

First Step: Solve for Translation (1)

- This is trivial, because we can pick a simple origin.
 - World origin is arbitrary.
 - Example: We can assume first point is at origin.
 Rotation then doesn't effect that point.
 - All its motion is translation.
 - Better to pick center of mass as origin.
 - Average of all points.
 - This also averages all noise.

Specifically, we can never tell where the world points were to begin with. Adding one to every x coordinate in P and then subtracting 1 in every tx is undetectable.

So, wlog we can assume that sum(P(k,:)) = 0 for k from 1 to 3, ie., $sum(x1 \dots xn) = 0$, $sum(y1\dots yn) = 0$, $sum(z1 \dots zn) = 0$.

Rotation doesn't move the origin, which is now the center of mass. Neither does scaled orthographic projection. So, this only moves from translation.

Explicitly, we assume sum(p) = $(0,0,0)^{T}$. Then: sum(s*R(p)) = s*R(sum(p)) = s*R(0,0,0)^T = $(0,0,0)^{T}$. (^T means transpose). More explicitly, suppose sum(p) = $(0,0,0,n)^{T}$. Then, sum(R⁺P) = R⁺(sum(P)) = R⁺(0,0,0,n)^T = $(0,0,0,n)^{T}$. Sum(T⁺R⁺P) = T⁺(0,0,0,n)^T = $(ntx,nty,ntz,n)^{T}$. (Or just look at the 2x4 projection matrix). If we subtract tx or ty from every row, then the residual is (s11,s12,s13;s21,s22,s23)^{*}P. I = s part of matrix + t part of matrix.

$s \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$	$\begin{array}{c} 0\\ 0\\ 0\\ \end{array} \begin{pmatrix} 1\\ 0\\ 0\\ 0 \end{array}$	0 0 1 0 0 1	$ t_{x} \begin{pmatrix} r_{1,1} \\ r_{2,1} \\ r_{3,1} \\ r_{3,1} \\ 0 \end{pmatrix} $	$r_{1,2}$ $r_{2,2}$ $r_{3,2}$	$r_{1,3}$ $r_{2,3}$ $r_{3,3}$	$\begin{pmatrix} 0\\0\\0\\1 \end{pmatrix}$
	(0	0 1	[*] { 0	0	0	1

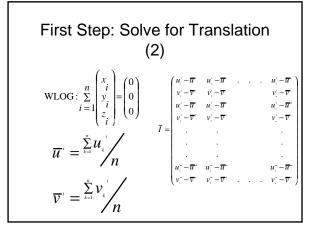
Even more explicitly. Consider the first row of the image matrix I. Average together all the entries in this row. This gives us:

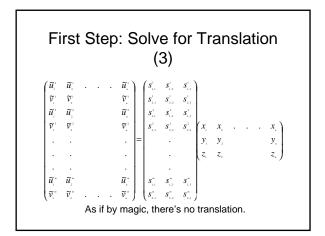
 $sum(\ (s\{1,1\},s\{1,2\},s\{1,3\})^*(x_i,y_i,z_i) + tx)/n$

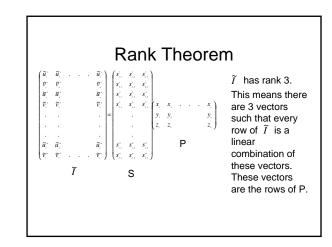
 $= (s\{1,1\},s\{1,2\},s\{1,3\})^*sum(x_i,y_i,z_i)/n + tx$

 $=(s\{1,1\},s\{1,2\},s\{1,3\})^*(0,0,0)+tx=tx.$

So we've solved for tx. If we subtract tx from every element in the first row of I, we remove the effects of translation.







Solve for S

- SVD is made to do this.
- $\widetilde{I} = UDV$ D is diagonal with non-increasing values.

U and V have orthonormal rows.

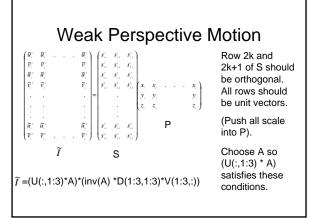
Ignoring values that get set to 0, we have U(:,1:3) for S, and $D(1:3,1:3)^*V(1:3,:) \mbox{ for P}.$

Linear Ambiguity (as before) $\tilde{I} = U(:,1:3) * D(1:3,1:3) * V(1:3,:)$

 $\tilde{I} = (U(:,1:3) * A) * (inv(A) *D(1:3,1:3) * V(1:3,:))$

Noise

- Ĩ has full rank.
- Best solution is to estimate I that's as near to \tilde{I} as possible, with estimate of I having rank 3.
- Our current method does this.



Related problems we won't cover

- Missing data.
- Points with different, known noise.
- Multiple moving objects.

Final Messages

- Structure-from-motion for points can be reduced to linear algebra.
- Epipolar constraint reemerges.
- SVD important.
- Rank Theorem says the images a scene produces aren't complicated (also important for recognition).

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