Problem Set 8
CMSC 426
Due, May 7, 2014

1. **Motion:** Suppose we have a camera with a focal point at (0,0,0) and an image plane of z = 1.
    a. **4 points:** Suppose also that there is a point in the scene located at (5, 2, 5). What are the image coordinates where this point will be seen?
    b. **4 points:** Suppose we translate the camera forward, so that the focal point is located at (0,0,1). What are the image coordinates now for the scene point at (5,2,5)? What is the flow vector (ie., the difference between the coordinates found in (a) and in (b))?
    c. **4 points:** Suppose we translate the camera so that the focal point moves from (0,0,0) to (1,0,1). What are the image coordinates now for the scene point at (5,2,5)? What is the flow vector?

2. **Motion:** Suppose a camera with a focal point at (0,0,0) and an image plane of z=1 takes a picture, translates, and then takes a second picture. In the first picture, a point appears in location (3,0). In the second picture, the same point appears at (2,0).
    a. **4 points:** Give an example of a possible 3D scene location for the point and a possible translation of the camera that would explain this.
    b. **4 points:** True or false: the camera must have zero translation in the y direction. That is, the camera's translation has the form: (x, 0, z). Explain your answer.

3. Suppose the following two images, H and I, are taken one second apart.

H

4	4	4	5	6	7	8	9	10	11
5	5	5	6	7	8	9	10	11	12
6	6	6	7	8	9	10	11	12	13
7	7	7	8	9	10	11	12	13	14
8	8	8	9	10	11	12	13	14	15
9	9	9	10	11	12	13	14	15	16
10	10	10	11	12	13	14	15	16	17
11	11	11	12	13	14	15	16	17	18

I

3	3	3	3	3	4	5	6	7	8
4	4	4	4	4	5	6	7	8	9
5	5	5	5	5	6X	7	8	9	10
6	6	6O	6	6	7	8	9	10	11
7	7	7	7	7	8	9	10	11	12
8	8	8	8	8	9	10	11	12	13
9	9	9	9	9	10	11	12	13	14
10	10	10	10	10	11	12	13	14	15

Recall the optical flow equation derived in class:

$$0 = I_t + \nabla I \cdot (u, v)$$

a. **4 points:** Apply the optical flow equation to the point marked with an X in image I to find an equation that describes the optical flow at that point.

b. **4 points:** Apply the optical flow equation to the point marked with an O in image I to find an equation that describes the optical flow at that point.

c. **4 points:** Assuming that the flow is constant throughout the image, use these equations to solve for the optical flow of the entire image.

4. Suppose we take two images with a camera that has translated (but not rotated) between images. We see three points in each image. The first set of points have coordinates (3,6), (9,0) and (3,3). The same points in the second image have the coordinates, respectively, of (3,8), (11,0) and (3,4).

a. **2 points** What is the essential matrix that relates the two images?

b. **2 points** What is the translation between the two images?

c. **2 points** What is the epipole in the second image? Give this in image coordinates.

d. **2 points** Give two examples of pairs of conjugate epipolar lines for the two images (that is, give two lines in each image). Give these in image coordinates.

5. Suppose we take a picture using a camera that has a focal length of 1. For the first picture, the camera's focal point is at (0,0,0) and the image plane is z=1. For the second picture, the camera's focal point is at (10,0,10), and the image plane is at x=9. Furthermore, the second camera has rotated so that the x axis in the image is now pointing in the z direction. So a point at (8, 0 12) in the world will appear on the second image plane with world coordinates (9,0,11), and with coordinates in the image of (1,0,1).
    a. **3 points.** What is the translation that relates the two camera positions? (This should be easy, but will lead into other questions).
    b. **3 points.** Give a rotation matrix that relates the two cameras. (Technically, we haven't talked much about the representation of 3D rotations. However, you should be able to piece this together. It may help to consider the case of a second camera position with the same rotation but no translation. Or, it might help to think about what the rotation matrix would have to look like in order to rotate the z axis so it is pointing in the direction of the –x axis.)
    c. **3 points.** What is the essential matrix, E, that relates the two cameras?
    d. **3 points.** To verify this, consider the point in the scene with location (0,0,10). Where does this appear in the two images? Use the location of these two corresponding points to verify that the equation p^TEp' holds. (Keep in mind, that you need to write the coordinates of p and p' relative to the center of the camera. For example, if a camera has a focal point of (3,7,3) and an image plane of x=4, then the center of the image will be at (4,7,3), and a point there will have image coordinates (0,0,1). A point that appears in the image plane at (3,8,4) will have image coordinates (0,1,1)).
    e. **3 points.** To verify that you have the right E with another example, consider the point in the scene with location (5,5,5). Where does this appear in the two images? Use the location of these two corresponding points to verify that the equation p^TEp' holds.

6. **Challenge problem 10 points:** Q1, below, contains a set of points found in the first image. That is, the first row of Q1 contains the first point found. Q2 contains the corresponding points found in the second image. That is, the first row of Q2 contains a point that matches the point found in the first row of Q1. Using these points, determine the Essential matrix that relates the two images. Determine the rotation and translation that relates the two images. Then compute the 3D location of the 8 points. You may use Matlab in performing these calculations.

Q1 =

10	2
2	1
1	2
6	1
3	6
9	3
3	3
10	5

Q2 =

0.8830	0.8038
0.3471	0.3592
-0.3204	0.3592
0.8824	0.6749
-0.4093	0.7364
0.6650	0.8038
0.0433	0.6536
0.4933	0.8667