Practice Final

CMSC 426

The final will be cumulative for the whole semester. You should consider the 1st and 2nd Practice midterms as providing notes and practice for the final. In addition, the final will cover the following material, which we have discussed since the second midterm.

- Stereo Matching
  - Understand what a disparity map is. Understand constraints on matching, including the epipolar constraint, the ordering constraint, the non-negative disparity constraint, the uniqueness constraint, the photometric constraint (matching pixels have similar intensity) and the smoothness constraint (neighboring pixels have similar disparity).
  - Understand how dynamic programming can be used for stereo matching.
  - Understand how window-based correlation can be used for stereo matching.
- Motion Flow Fields
  - Understand the flow fields caused by simple camera motions. Eg., what is the flow field when the camera just rotates about the x, y, or z axis, or just translates.
  - Focus of expansion and its relationship to the epipole.
- Matching with Optical Flow
  - Understand the optical flow equation and how to use it to derive a constraint (linear equation) on the motion of a point.
  - Understand the Lucas-Kanade algorithm, and how it uses an assumption of constant flow in a small window to solve for the flow.
  - The aperture problem. When is this present? What is its relation to the Lucas-Kanade algorithm, and to corner detection.
- The Essential Matrix, and epipolar geometry.
  - Understand what the essential matrix means. How can it be used to determine whether two image points come from the same scene point. What is its relationship to the epipolar constraint?
  - Understand how to build the essential matrix when the camera translates but does not rotate.
  - Understand how to build an essential matrix when there is also rotation.
- Image coordinates and world coordinates. You should understand what we mean when we represent an image point with world coordinates, and when we represent an image with image coordinates, relative to the camera.
- 3D Rotation Matrices
  - Know that a 3D rotation matrix has rows (and columns) that are orthonormal.
  - Be able to construct a 3D rotation matrix for simple rotations.
- The final will NOT include other material on linear lighting and motion, object detection or object recognition.

This is also a good time to take stock of some themes that run throughout the course. There are a few main concepts, mostly mathematical, that underlie much of the material we have discussed:

- **Correlation and Convolution**
  These show up in many ways throughout the semester. First, we discussed how to use convolution to smooth an image, or to find image derivatives. Then, in the texture problem set, we saw that correlation has a close relationship with matching based on the sum-of-square distances. SSD matching shows up again when we are matching between images, as in stereo matching.
- **Image Gradients**
  Image gradients are a fundamental way in which we measure how an image is changing. Understanding how to compute an image gradient, and getting an intuition for its properties is very important. This includes, for example, understanding that the direction of the image gradient encodes the direction in which the image is changing most rapidly, while the magnitude of the gradient tells us how rapidly the image is changing. Image gradients are basic to edge detection, but they also show up in SIFT descriptors, and in optical flow.
- **Histograms**
  Histograms are a way of capturing the aggregate properties of an image, or a region of an image. They are also a way of building a probability distribution to model what is going on in an image. In addition to constructing a histogram from image information, we have also learned about smoothing histograms using Kernel Density Estimation (KDE) to build a probability distribution, and we have talked about using vector quantization to build a discrete histogram in a high-dimensional space. Histograms are used in image processing, such as in histogram equalization. Vector quantization is used for color quantization, and also to build histograms that describe textures, or for classification with bag-of-words approaches. Histograms are also used in building SIFT descriptors, while histograms combined with KDE are used for background subtraction.
- **Representing Motion with Matrices**
  We have talked about how to represent rotation and translation in a matrix. This includes understanding what a rotation matrix is and how to build one. We've also talked about affine transformations, similarity transformations, and scaled orthographic projection with matrices. 2D matrix operations are used to align images, as in mosaicing. 3D matrices are needed for 3D operations, including 3D structure-from-motion and the Essential matrix.
- **3D Geometry: Perspective projection and epipolar geometry**
  The core of this is understanding the pinhole camera model, and how it can be used to determine the relationship between a camera, a 3D point, and its 2D image. This is fundamental in any vision task that attempts to recover the 3D structure of the world. Next, we have talked about the relationship between a 3D scene and two cameras. This gives rise to epipolar geometry, which we have used to constrain stereo matching, understand flow fields and the focus of expansion, and build the Essential matrix.

Practice Problems

1. Stereo Matching
   a. Which of the following disparity maps violate the ordering constraint or the uniqueness constraint.

[0 0 1 1 1 2 2 2 2 3]
[0 0 X 1 1 X 2 2 X 3 3]
[0 1 1 X X 3 3]
[X X 1 X X 5]

   b. Consider the following 1D images, assuming an occlusion cost and photometric cost as used in the problem set. What are the best matches, if we enforce the ordering constaint, the uniqueness constraint, and the non-negative disparity constraint? What if we enforce any two of these constraints? What if we enforce any one of these constraints? What if we enforce none?

L: [1 1]
R: [1 1]

2. Motion Flow Fields
   a. How can you tell the difference between the motion flow caused by a translation in the x direction, and a rotation about the y axis?
   b. Suppose a camera is translating in the direction (1,1,1). Draw the resulting flow field. How would this differ from the flow caused by a translation of (2,2,2)?

3. Matching with optical flow.
   a. Suppose a first image has intensities described by $H(x,y) = x^2+y^2$. A second image has intensities described by $H(x,y) = (x-1)^2+(y-1)^2$. Use the optical flow equation to determine a linear constraint on the optical flow at the points (7,3) and (3,7), and then assume that the flow is the same at both points and solve for the flow.
4. The Essential Matrix
   a. Suppose you see a point at (2,0) in the first image and at (4,0) in the second image. You see another point at (5,4) in the first image and at (5,2) in the second image. Assume the camera is translating, but not rotating.
      i. What are the flow vectors for each point? Where do they intersect? What is the focus of expansion? What does this tell us about the direction of the motion?
      ii. Notice that the points are moving towards the focus of expansion. What does this tell us about the motion?
      iii. Use the direction of the motion to determine the Essential matrix.

iv. Use two equations derived from the points to determine the essential matrix.

5. Coordinate systems.
   a. Suppose a camera has a focal point of (1,0,1) and an image plane of z = 2. Suppose a point appears in the image with world coordinates (3,4,2). What are its image coordinates?
   b. Suppose you have a camera with a focal point of (3,7,2) and an image plane of y = 6, and another camera with a focal point of (3,7, 12) and an image plane of y = 6. There is a point in the scene at the origin. Where does this appear in the two cameras, in image coordinates. What is the disparity?
6. 3D rotations
   a. Which of the following matrices represent 3D rotations? Explain your answer for each.

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1/\sqrt{3} & 1/\sqrt{3} & 1/\sqrt{3} \\ 0 & 1/\sqrt{2} & -1/\sqrt{2} \\ -1/\sqrt{2} & 1/\sqrt{2} & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1/\sqrt{3} & 1/\sqrt{3} & 1/\sqrt{3} \\ 0 & 1/\sqrt{2} & -1/\sqrt{2} \\ -\sqrt{2}/\sqrt{3} & 1/\sqrt{6} & 1/\sqrt{6} \end{pmatrix}$$