

Problem Set 8  
CMSC 426  
Due December 6, 2012

1. **Motion:** Suppose we have a camera with a focal point at  $(0,0,0)$  and an image plane of  $z = 1$ .
  - a. **4 points:** Suppose also that there is a point in the scene located at  $(5, 2, 5)$ . What are the image coordinates where this point will be seen?
  - b. **4 points:** Suppose we translate the camera forward, so that the focal point is located at  $(0,0,1)$ . What are the image coordinates now for the scene point at  $(5,2,5)$ ? What is the flow vector (ie., the difference between the coordinates found in (a) and in (b))?
  - c. **4 points:** Suppose we translate the camera so that the focal point moves from  $(0,0,0)$  to  $(1,0,1)$ . What are the image coordinates now for the scene point at  $(5,2,5)$ ? What is the flow vector?
2. **6 points:** Suppose we translate a camera so that the focal point moves from  $(0,0,0)$  to  $(1,0,1)$ . Find a scene point that appears in the same location before and after the camera moves. Where does it appear in the image?
3. Suppose a camera with a focal point at  $(0,0,0)$  and an image plane of  $z=1$  takes a picture, translates, and then takes a second picture. In the first picture, a point appears in location  $(3,0)$ . In the second picture, the same point appears at  $(2,0)$ .
  - a. **4 points:** Give an example of a possible 3D scene location for the point and a possible translation of the camera that would explain this.
  - b. **4 points:** True or false: the camera must have zero translation in the  $y$  direction. That is, the camera's translation has the form:  $(x, 0, z)$ . Explain your answer.
  - c. Suppose there is also a second point that appears in the location  $(5,2)$  in the first image, and in location  $(5,3)$  in the second image.
    - i. **4 points:** True or false: the camera must have zero translation in the  $x$  direction. Explain your answer.
    - ii. **4 points:** Using these two points, can we determine the direction the camera is moving in? If yes, give the direction. If no, explain why?
    - iii. **4 points:** Using these two points, can we determine the distance the camera has moved? If yes, give the distance. If no, explain why?

4. Suppose the following two images, H and I, are taken one second apart.

H

4	4	4	5	6	7	8	9	10	11
5	5	5	6	7	8	9	10	11	12
6	6	6	7	8	9	10	11	12	13
7	7	7	8	9	10	11	12	13	14
8	8	8	9	10	11	12	13	14	15
9	9	9	10	11	12	13	14	15	16
10	10	10	11	12	13	14	15	16	17
11	11	11	12	13	14	15	16	17	18

I

3	3	3	3	3	4	5	6	7	8
4	4	4	4	4	5	6	7	8	9
5	5	5	5	5	6X	7	8	9	10
6	6	6O	6	6	7	8	9	10	11
7	7	7	7	7	8	9	10	11	12
8	8	8	8	8	9	10	11	12	13
9	9	9	9	9	10	11	12	13	14
10	10	10	10	10	11	12	13	14	15

Recall the optical flow equation derived in class:

$$0 = I_t + \nabla I \cdot (u, v)$$

- 4 points:** Apply the optical flow equation to the point marked with an X in image I to find an equation that describes the optical flow at that point.
  - 4 points:** Apply the optical flow equation to the point marked with an O in image I to find an equation that describes the optical flow at that point.
  - 4 points:** Assuming that the flow is constant throughout the image, use these equations to solve for the optical flow of the entire image.
5. **Challenge Problem, 5 points:** You are looking at a wall. Your camera has a focal point of (0,0,0) and an image plane of  $z = 1$ . The wall is in the plane  $z = 10$ . It has a bulls-eye like pattern on it. The intensity at the point  $(x,y,10)$  is given by:  $\sqrt{x^2 + y^2}$ . Suppose the camera translates by  $(t_x, t_y, t_z)$ . Write down the optical flow equation that you will obtain at every point in the image (is it possible to determine the optical flow at every point in the image? If not, give an equation that applies to most points and explain why you can't find the optical flow everywhere). That is, how will  $I_t$  and  $\nabla I$  depend on  $(t_x, t_y, t_z)$ ? Using this equation, show how you would derive expressions for the direction of motion and the amount of time it will take for you to hit the wall

(you can assume is  $t_z > 0$ ). If everything works out right, you will have derived the equations for time to collision and shown that they are correct; don't just write the time to collision using  $t_z$ .