Mosaics

The goal of this problem set is to write code to form a mosaic of two images. We begin with two images that have overlapping fields of view (see below) and stitch them together into a single, larger image. Our strategy is to use SIFT to locate feature points and their descriptors. Each feature in one image is matched to a feature in the second image with the most similar descriptor. Note that many, but not all of these matches will be correct. To find a good set of matches we use RANSAC. We randomly pick three matches, compute the affine transformation that relates these matches, and then apply these to all the feature points. We repeat many times and pick the transformation that maps the most feature points in one image near their matching feature point in the other image. Finally, we use this transformation to combine the images.

We will give you code that computes SIFT features and descriptors for a single image. (Actually, we give you instructions on retrieving this code). This code includes a Matlab wrapper you can use to call SIFT. We also give you two test images, LR1 and LR2 (above), to use in testing your code.

DOWNLOAD SIFT CODE:

The code for finding SIFT features is due to Andrea Vedaldi. Go to www.vlfeat.org and download the latest version of vlfeat code (0.9.16). Download the binary package. Then follow the instructions under Matlab install on the menu on the left. Note that there is a lot of documentation for everything, including all functions in VLfeat. You should just need to execute the command:

```matlab
run('VLFEATROOT/toolbox/vl_setup')
```
We include some skeleton code and a helper function, in the file ps6.m.

Problems:

1. **10 points:** Run the skeleton code to detect SIFT features in a white square on a black background, and in the two images above. The results should look like this:

   ![Image](image1.png)

   ![Image](image2.png)

   Essentially all the code you need to do this is given to you. The point of this problem is just to get you started running SIFT. Include the resulting images in your write-up.

2. **10 Points: Find Best Match.** vl_sift returns two values, F and D. Each column of F contains the x and y coordinates, orientation and scale of each feature. Each corresponding column of D contains a descriptor for that feature. Write code that takes every SIFT feature in image 2 and finds the feature in image 1 with the most similar descriptor. You should compare two SIFT descriptors (which are histograms) using SSD. That is, just compute the SSD of the appropriate columns of the D that is computed for each image. If there are \( n \) features in image 2, the output of this might be an \( nx5 \) array, in which each row contains the (x,y) coordinates of a point in image 2, the point in image 1 that matches it best, and the score of this match.

   Now, using this code, write a function, find_best_match, which finds the three matches that have the lowest SSD. Display both images with these points shown.
in three different colors. That is, the images might each have a white disk, indicating the location of the two points that match best, a green disk, showing the points that match second best, and a blue disk, showing the points that match 3rd best. Run this on LR1 and LR2. Include the resulting images in your write-up. The results should look like this (it’s a little hard to see the blue dot without enlarging the image):

3. **10 points: Affine Transform.** Write a function that takes three matching points as input, and computes an affine transformation that maps three points from one image to the matching three points in the second image. So executing:

   \[ A = \text{affine\_transformation}(p1,p2); \]

computes a 2x3 affine transformation, A, that maps the points in p2, represented as a 2x3 matrix in which each column is a point, so that they match the points in p1. Test this by running the code in ps6. You might get results like:

   \[
   >> \text{ps6}(3)
   \]

   \[
   \text{ans} =
   \begin{bmatrix}
   0.3331 & 0.2220 & 0.1110 \\
   -0.0555 & -0.2220 & -0.4441
   \end{bmatrix}
   \]

4. **30 Points: RANSAC.** Now, perform RANSAC to find the best affine transformation that maps the most points from image 2 to image 1. To do this, perform the following steps:
   a. Randomly pick three matching points, in the format computed in part 2.
   b. Compute the affine transformation, A, that relates them, using part 3.
   c. Apply A to all points in image 2.
d. For each point, p, in image 2, that has been matched to point q in image 1, find out whether \( Ap \) is close to q ("close" might mean their Euclidean distance is less than 2).

e. Count the number of points that are mapped by \( A \) to be close to their matching point.

f. Repeat steps a-e many times (maybe a thousand?) and choose the \( A \) with the highest total.

Test your code using part 4 of ps6. In this code, I construct a test case with random points in \( p1 \) and \( p2 \), some of which are related by an affine transformation. For example, you might get:

\[
A =
\begin{bmatrix}
70.1550 & 77.2314 & 14.4582 \\
36.9972 & 9.5638 & 9.4105
\end{bmatrix}
\]

\[
A_{res} =
\begin{bmatrix}
70.1550 & 77.2314 & 14.4582 \\
36.9972 & 9.5638 & 9.4105
\end{bmatrix}
\]

5. **20 Points: Stitching.** Now write a function, \( \text{stitch}(J, K, A) \), that will stitch two images, \( J \) and \( K \), together, using the affine transformation \( A \). For this problem, just do this in a very simple way. For example, for every pixel in image 2, apply \( A \) to find its transformed location. Round off this location to an integer value. Place this value in the new image at this location. At the same time, place all values from image 1 into the new image at their original location. If a pixel doesn’t get a value from image 1 or image 2, make it black. If a pixel gets two or more values, you can combine them in any way you want. That is, if image 1 and image 2 overlap, you might use the average, or always use image 1’s value, or always use image 2’s value. Doing this may produce some artifacts, since there may be pixels in the middle of the new, target image, that don’t get any value assigned from either of the original images. You can fix this for extra credit as part of the challenge problem. Test your code on \( LR1 \) and \( LR2 \) using two affine transformations. First, create an affine transformation that will translate \( LR2 \) 100 pixels in the x direction, to the right of \( LR1 \). Next, create an affine transformation that will rotate \( LR2 \) by \( \pi/16 \), scale it by 0.8, and translate it 100 in the x direction and 50 in the y direction. The results of my program are shown below:
Hint: Suppose I and J are the first and second image, and A is the affine transformation. For a point, (x,y), let (x’,y’) = round(A(x,y)). Then we could just set I(x’,y’) = J(x,y). One of the things that might make this tricky is that x’ or y’ might be less than 1. However, the examples in this project are constructed so that you won’t have to worry about that. This is something you might want to handle as part of the challenge problem.

Note: Combining two images can be done much more efficiently using the Matlab function imtransform. I’m not recommending that, because I think it will be trickier, though. However, you are free to use imtransform if you want.

6. **10 points** Now write a mosaic program that puts this all together. It runs SIFT on each image, finds the best matching points, runs RANSAC to compute an affine
transformation, and uses this transformation to stitch the images together. The results of my program are shown below.

7. **10 points:** Now, take your own photos and run your program on them. Notice that the affine transformation you computed in part 4 is almost a pure translation. This is because I moved the camera very little between pictures. If you move the camera too much, even an affine transformation won’t work well. See if you can take two pictures that are well aligned by a more complicated affine transformation. Show the pictures you started with, the result, and the affine transformation that was computed.

8. **Up to 20 Points:** Challenge problem. Enhance this in a significant way. Suggestions:

   a. Improve your program so that you can provide it with many images, and it will stitch them all together.
   b. Improve the program so that you use bilinear interpolation to fill up all the pixels in the image. Show an example where this improves the results. For example, if the second image is smaller than the first image, this will be necessary.
   c. Often, due to automatic gain control or other factors, the overall lightness of the images will be different. This looks bad when they are stitched together. Even in the result in Part 5 this is a factor. You can see a slight line where the images overlap, because the left image is a little lighter. Find a way to normalize the images to remove this effect.
d. If you know about projective transformations, enhance the program so that it uses a full homography to align the images.
e. Make up your own improvement. You can check the paper by Brown and Lowe (http://cvlab.epfl.ch/~brown/papers/ijcv2007.pdf) for ideas.

Prizes: I will give two prizes. One will be for the nicest looking mosaic produced with the regular program (part 6) and one for the nicest looking mosaic produced with an enhanced system (part 7).