4, 1)$. Actually, this problem would lead into the next a little better if I had made the point at $(-4, 3, 5)$, so it would have been on the first image plane.

- b. **10 points:** In the same setting, suppose we want to find the intensity that will be in our new image at $(1, 2, 1)$. Where would we look in the image that we took?

We want to know the intensity of light that is traveling through $(1, 2, 1)$ to the focal point. That is, the light is going along the line: $(x, y, z) = t(1, 2, 1)$. We want to know where this ray of light intersected our actual image plane, $x + z = 1$. Substituting we get $t + t = 1$, $t = 1/2$, so we have the point $(1/2, 1, 1/2)$.