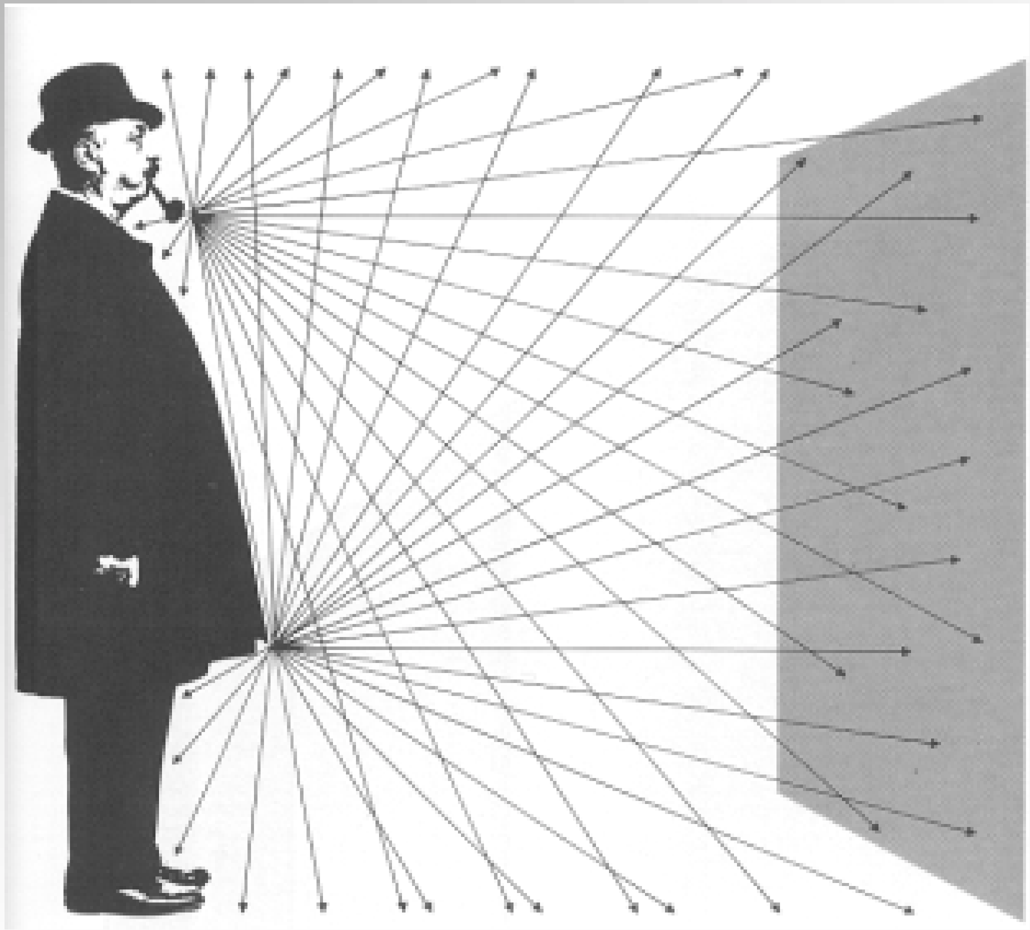
# Image Formation and Camera Design

Spring 2003 CMSC 426

Jan Neumann

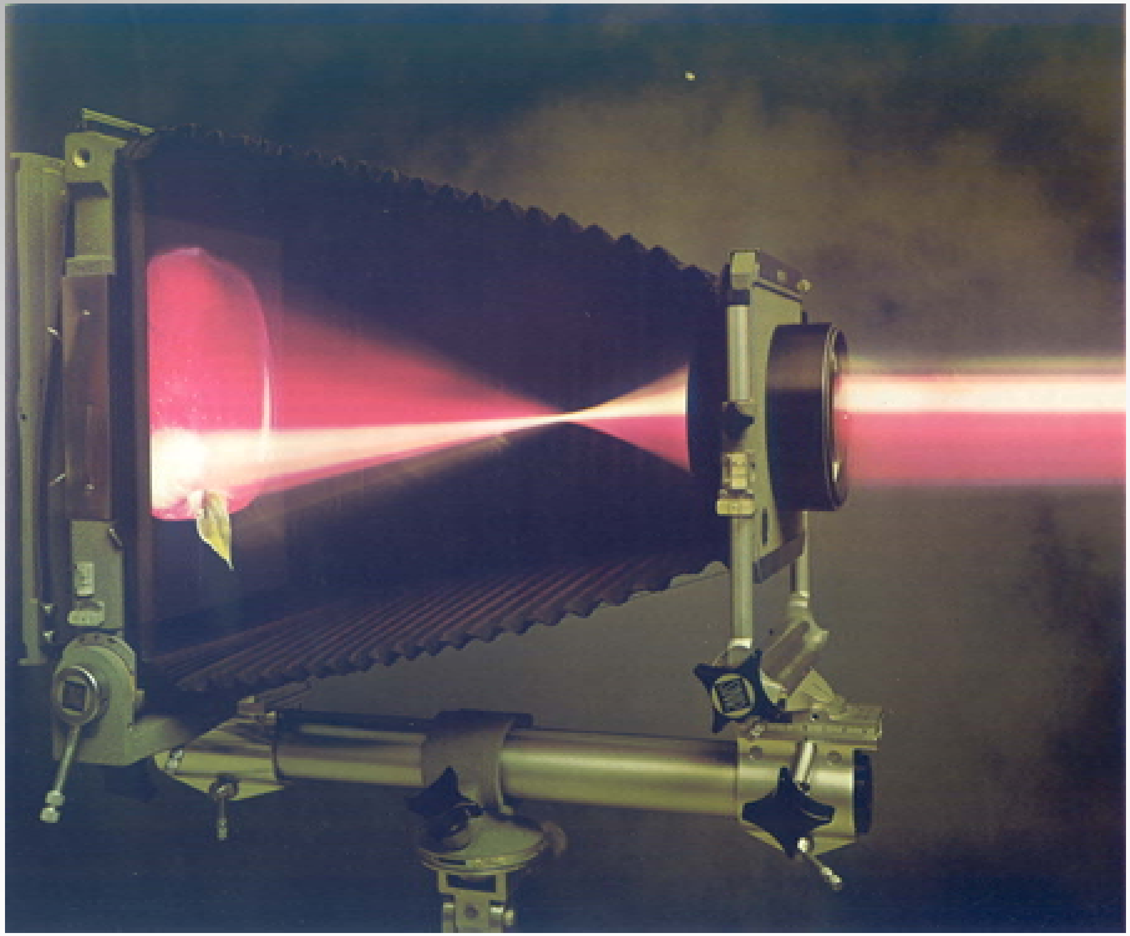
2/20/03

# Light is all around us!



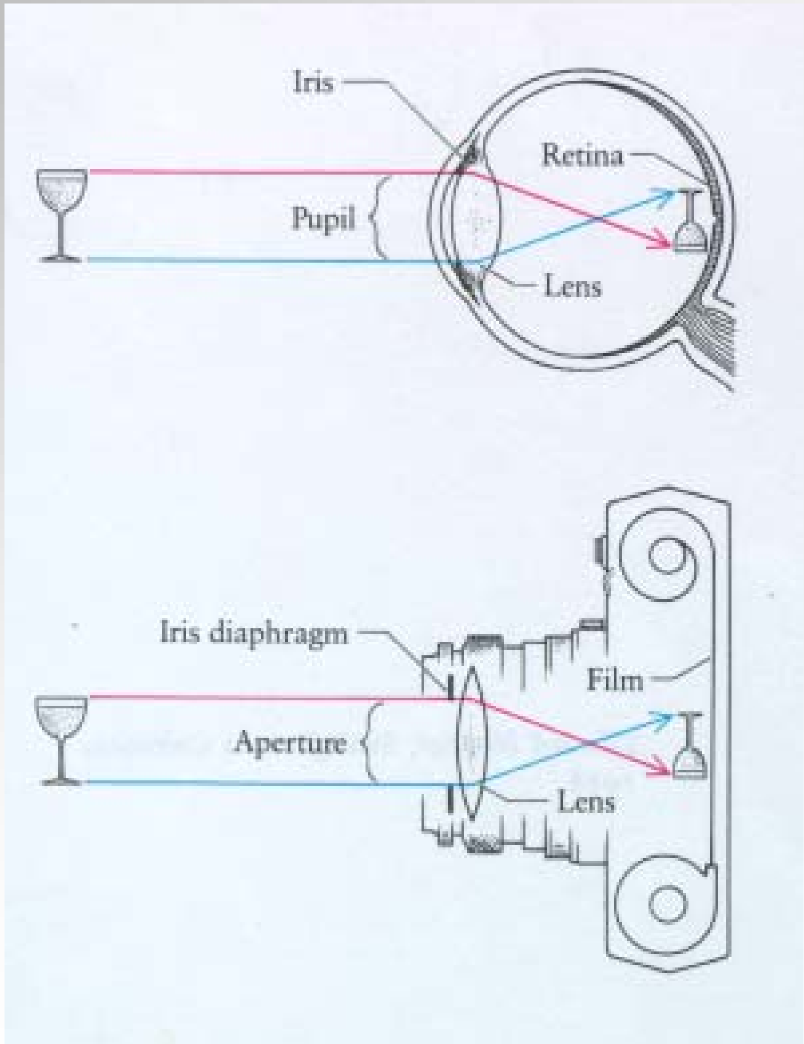
From London & Upton, “Photography”

# Conventional camera design ...



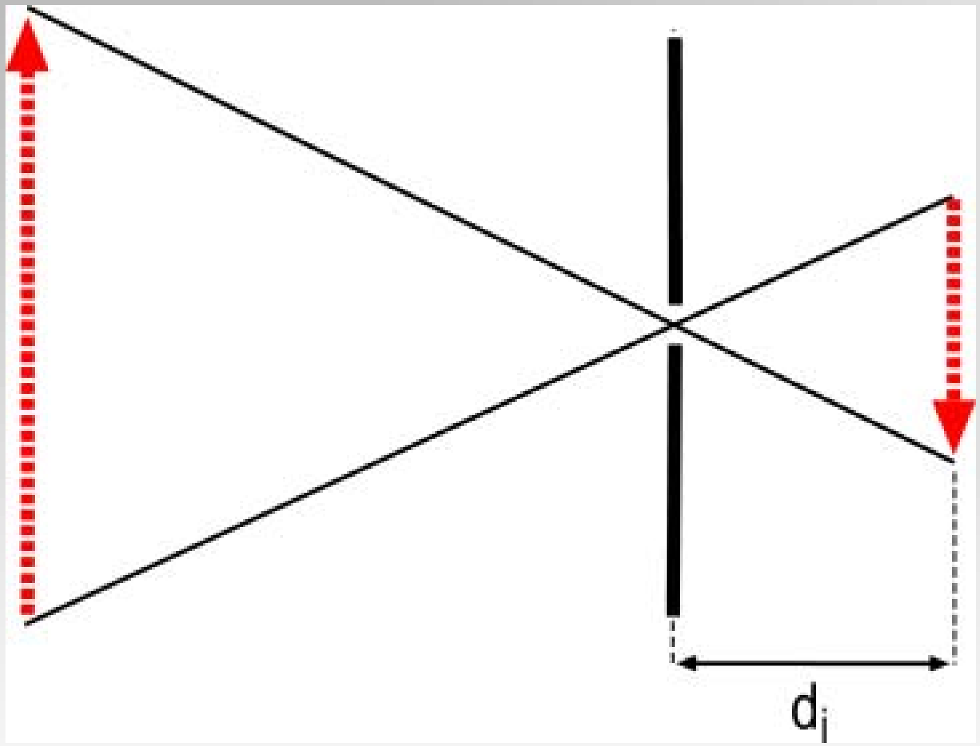
Ken Kay, 1969 in “Light & Film”, TimeLife Publ.

# ... is copying from human eyes!




From I.Rock, "Perception"

# Model: Pinhole Camera



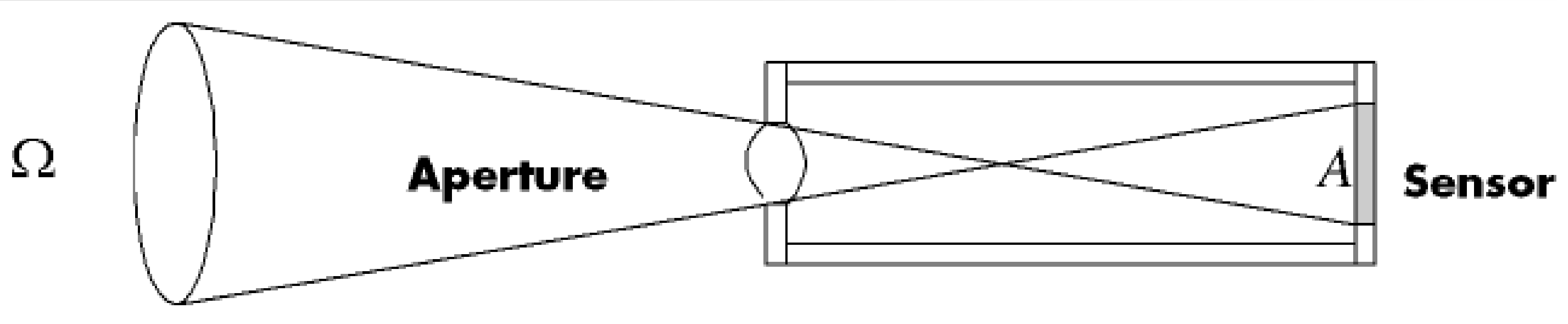

- Infinitely small aperture
- infinite field of depth
- Image is dim
- Projection Equation:

$$\mathbf{p} = \frac{d_i \mathbf{P}}{\hat{\mathbf{z}} \cdot \mathbf{P}}$$

$$\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & \frac{1}{d_i} & 0 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

# Sensor Response

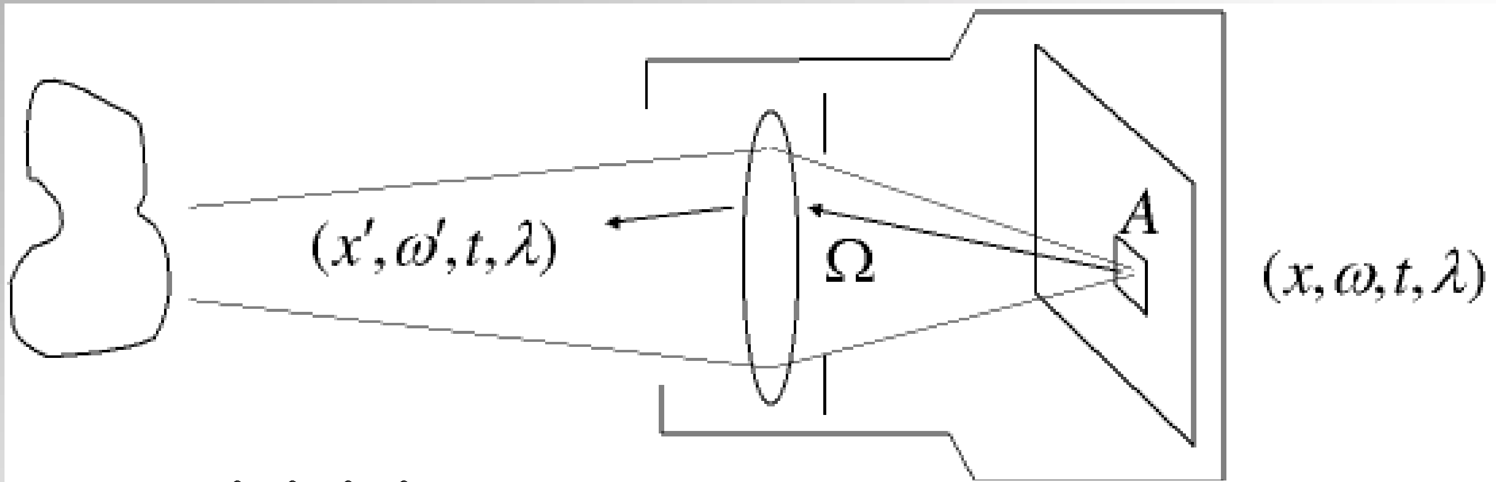
- The response of a sensor is proportional to the radiance visible to the sensor.




$$R = \int_{\Omega} \int_{A} L \, d\omega \, dA = \hat{L} T$$

$$T = \int_{\Omega} \int_{A} d\omega \, dA$$

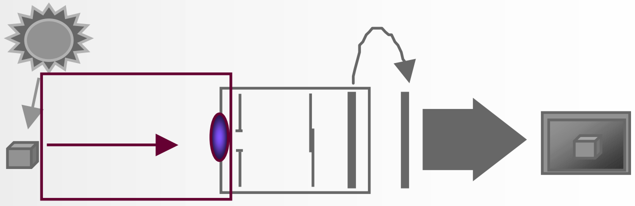
# Measurement Equation

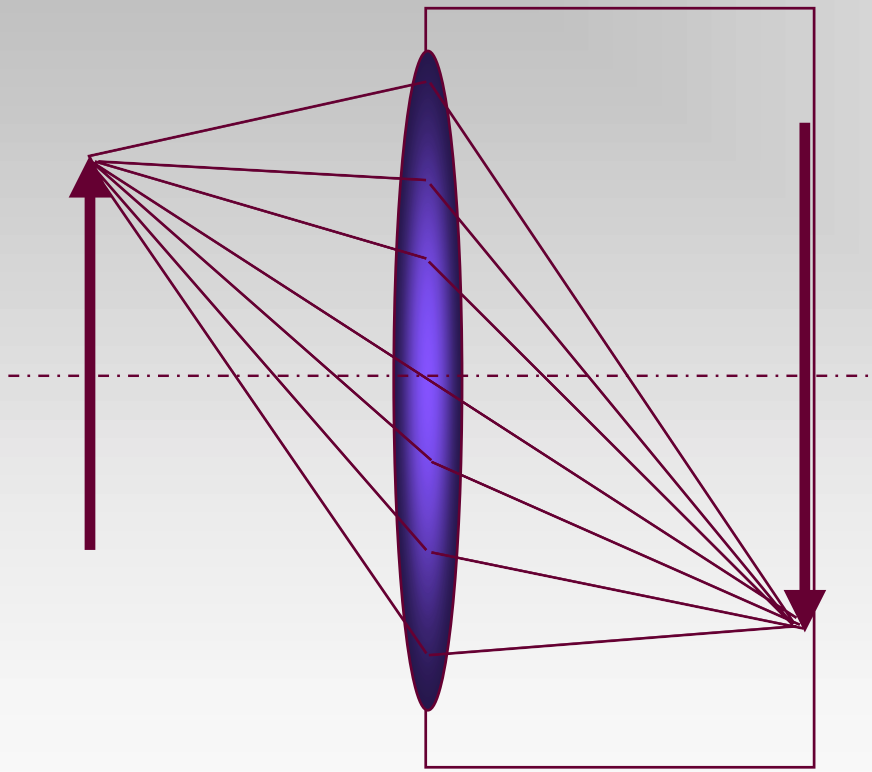



$$I(\mathbf{x}) = \int_A \int_\Omega \int_{T_s} \int_\Lambda L(T(\mathbf{x},\omega,\lambda),t,\lambda) P(\mathbf{x},\lambda) S(\mathbf{x},\omega,t)\, dA\, d\omega\, dt\, d\lambda$$

- Scene Radiance $L(\mathbf{x},\omega,t,\lambda)$
- Optics $(\mathbf{x}',\omega') = T(\mathbf{x},\omega,\lambda)$
- Pixel Response $P(\mathbf{x},\lambda)$
- Shutter $S(\mathbf{x},\omega,t)$

# Collection

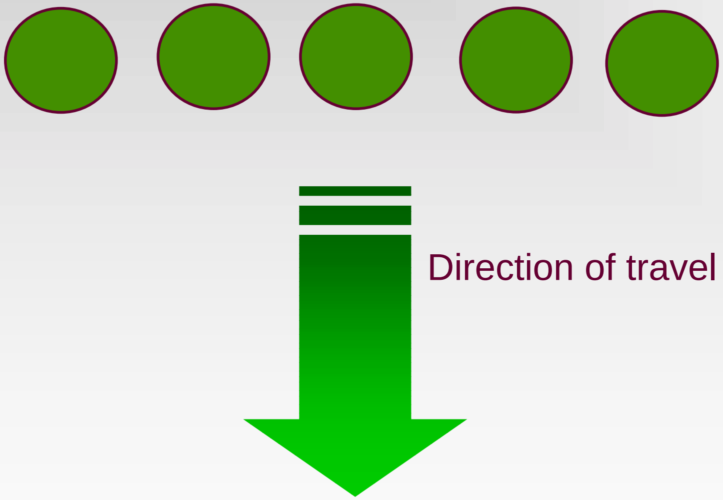




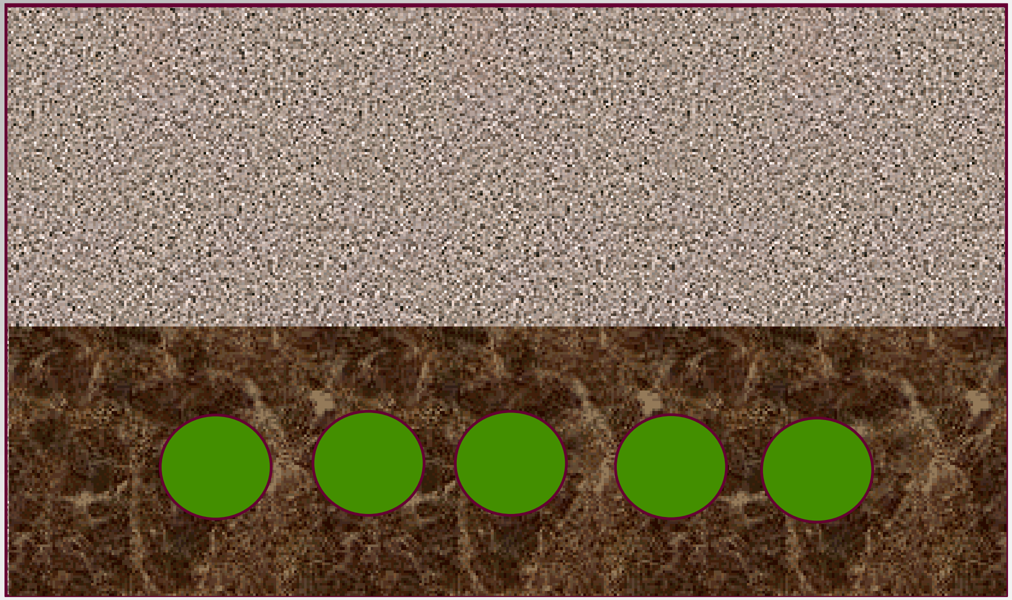
- For a camera to be efficient, the pinhole is replaced by a lens.
- The lens *redirects* light rays emanating from the object.

# Refraction

- Light slows down in materials.
- Imagine a line of marching Girl Scouts . . .



# "Girl Scouts in the Mud"

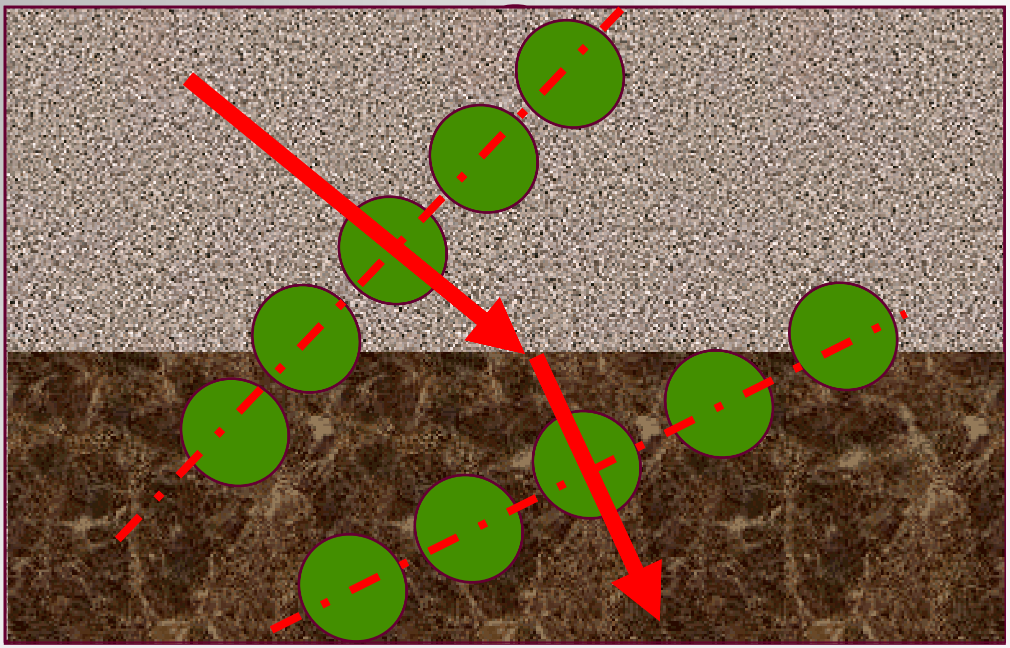


Mud

- As the marching line steps into the mud, they will slow down, depending on how thick the mud is.

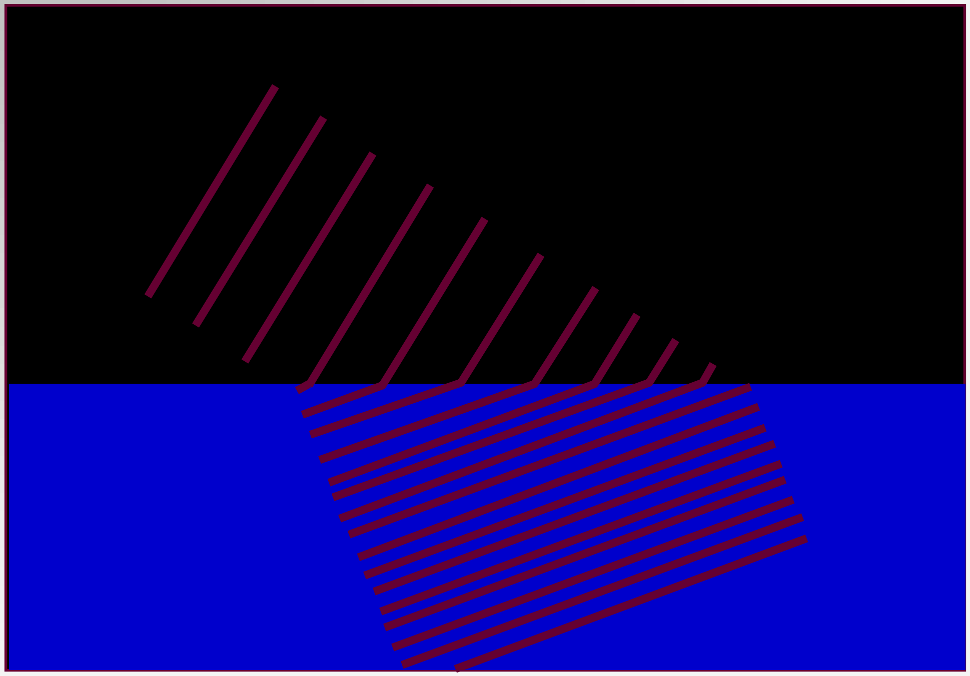
# Wavefronts at Normal Angle of Incidence

# "Girl Scouts in Mud" at an Angle

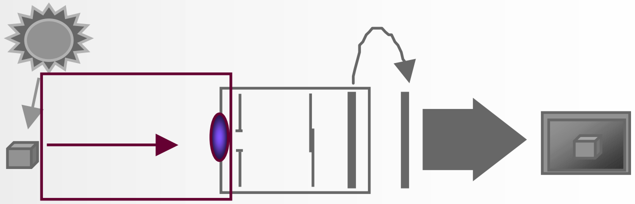


- The direction of travel changes when the marching line hits the mud at a non-normal angle.

# Wavefront at Non-Normal Angle of Incidence



# Index of Refraction

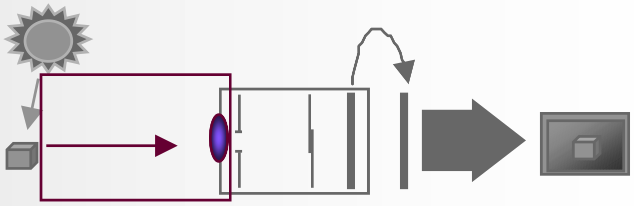


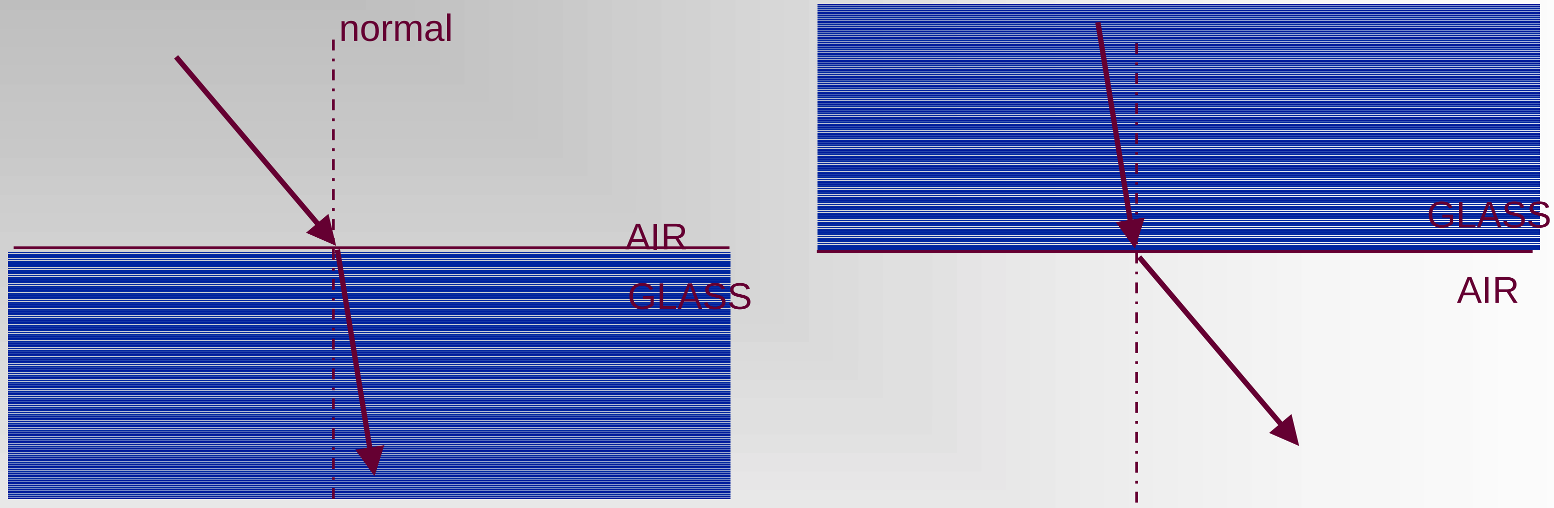
- Index of Refraction (n) is the ratio between the speed of light in vacuum (c) and the speed of light in the medium (v).

$$n = c/v$$

| Medium | Index of Refraction |
|---|---|
| Vacuum | 1 (exactly) |
| Air | 1.0003 |
| Water | 1.33 |
| Glass | 1.5 |
| Diamond | 2.4 |

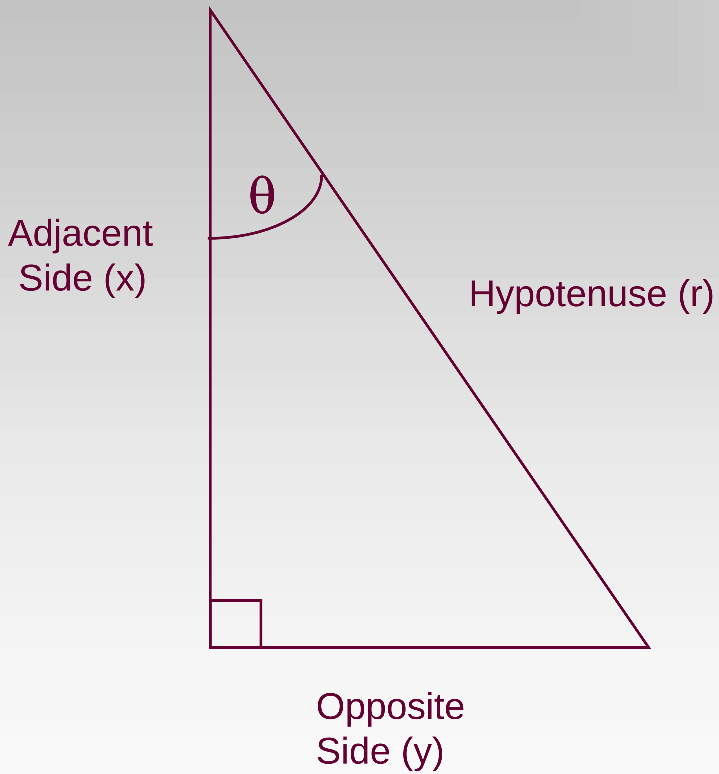
# Snell’s Law






- This change in direction is described by Snell’s Law

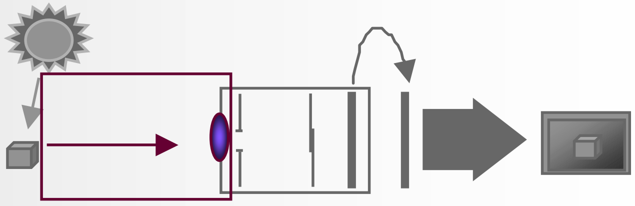
# Trigonometry Review

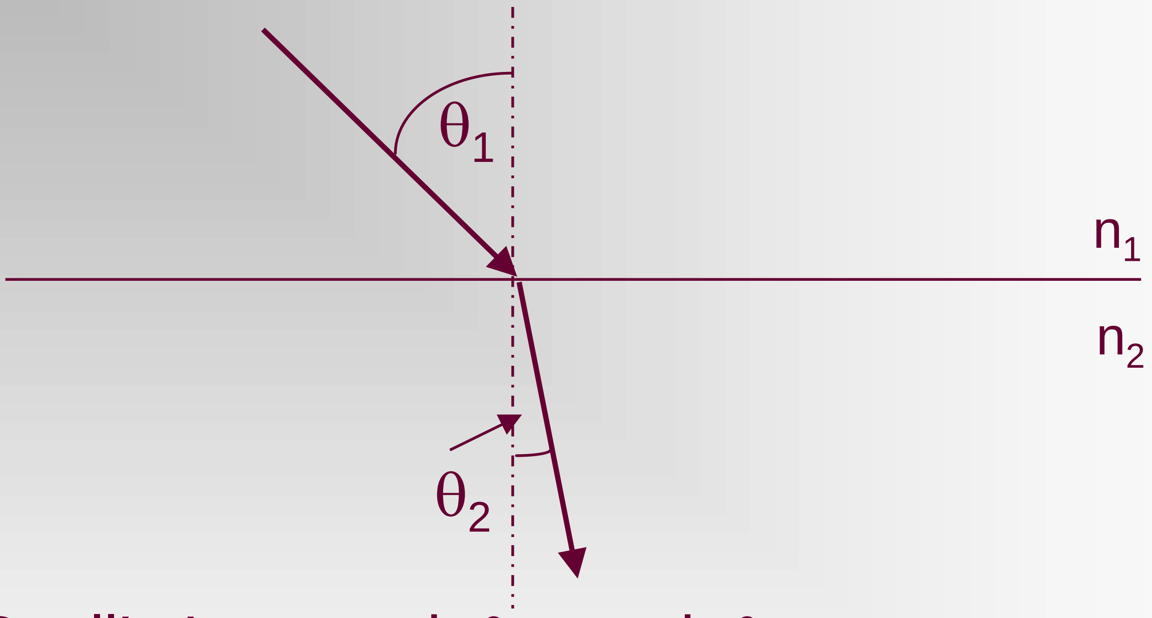



RULES THAT DEFINE SIN, COS, TAN of an ANGLE:

- sin(θ) = y/r (opp/hyp)
- cos(θ) = x/r (adj/hyp)
- tan(θ) = y/x (opp/adj)

# Snell's Law

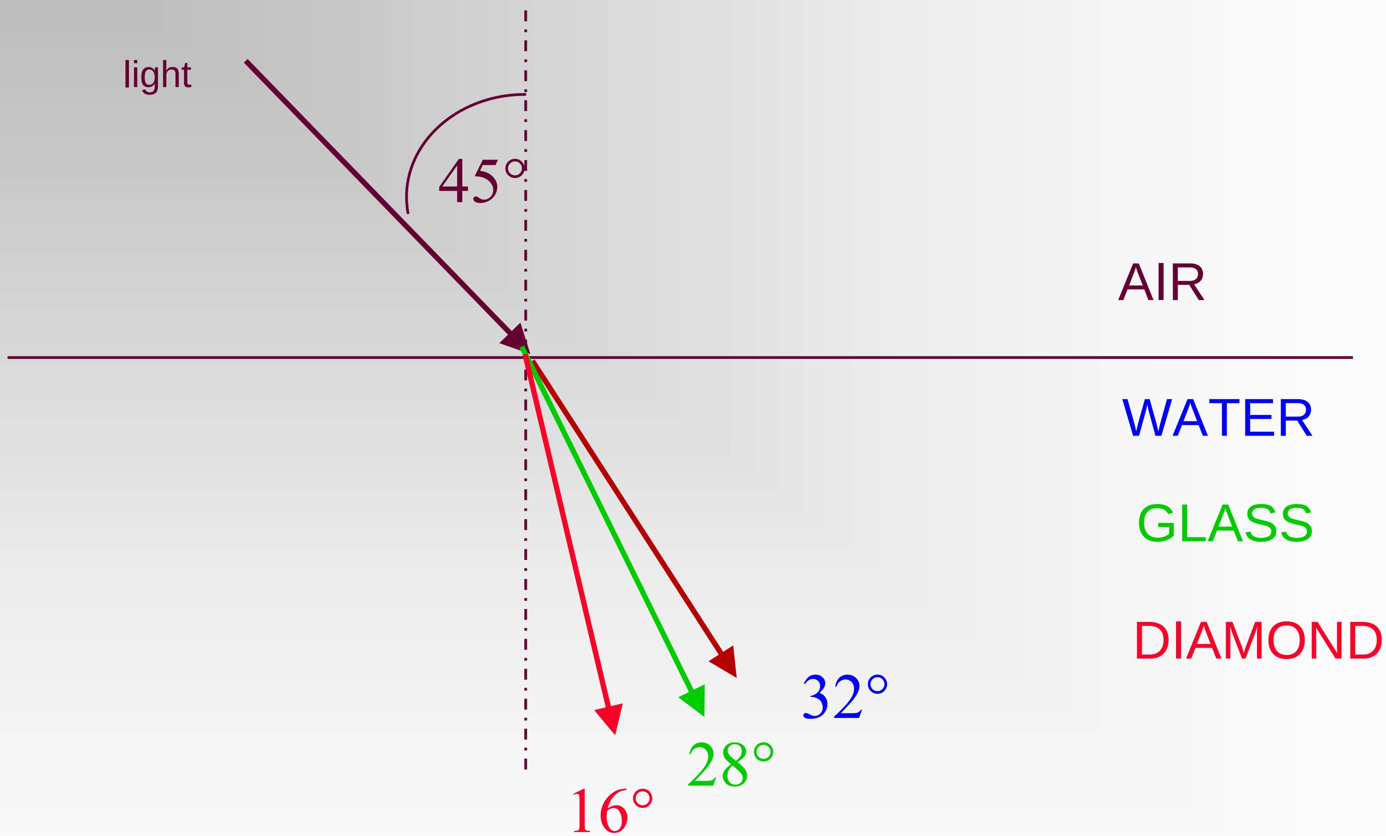




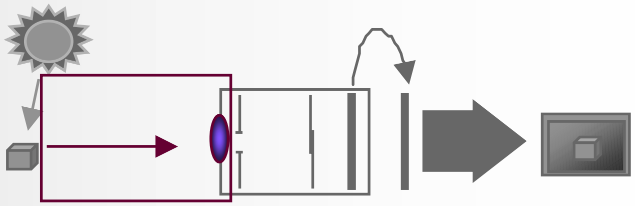

Snell's Law: $n_1 \sin\theta_1 = n_2 \sin\theta_2$

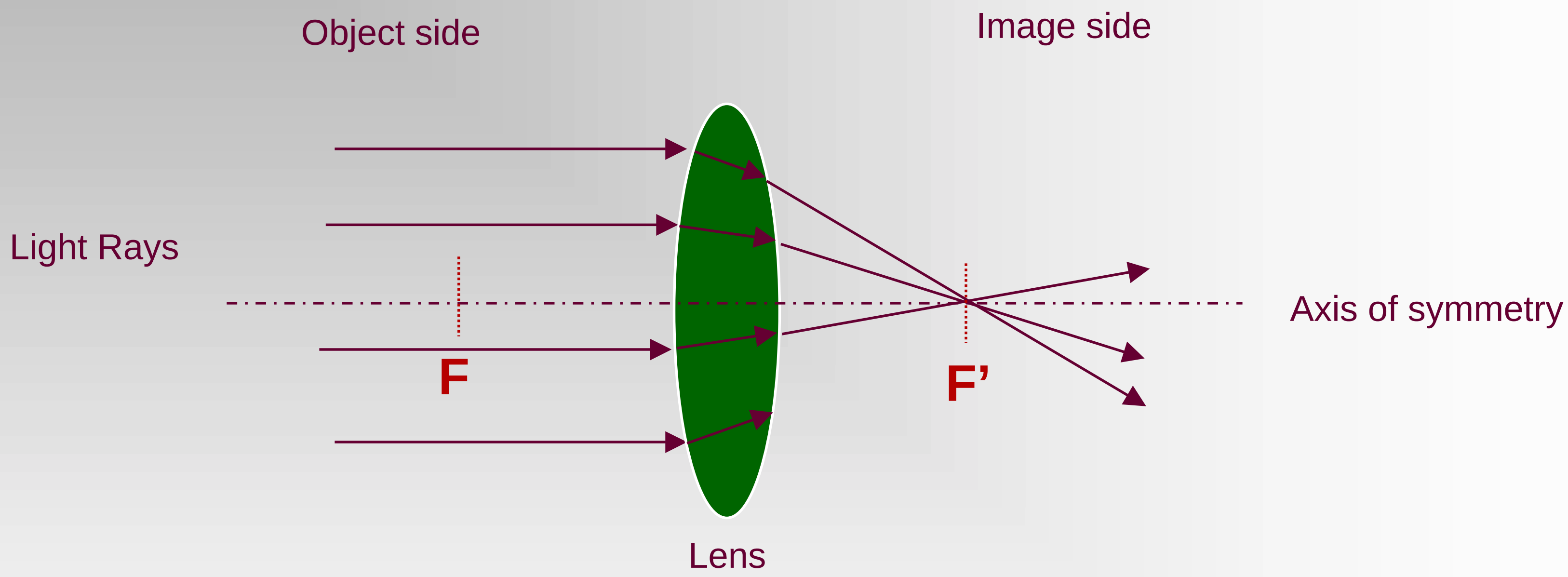
If $\theta_1$ and $\theta_2$ are small use 1st order approximation
$n_1\theta_1 = n_2\theta_2$ (known as paraxial raytracing)

# Refraction for Different Materials



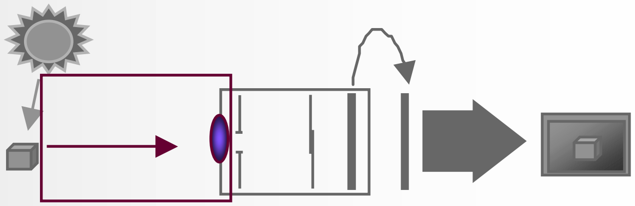
# Convex Lens

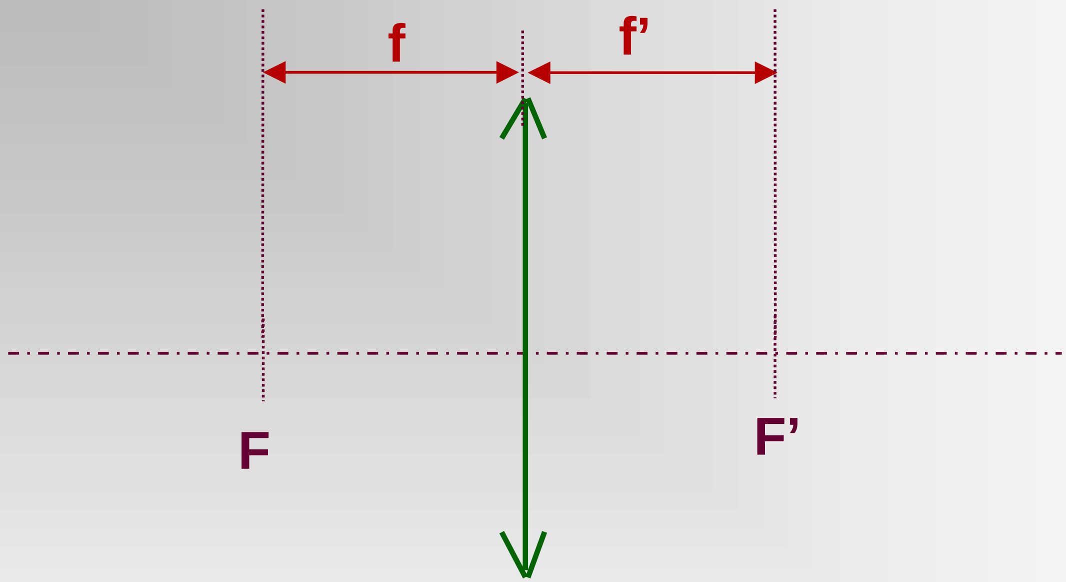





- Image focal point, F’, is half the distance to the effective center of curvature of the lens.
- Object focal point, F, is exactly the same distance on the object side of the lens.

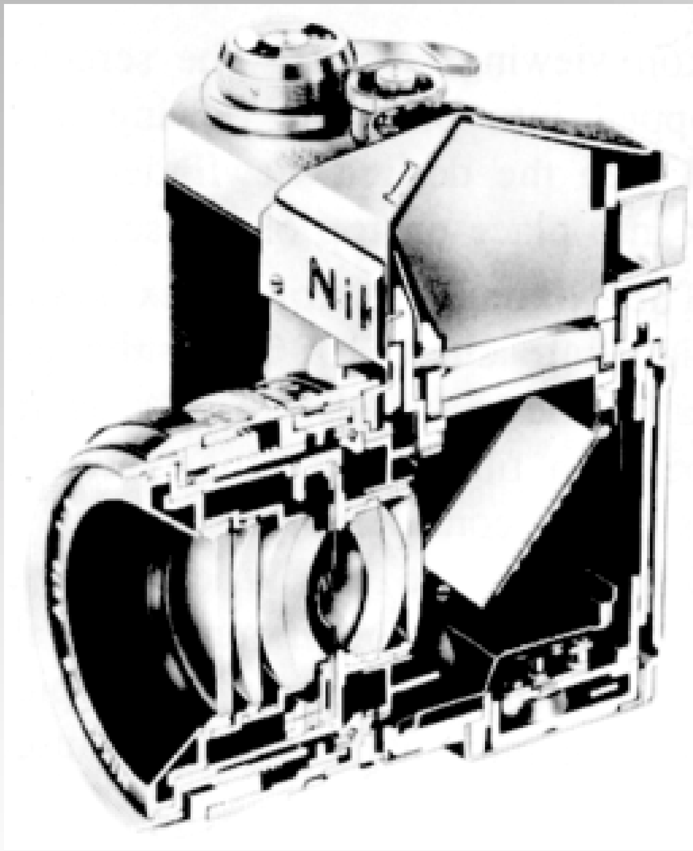
# Convex Lens





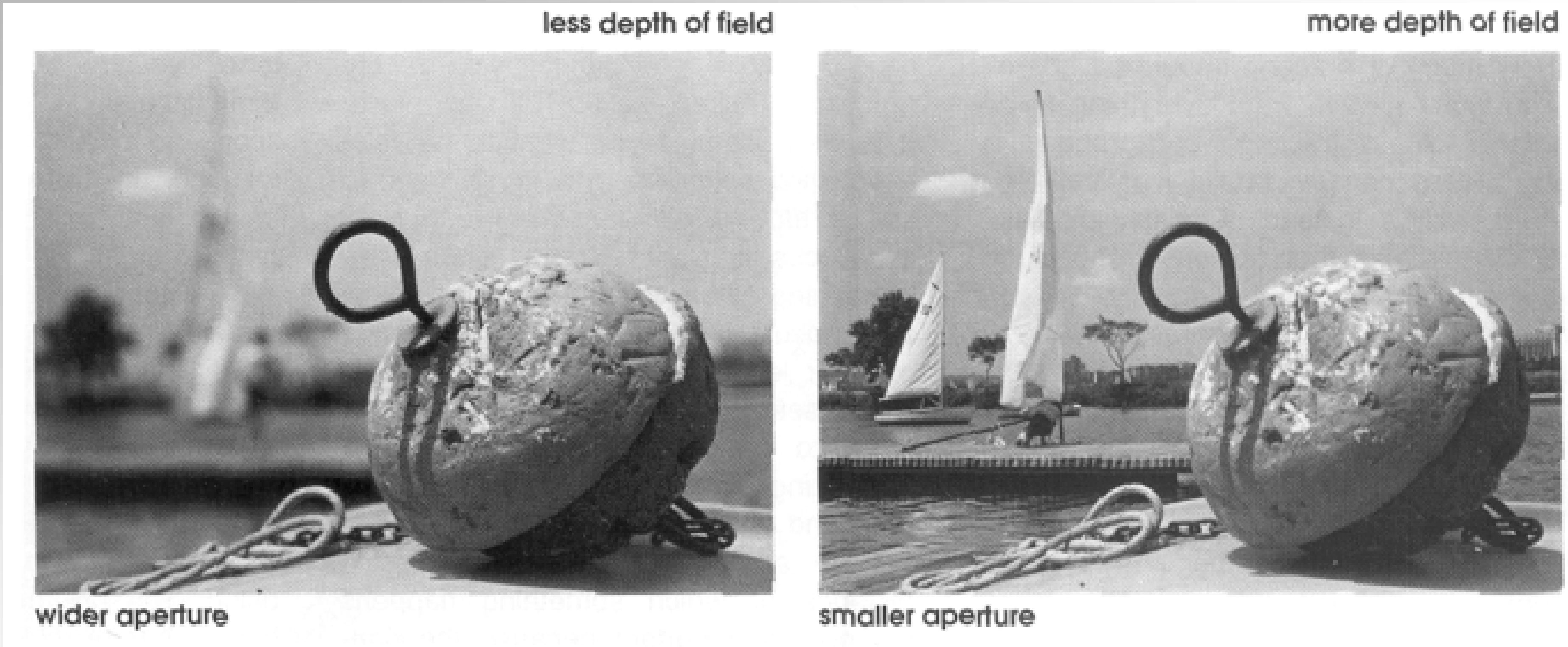

- Image focal length, f’, is the distance from the lens to the image focal point.
- Object focal length, f, is the distance from the lens to the object focal point.

# Camera Parameters



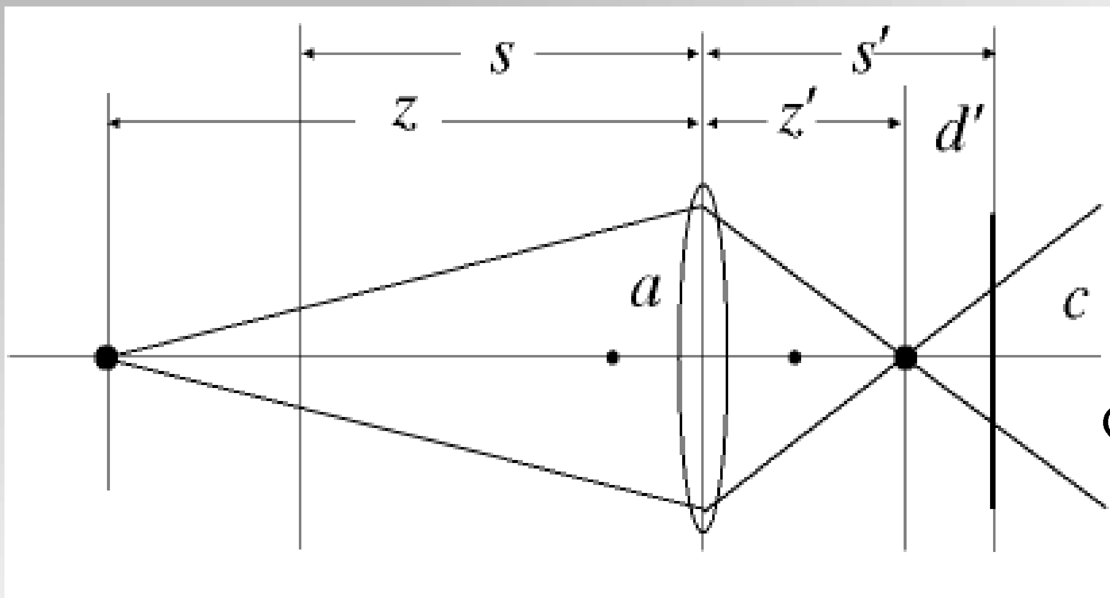
- Field of view (film size, stops and pupils)
- Depth of field (aperture, focal length)
- Motion blur (shutter)
- Exposure (film speed, aperture, shutter)

# Depth of Field




- From London and Upton

# Circle of Confusion

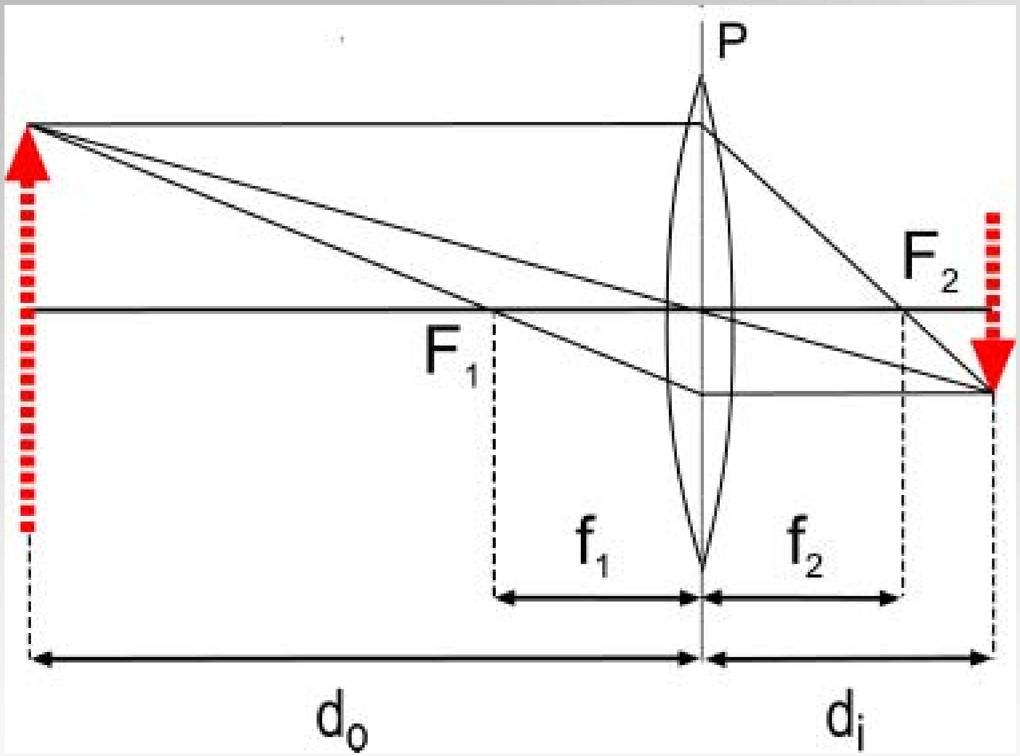



In Focus: $\frac{1}{s'} = -\frac{1}{s} + \frac{1}{f}$

Out-of Focus: $\frac{1}{z'} = -\frac{1}{z} + \frac{1}{f}$

Circle of Confusion: $\frac{c}{a} = \frac{d'}{z'} = \frac{s' - z'}{z'}$

# Thin Lens Camera




- Use lens to capture more light
- Limited Depth of Field
- Thin Lens Eq.:

$$\frac{1}{d_0} + \frac{1}{d_i} = \frac{1}{f}$$

- Circle of Confusion:

$$c = a \cdot d_i \left( \frac{1}{f} - \frac{1}{d_i} - \frac{1}{d_0} \right)$$

# Thin Lens Camera

- Thin Lens Eq.:

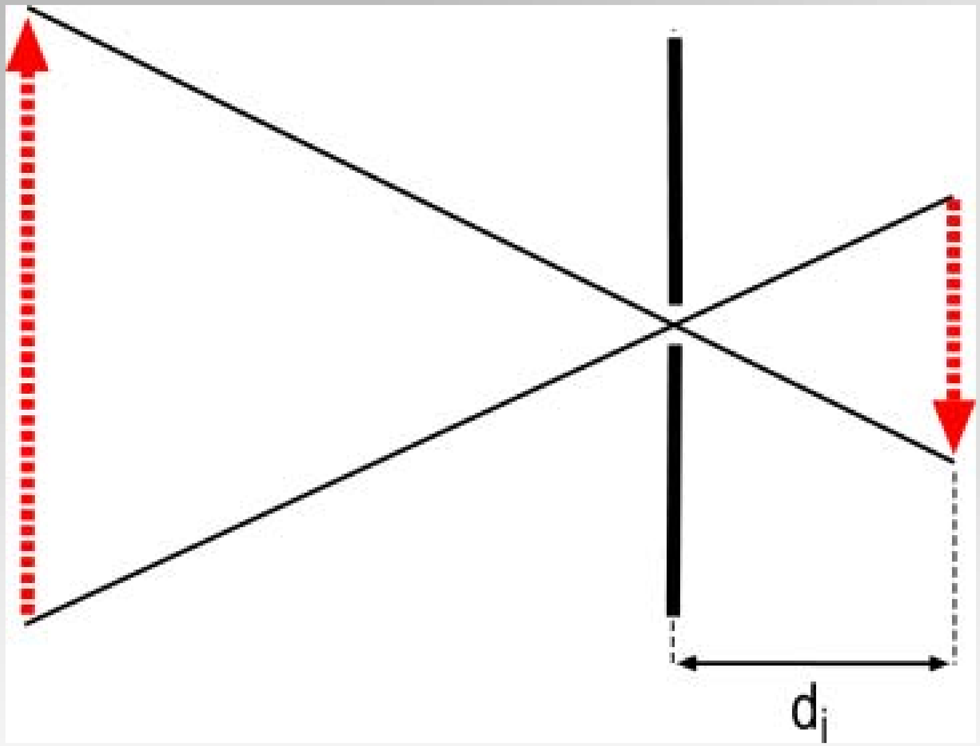
$$\frac{1}{d_0} + \frac{1}{d_i} = \frac{1}{f}$$

- Projection Eq.:

$$\frac{1}{d_0} + \frac{1}{d_i} = \frac{1}{f} \rightarrow d_i = \frac{f d_0}{d_0 - f}$$
$$\rightarrow x_i = \frac{f x}{d_0 - f}$$

$$\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & \frac{1}{f} & -1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

# Model: Pinhole Camera




- Infinitely small aperture
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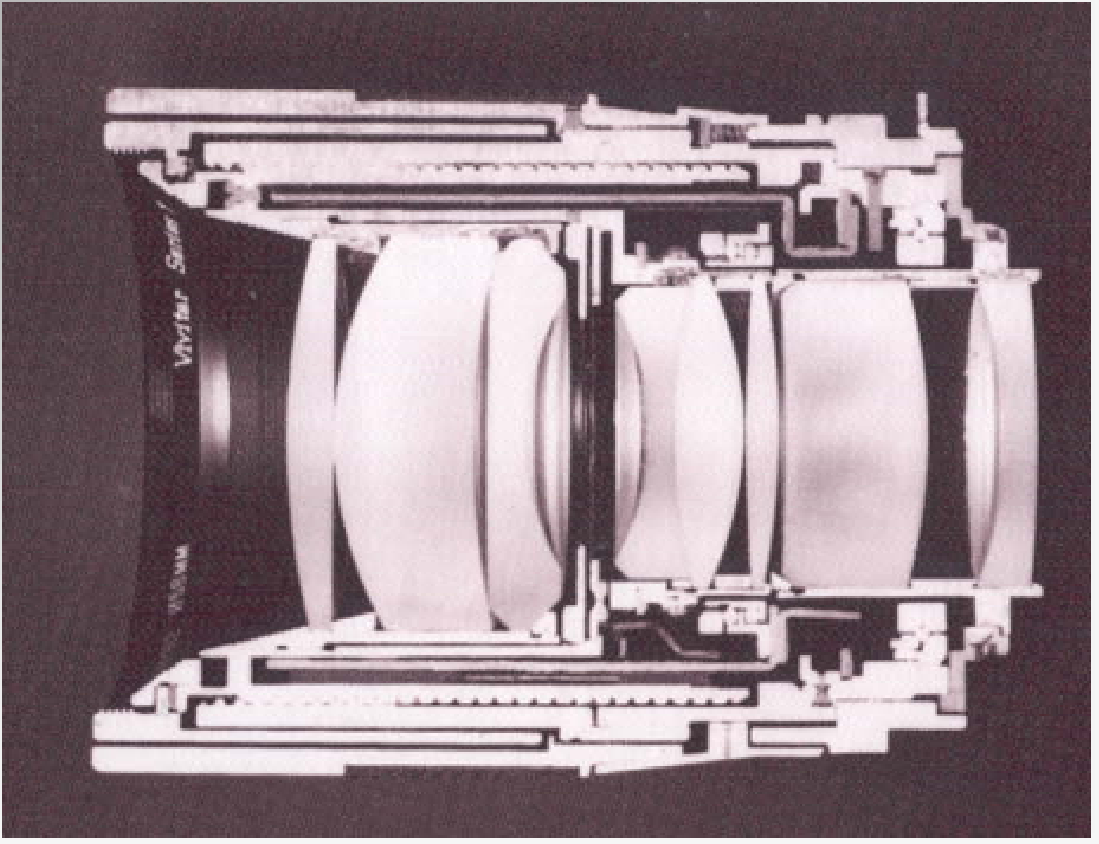
- Projection Equation:

$$\mathbf{p} = \frac{d_i \mathbf{P}}{\hat{\mathbf{z}} \cdot \mathbf{P}}$$

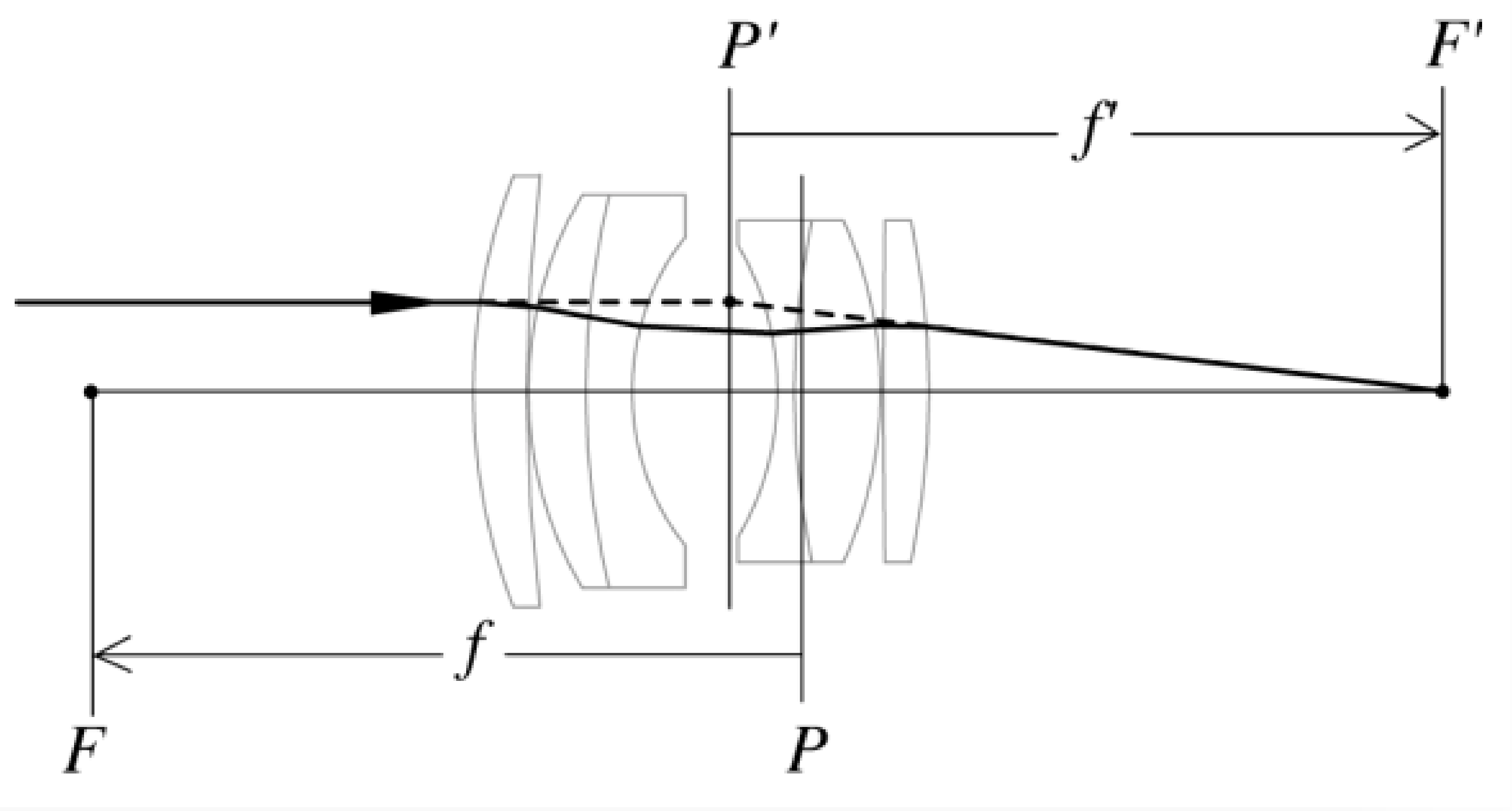
$$\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & \frac{1}{d_i} & 0 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

# Thick Lens



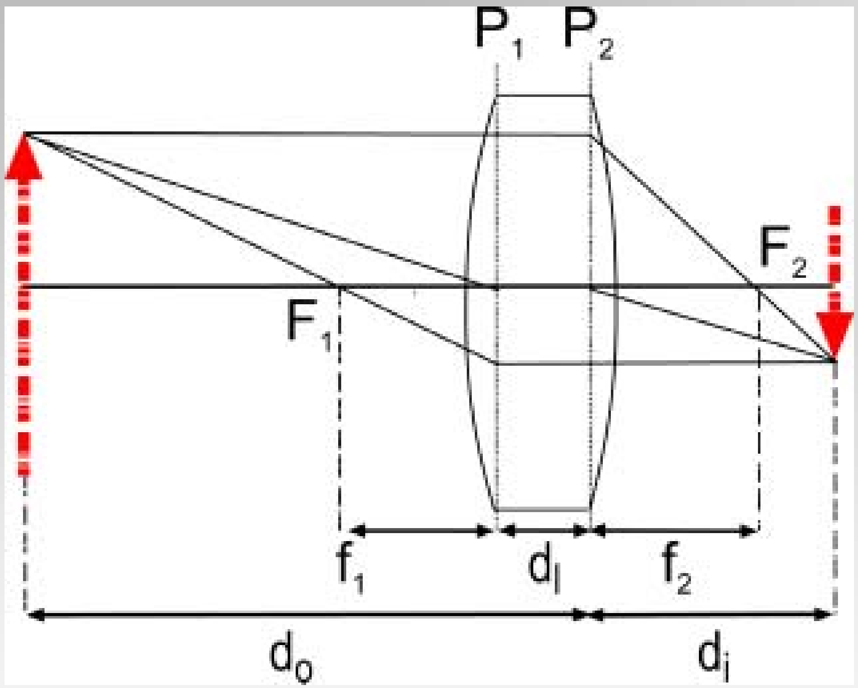
- Cutaway section of a Vivitar Series 1 90 mm f 1/2.5 lens
  Cover photo, Kingslake, *Optics in Photography*

# Thick Lens




Focal lengths are measured with respect to principal planes

# Thick Lens Camera



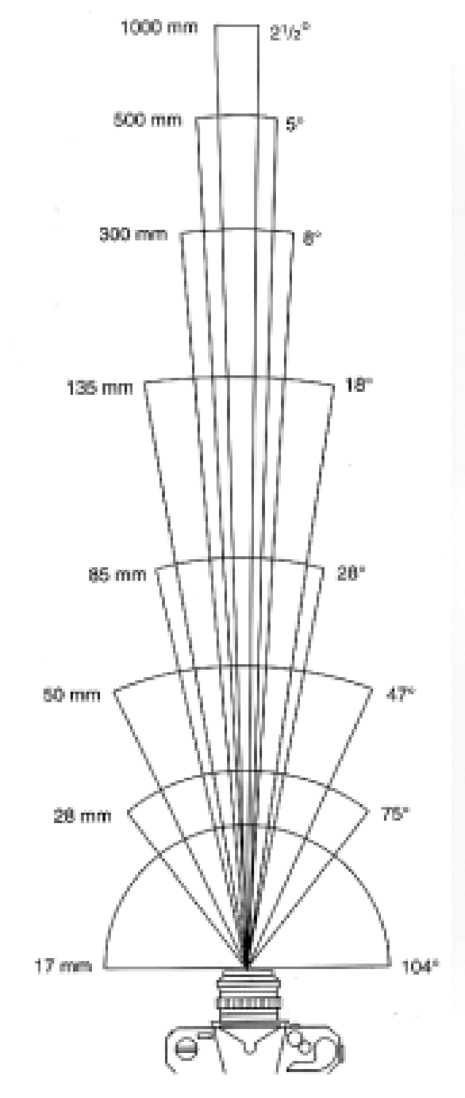

- A real lens has thickness

$$\frac{1}{d_0 - d_l} + \frac{1}{d_i} = \frac{1}{f}$$

$$\rightarrow d_i = \frac{f(d_o - d_l)}{d_o - d_l - f}$$

$$\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -d_l \\ 0 & 0 & \frac{1}{f} & -1 - \frac{d_l}{f} \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

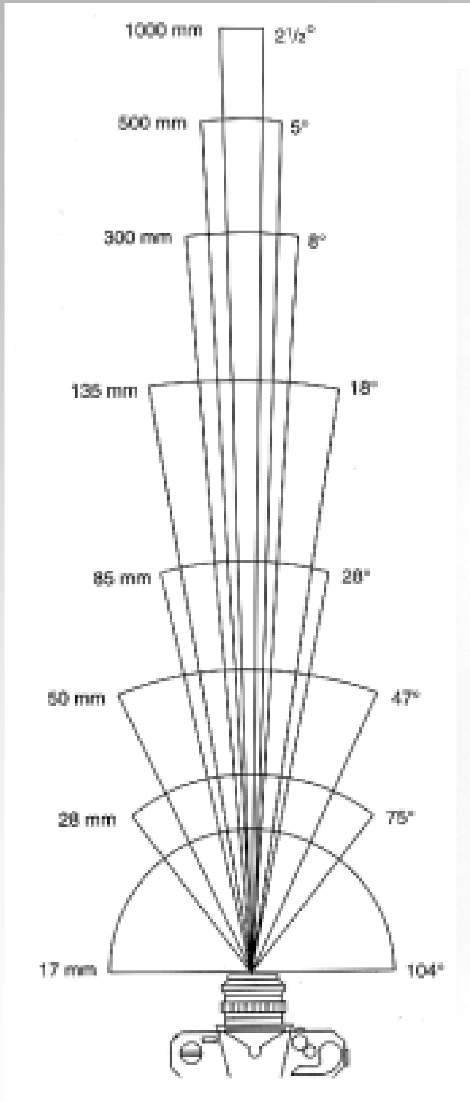
# Field of View

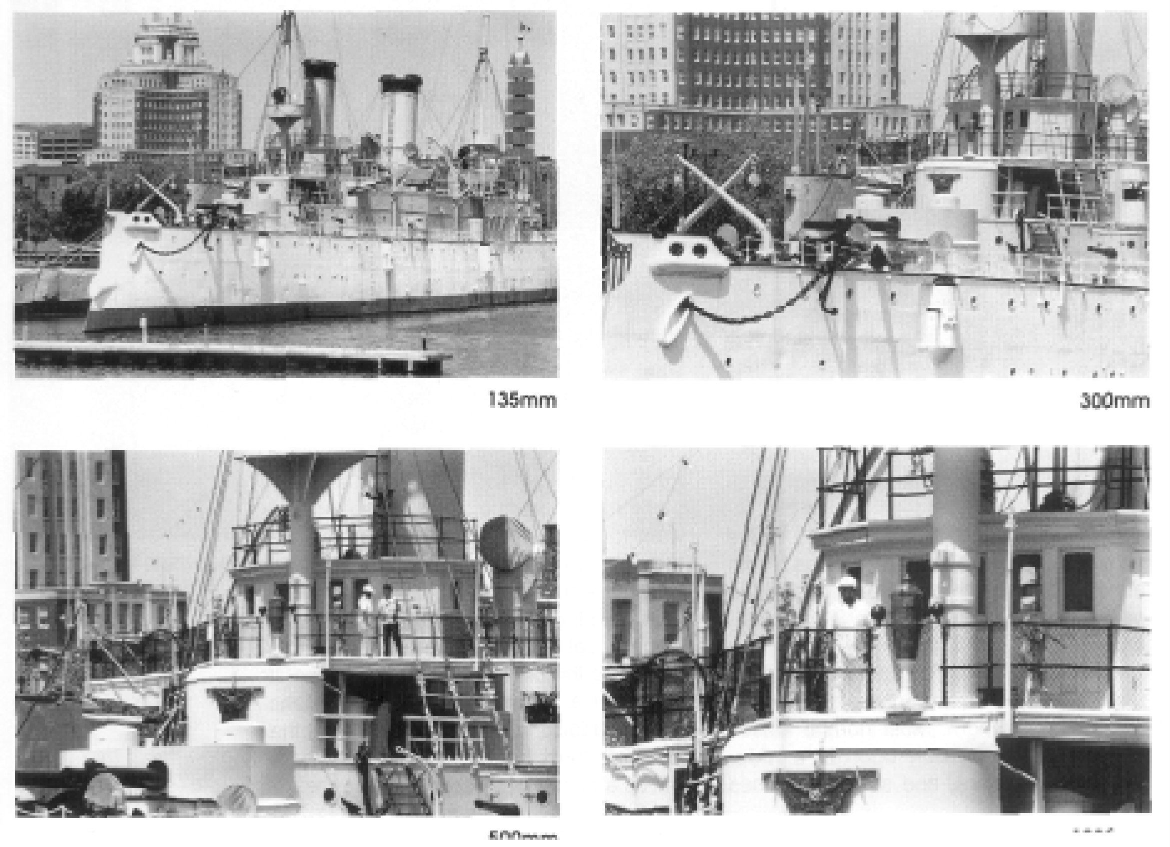




- From London and Upton

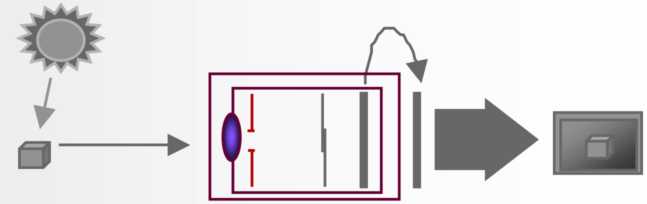
# Field of View

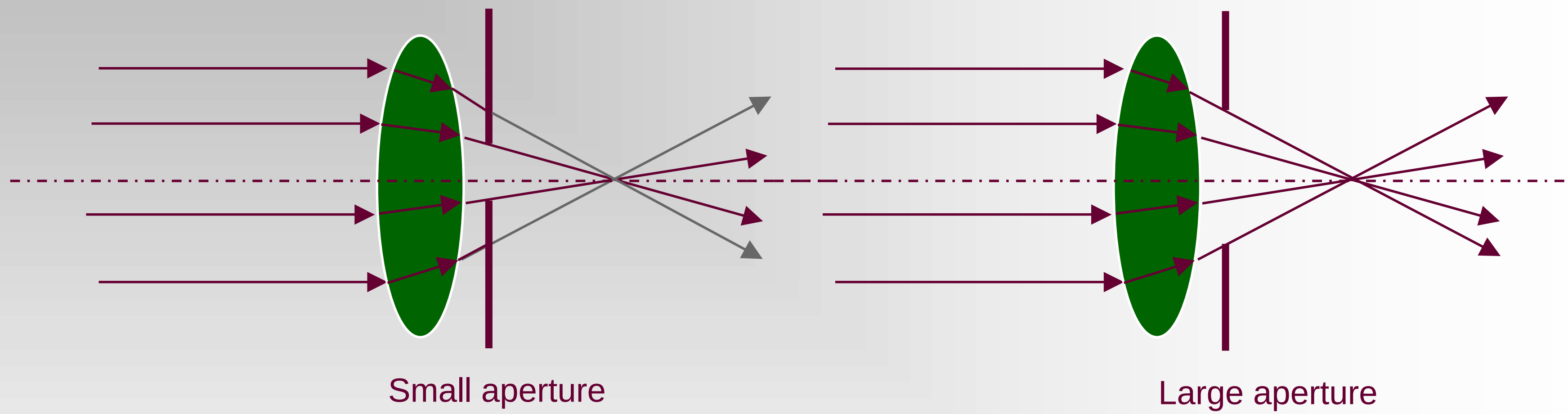



- From London and Upton

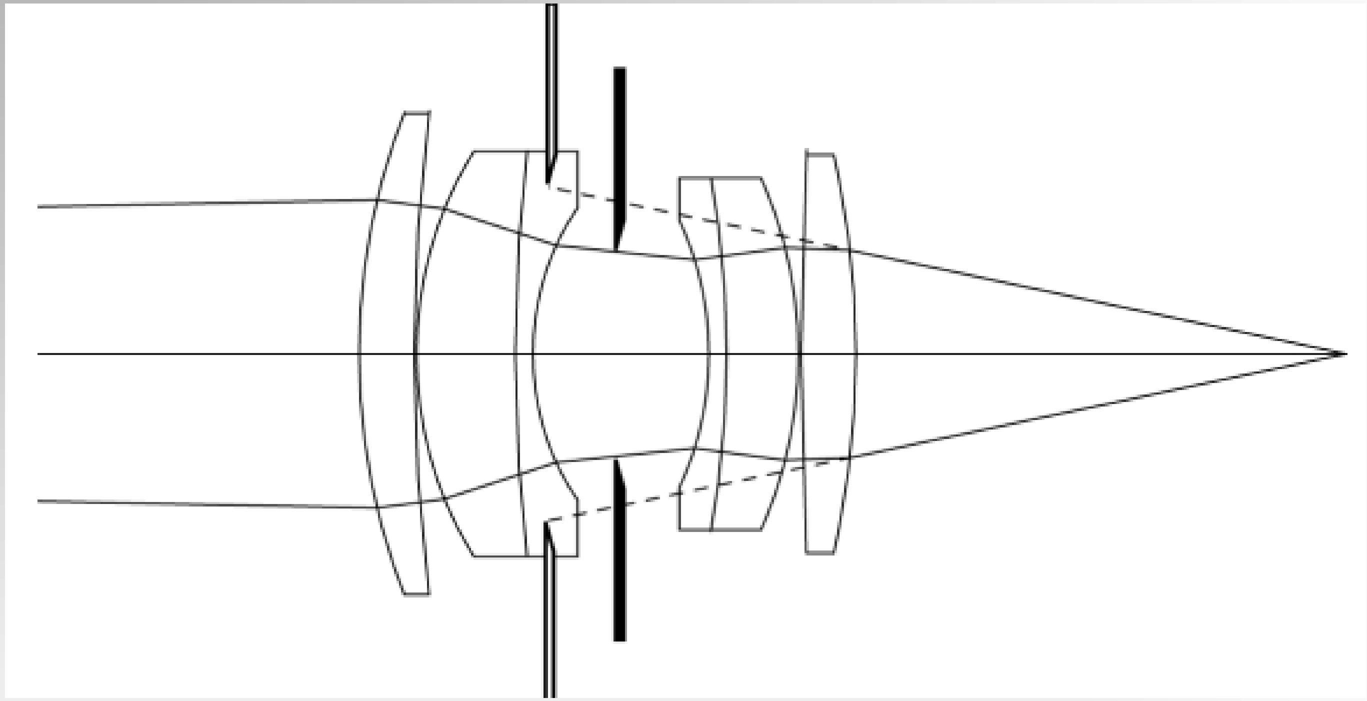
# Illuminance





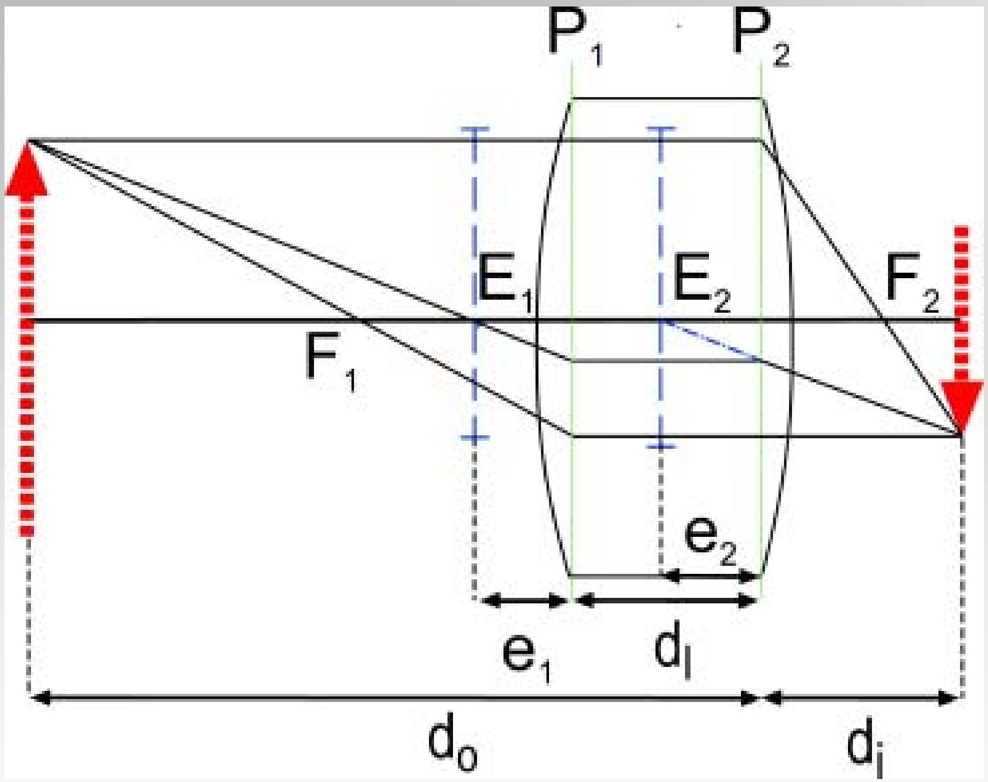

- Illuminance is the rate of light falling on a given area (i.e. energy per unit time).
- Illuminance is controlled by *aperture*: a larger aperture brings more light to the focus.

# Aperture



- Stops – physical limits
- Pupils – logical limits for entering and exiting rays

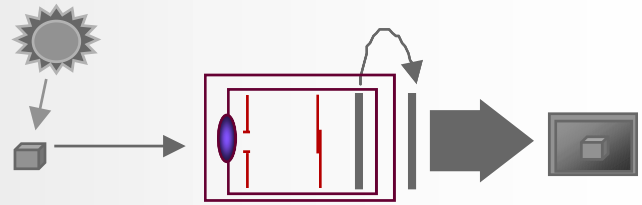
# Aperture

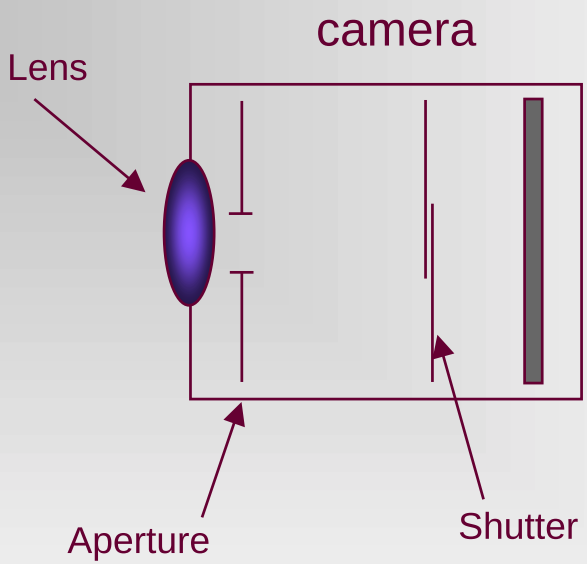



$$\mathbf{p} = \frac{-e_1}{d_o - d_l - e_1} \frac{d_i + e_2}{e_2} \mathbf{P}$$

$$\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -d_l \\ 0 & 0 & \frac{1}{f} & -1 - \frac{d_l}{f} \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

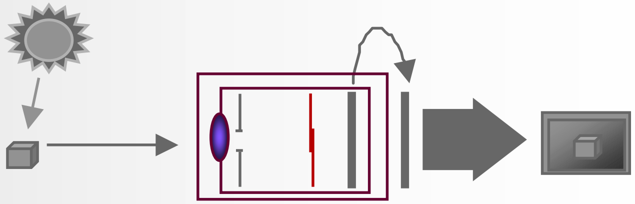
# Exposure

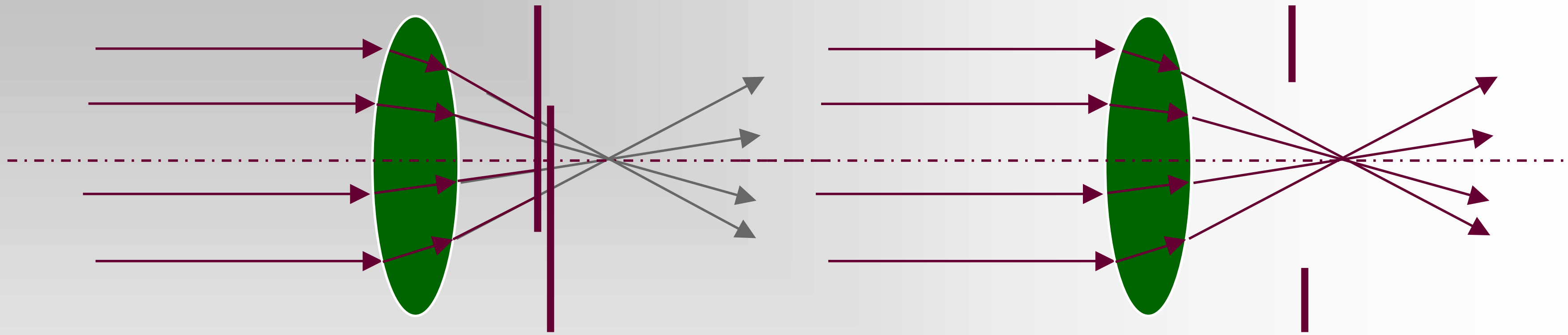





- Exposure is defined as the total amount of light falling on the film.
- Exposure = Illuminance * Time

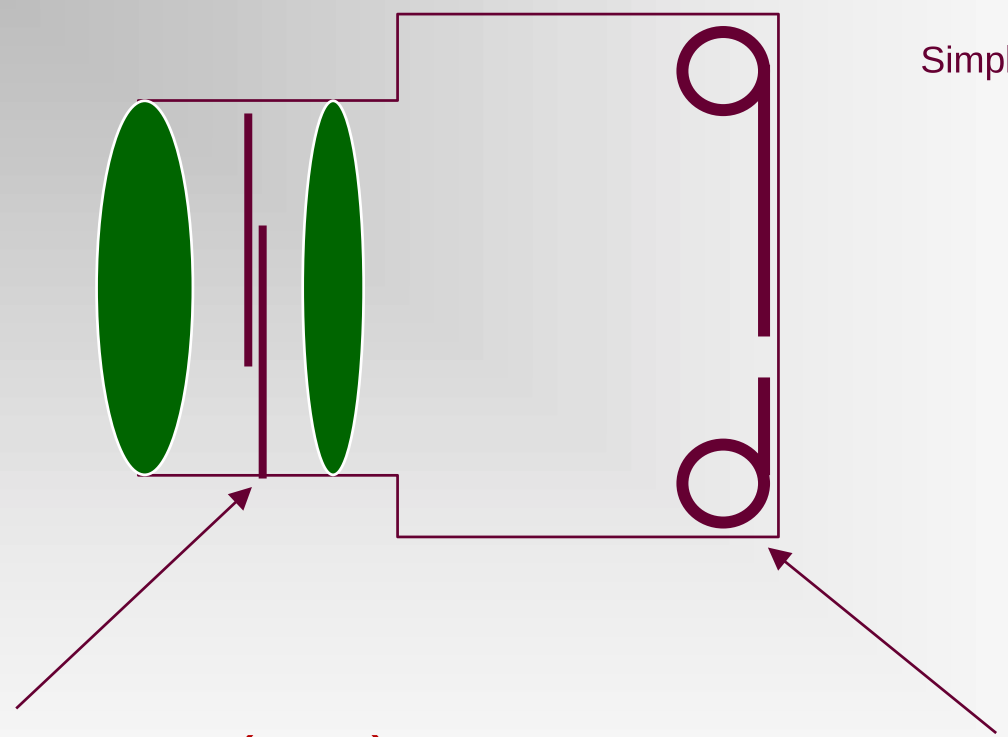
# Exposure Time



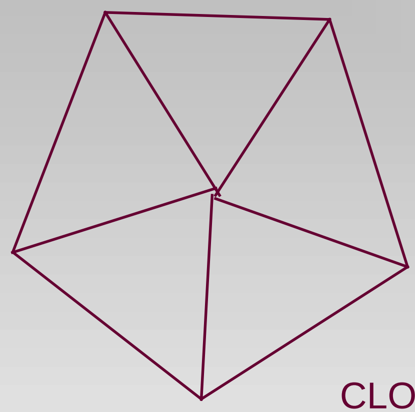



- Exposure time is controlled by the *shutter*: when closed, the film is not exposed to light.
- Exposure time is simply the time interval between opening and closing the shutter.

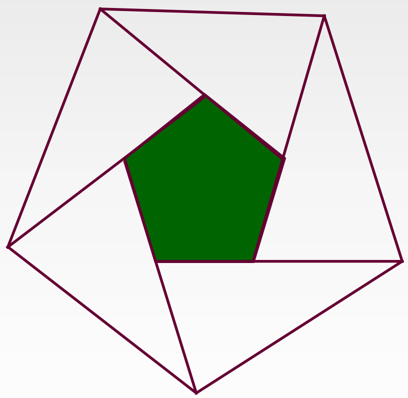
# Types of Shutters



# BTL or Leaf Shutter



CLOSED



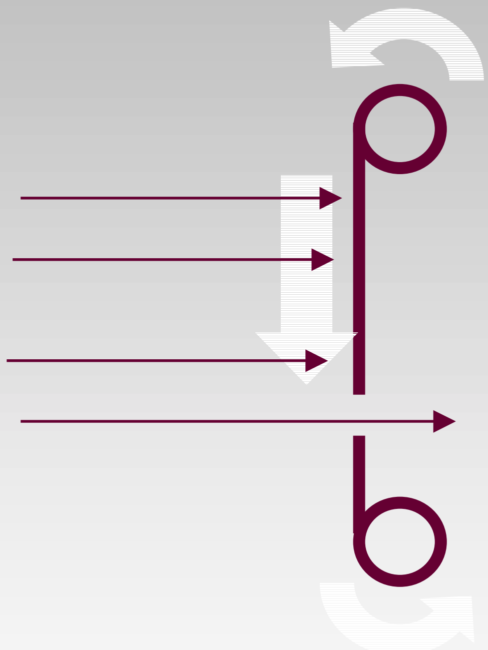
OPEN

- Made of overlapping "leaves" that slide out of the way when shutter opens.
- Located between the imaging lens elements.

# BTL or Leaf Shutter

- Advantages
  - Uniform illumination independent of film size.
  - Entire film frame illuminated at once.
- Disadvantages
  - Illumination of frame not constant over time.
  - Limitations on shutter speed.

# Focal Plane Shutter



- Metal or fabric with a narrow slit opening which traverses the area to be exposed.
- Located just before the detector (film) at the focal plane.

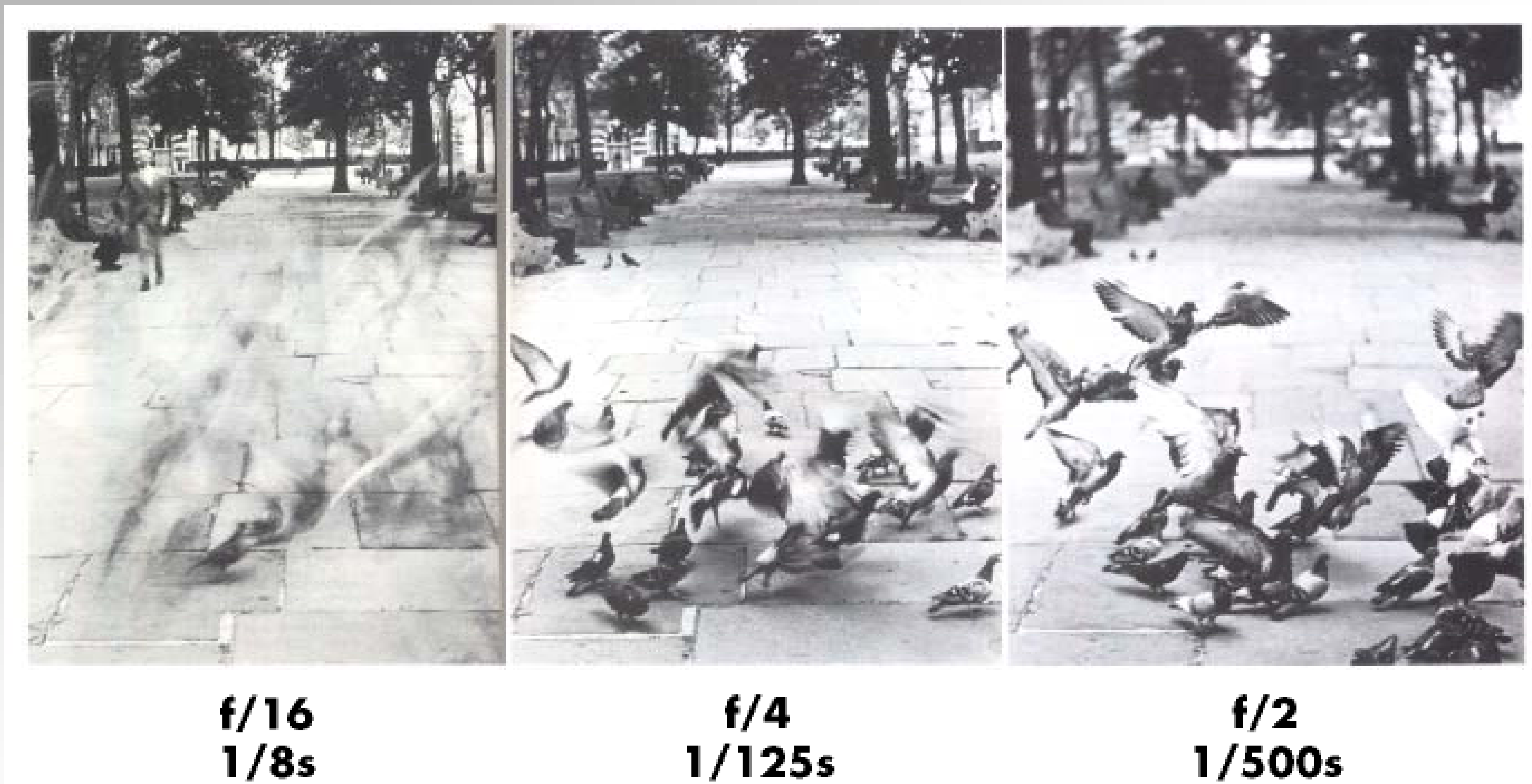
# Focal Plane Shutter

- Advantages
  - Cost effective (one shutter needed for all lenses - great for interchangeable lens systems)
  - Can achieve very fast shutter speeds (~1/10000 sec)
- Disadvantages
  - May cause time distortion if the film size is large (since the shutter slit must traverse the film)

# Why control exposure with aperture and shutter?

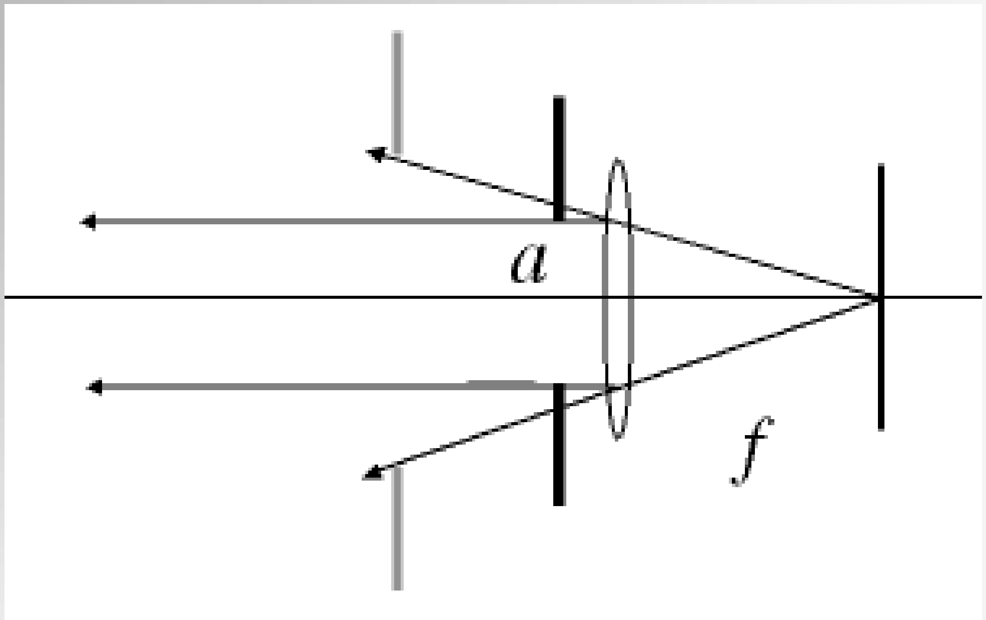
- Flexibility!
  - Fast shutter speed for freezing action (e.g. sports photography).
  - Slow shutter speed for low light levels (e.g. sunsets).
  - Small aperture for bright scenes or to enable longer exposures.
  - Large aperture for low light conditions (taking candle lit or moon lit pictures).

# Aperture vs Shutter




- From London and Upton

# Image Irradiance and Exposure




$$E = L\pi \sin^2\theta = L\frac{\pi}{4}\left(\frac{a}{f}\right)^2 = L\frac{\pi}{4}\frac{1}{N^2}$$

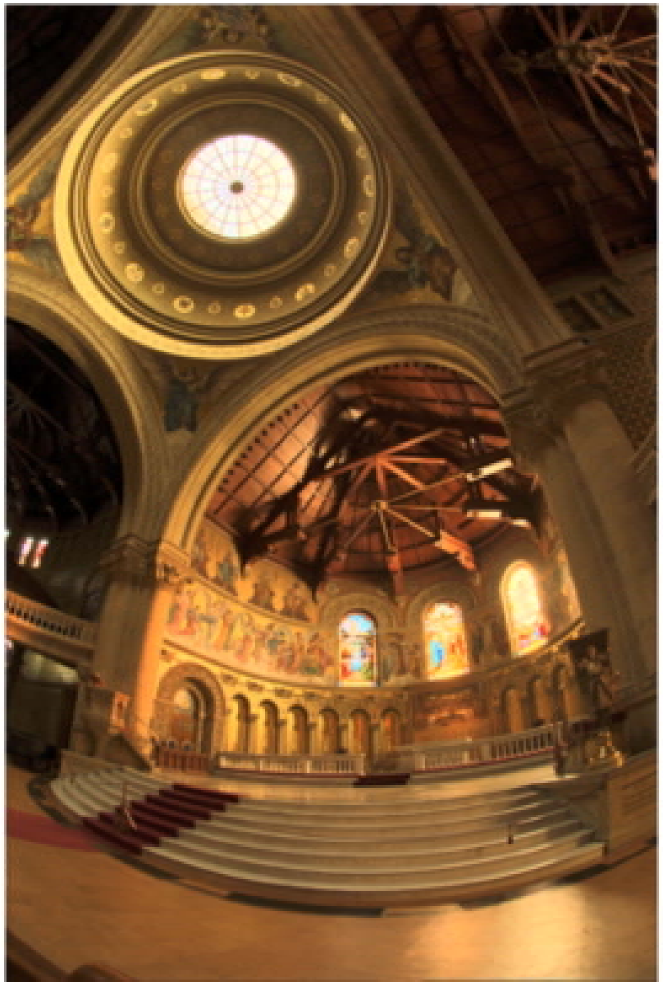
- 1 stop doubles exposure
  - interacts with depth of field
- Double shutter time doubles exposure
  - interacts with motion blur

F-Stop/F-Number: $N = \frac{f}{a}$

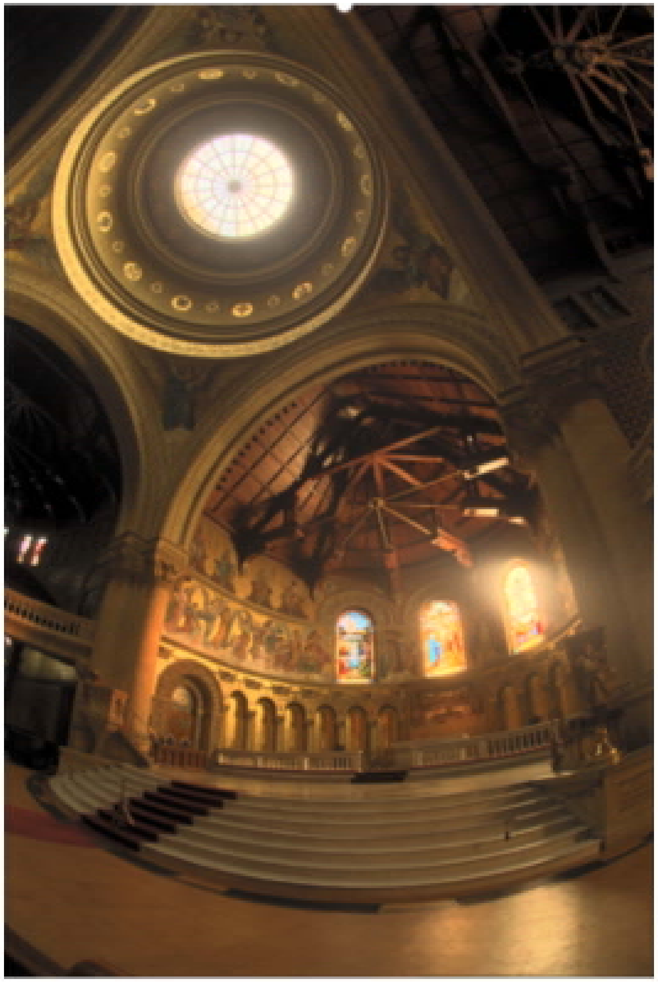
# High Dynamic Range Imaging

- 16 photographs of stanfords memorial cathedral at 1 stop increments from 30s to 1/1000s
- From Debevec and Malik, High Dynamic Range Imaging

# Simulated HDR Image



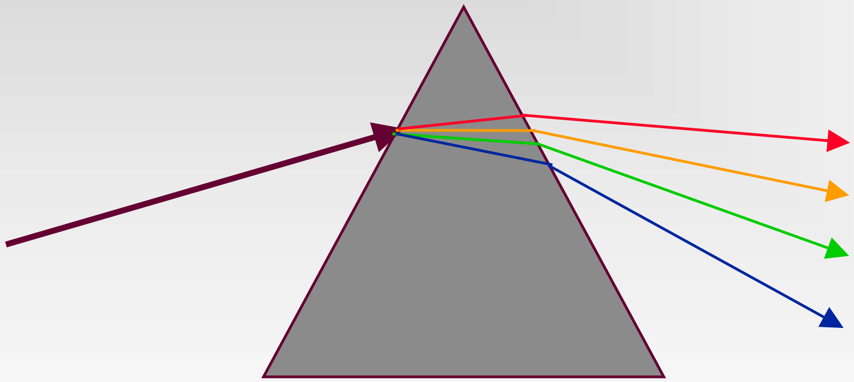
Adaptive histogram



With glare, contrast, blur

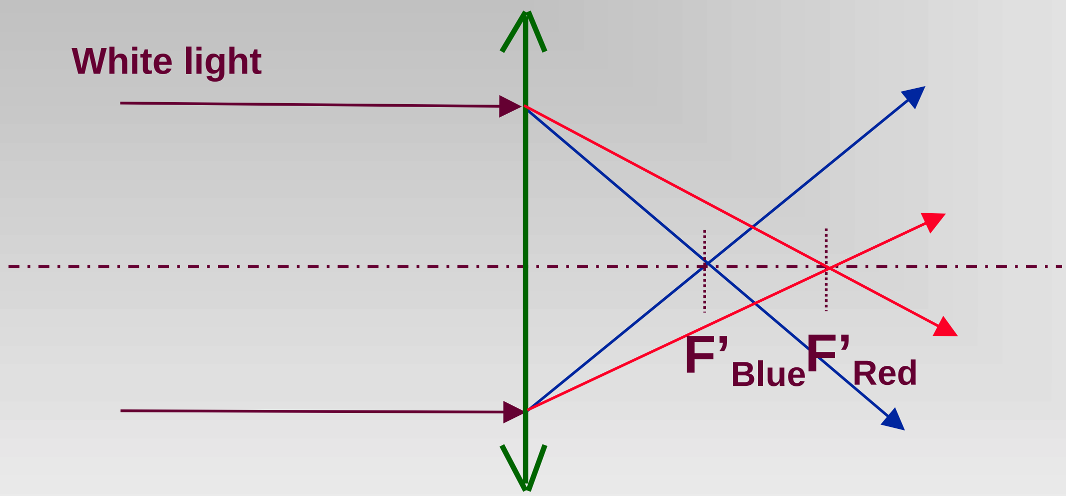
# Dispersion

- Dispersion - Index of refraction, n, depends on the frequency (wavelength) of light.



**Dispersion is responsible for the colors produced by a prism:**
**red light “bends” less within the prism, while blue light “bends” more.**

# Chromatic Aberration





Object (small dot)

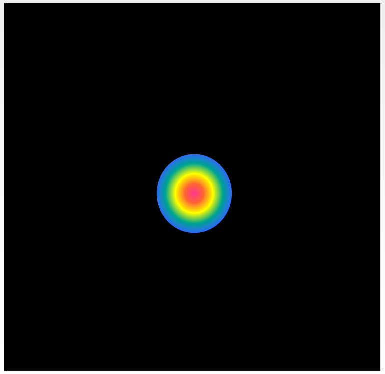
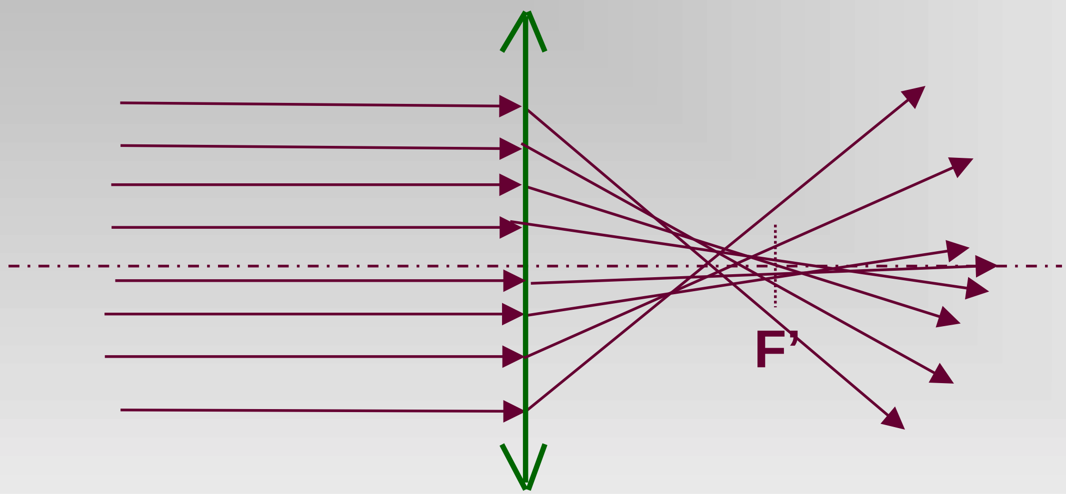


Image with chromatic aberration

- Dispersion results in a lens having different focal points for different wavelengths - this effect is called *chromatic aberration*.
- Results in a “halo” of colors.
- Solution: Use 2 lenses of different shape and material (“achromatic doublet”).

# Spherical Aberration




- All the rays do not bend toward the focal point, resulting in a blurred spot.
- Solution: use lenses with aspherical curvature, or use a compound lens.

**Object (small dot)**

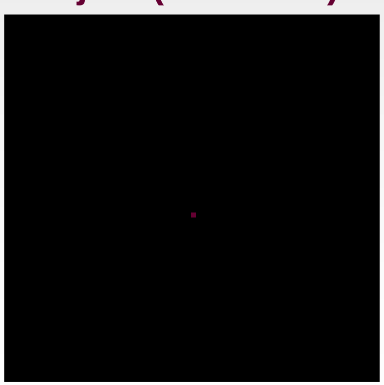
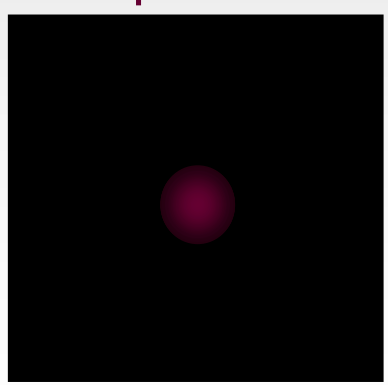
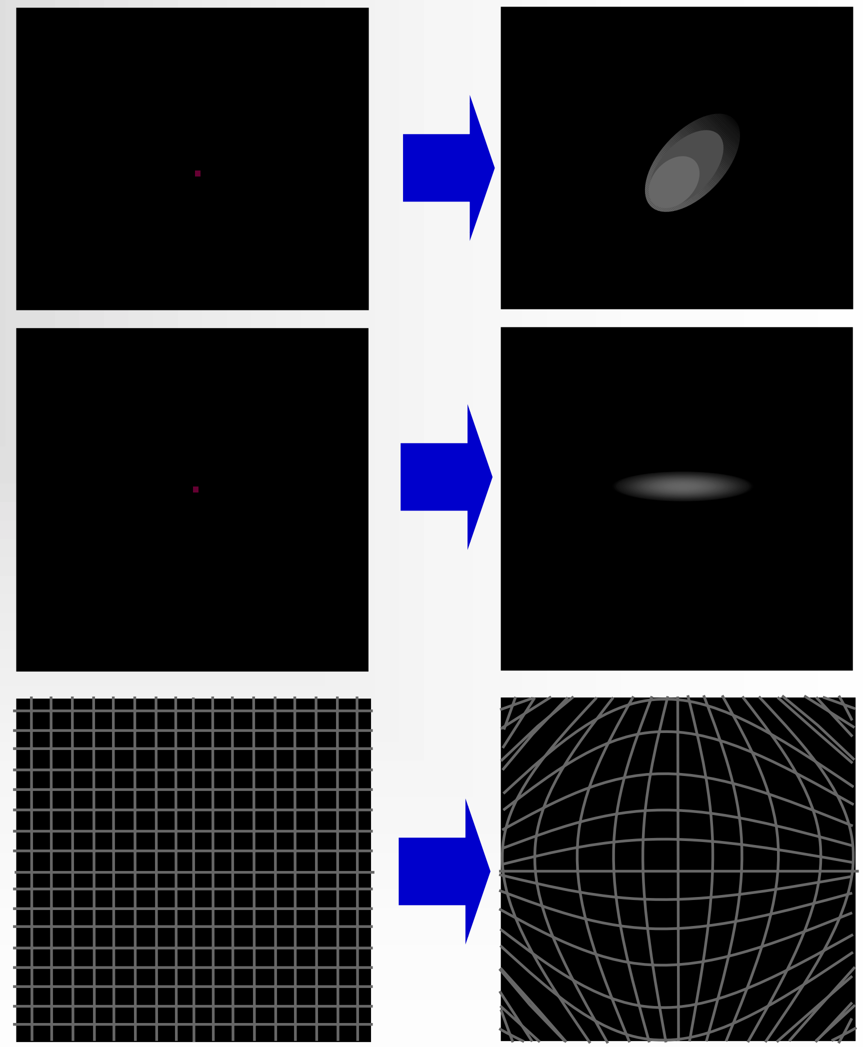


**Image with spherical aberration**

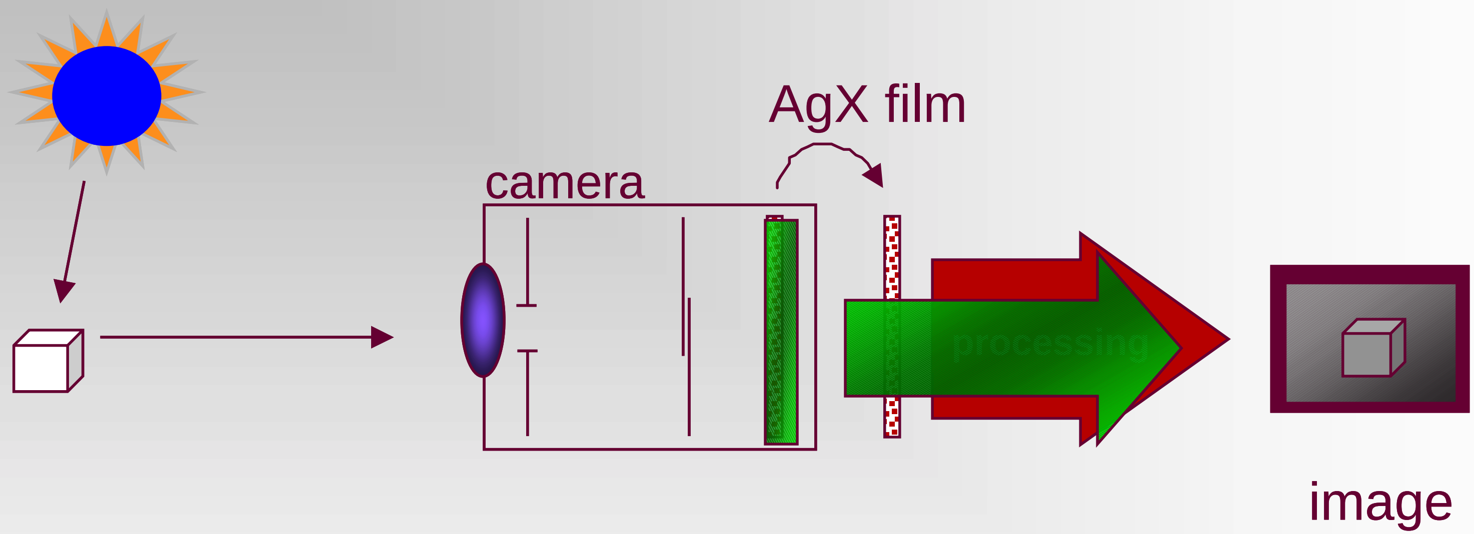


# Other Aberrations

- *Coma*
  - Off axis blur which looks like the "coma" of a comet.
- *Astigmatism*
  - Different focal lengths for different planes.
- *Distortion*
  - Images formed out of shape.



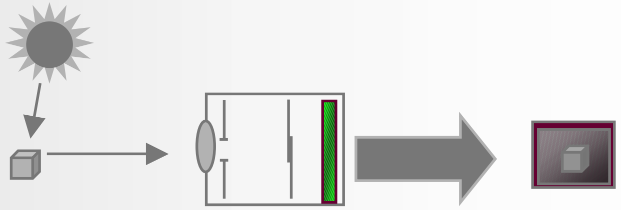
# So Far . . .

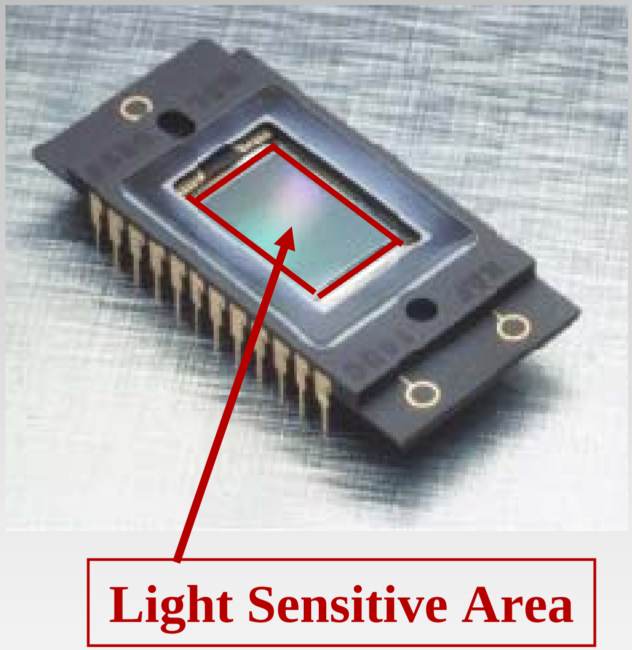



- AgX photographic film captures image formed by the optical elements (lens).
- Unfortunately, the processing for film is slow (among other disadvantages).

- Can we use something else to capture the image?

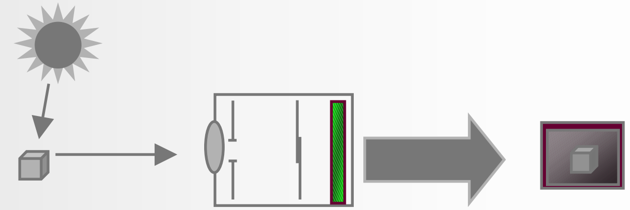
# Charge Coupled Device (CCD)



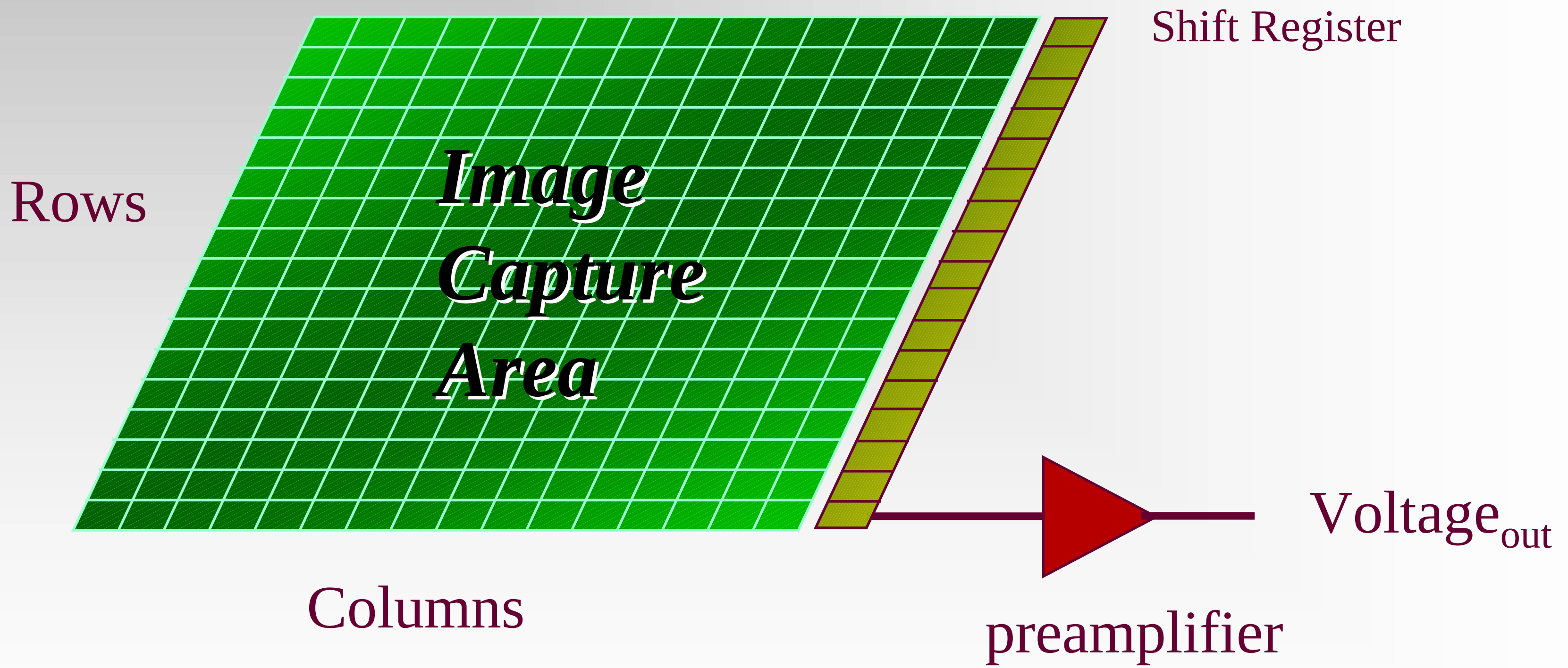



- CCD replaces AgX film
- Based on ***silicon chip***
- Disadvantages vs. AgX:
  - Difficulty/cost of CCD manufacture; large arrays are VERY expensive
  - “Young” technology; rapidly changing

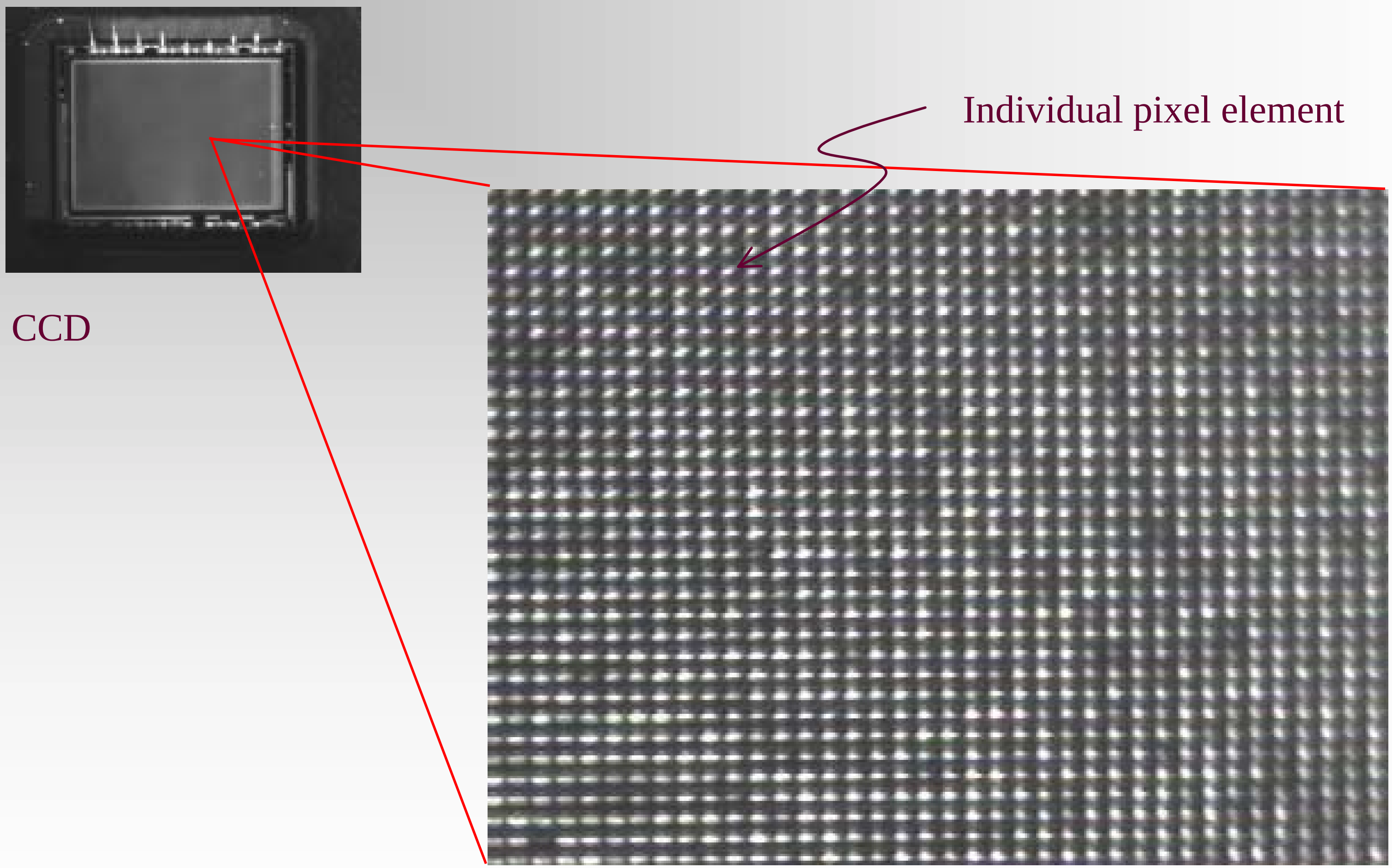
# Basic structure of CCD



Divided into small elements called pixels (*picture elements*).

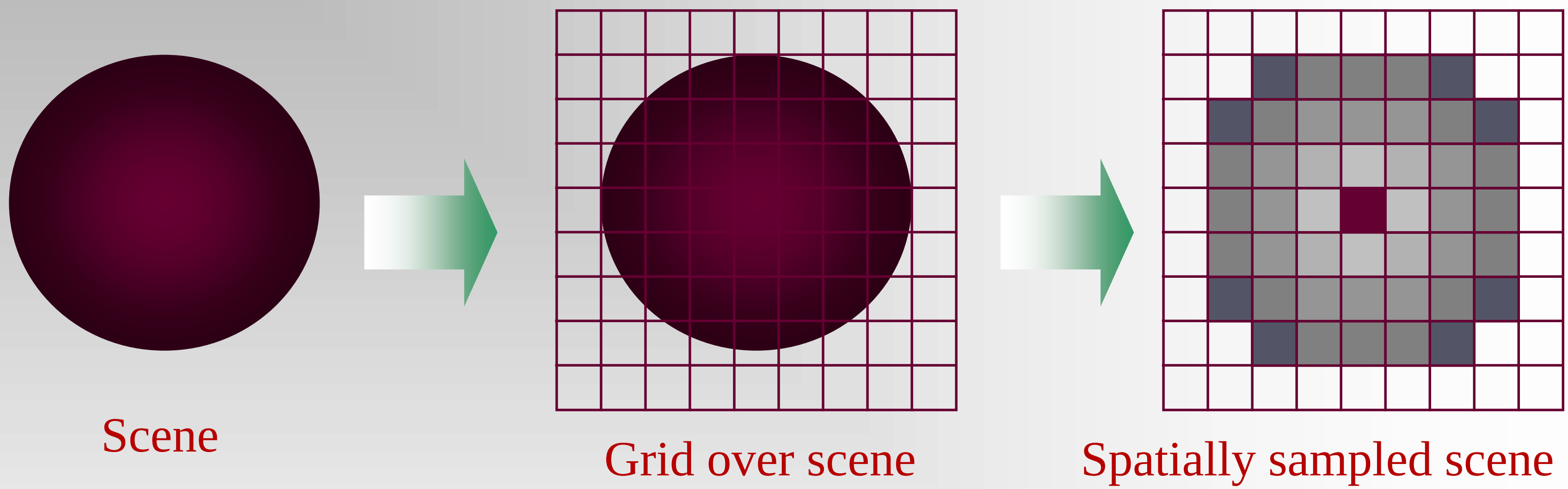


# Magnified View of a CCD Array



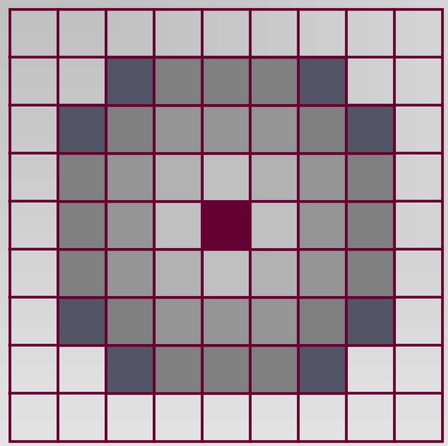

Close-up of a CCD Imaging Array

# Spatial Sampling




- When a continuous scene is imaged on the array (grid) formed by a CCD , the continuous image is divided into discrete elements.
- The picture elements (*pixels*) thus captured represent a spatially sampled version of the image.

# Quantization



Spatially sampled scene

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 25 | 40 | 40 | 40 | 25 | 0 | 0 |
| 0 | 25 | 40 | 64 | 64 | 64 | 40 | 25 | 0 |
| 0 | 40 | 64 | 97 | 97 | 97 | 64 | 40 | 0 |
| 0 | 40 | 64 | 97 | 150 | 97 | 64 | 40 | 0 |
| 0 | 40 | 64 | 97 | 97 | 97 | 64 | 40 | 0 |
| 0 | 25 | 40 | 64 | 64 | 64 | 40 | 25 | 0 |
| 0 | 0 | 25 | 40 | 40 | 40 | 25 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

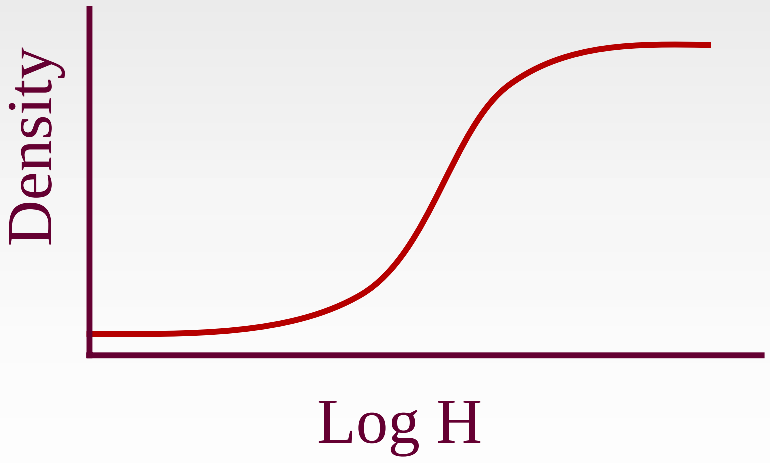
Numerical representation

- Spatially sampled image can now be turned into numbers according to the brightness of each pixel.

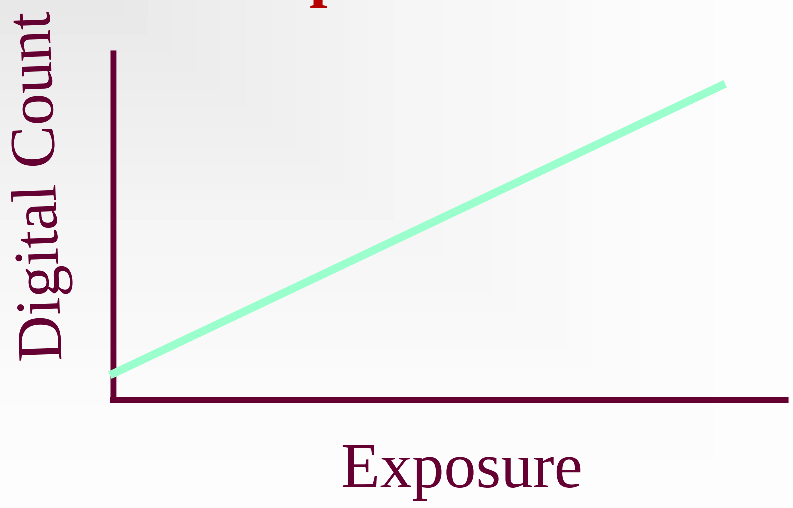
# Response of CCD

- The response of CCD is ***linear*** (*i.e.*, if 1000 captured photons corresponds to a digital count of ***4***, then 2000 photons captured yields a digital count of ***8***)
- Linearity is critical for scientific uses of CCD

**Response of photographic negative**



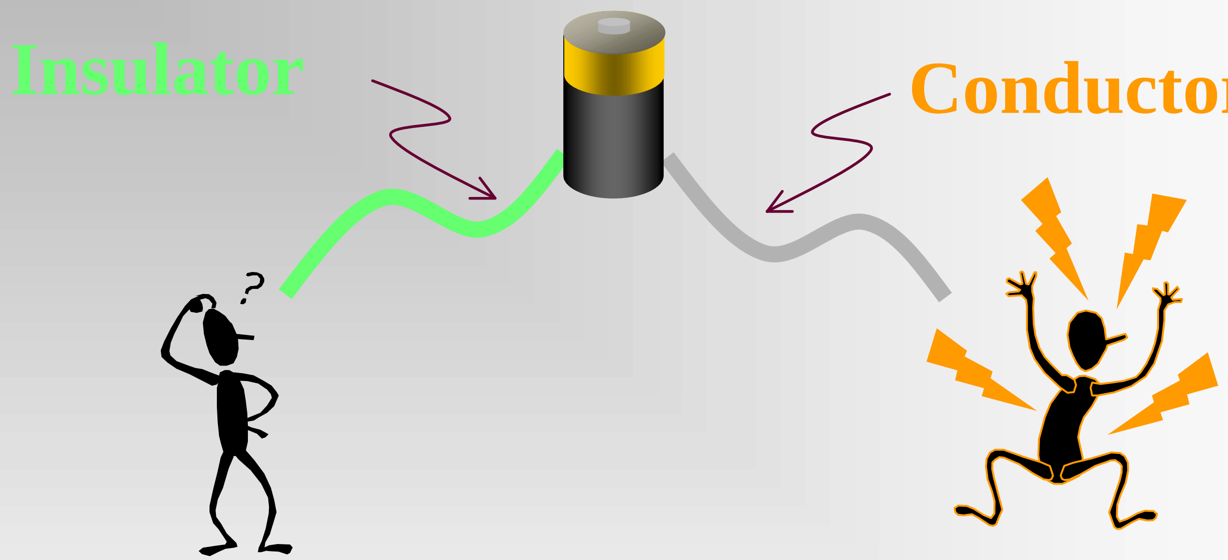

**Response of CCD**



# Image quality factors

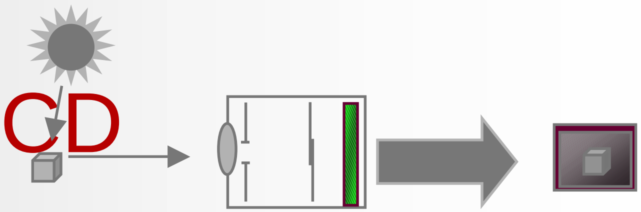
- Two major factors which determine image quality are:
  - *Spatial resolution --* controlled by spatial sampling.
  - *Color depth --* controlled by number of colors or grey levels allocated for each pixel
- Increasing either of these factors results in a larger image file size, which requires more storage space and more processing/display time.

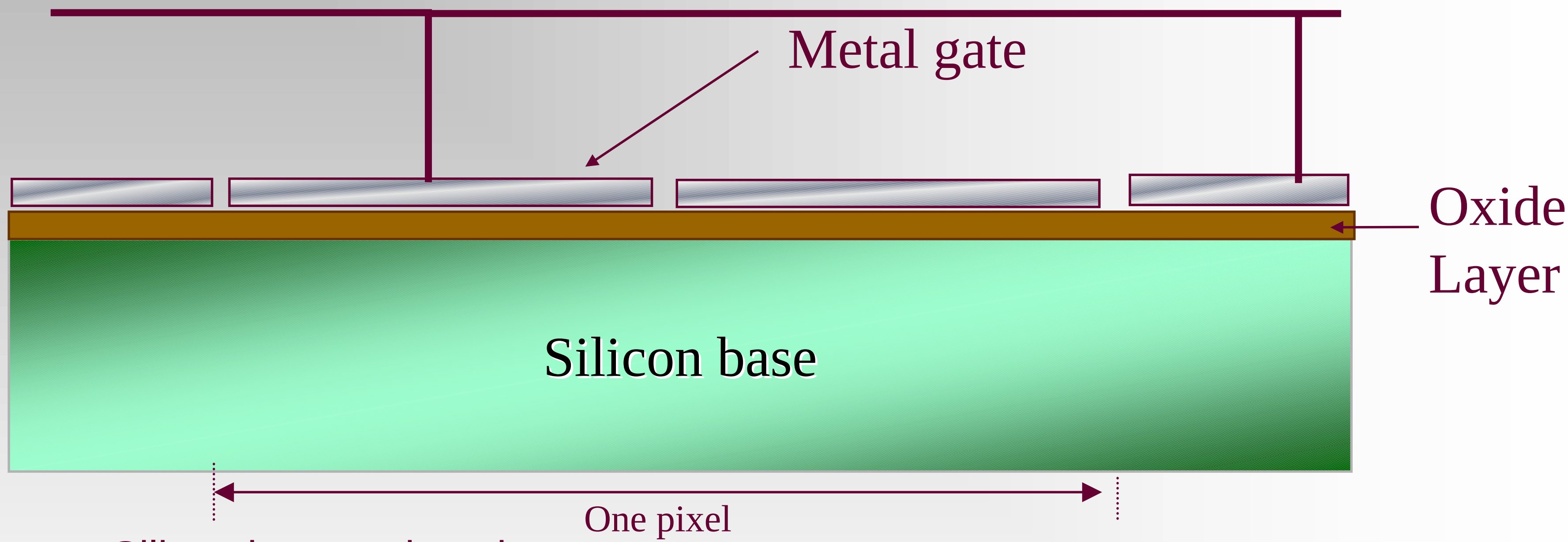
# CCDs as Semiconductors




- Conductors allow electricity to pass through. (Metals like copper and gold are conductors.)
- Insulators do not allow electricity to pass through. (Plastic, wood, and paper are insulators.)
- Some materials are halfway in between, called semiconductors.

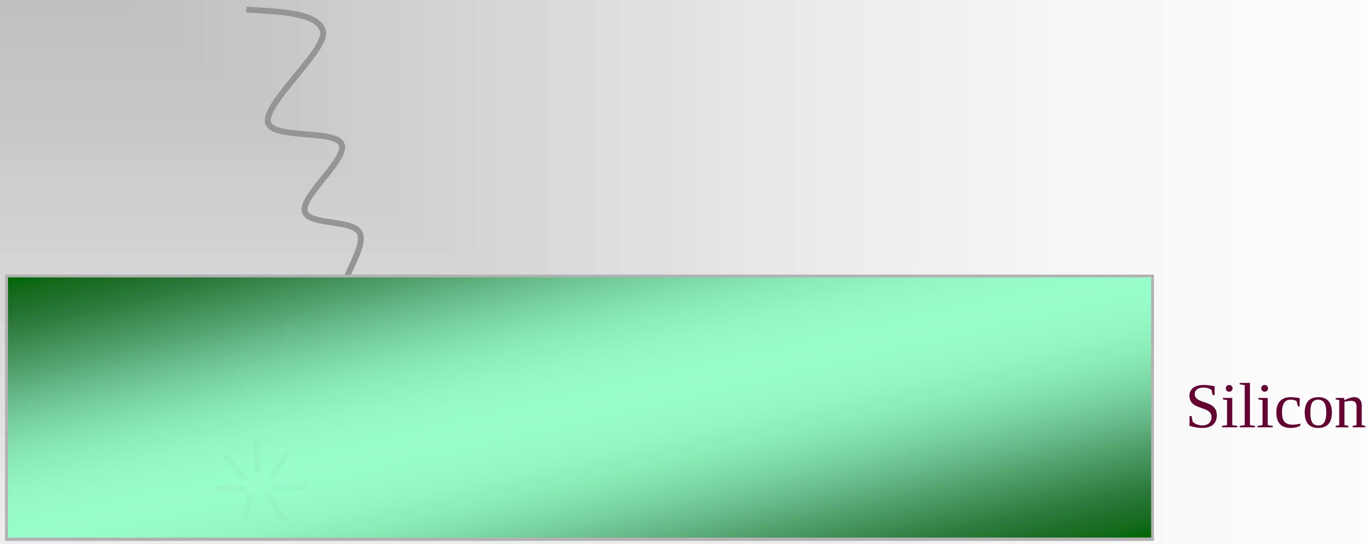
# Basic structure of a pixel in a CCD





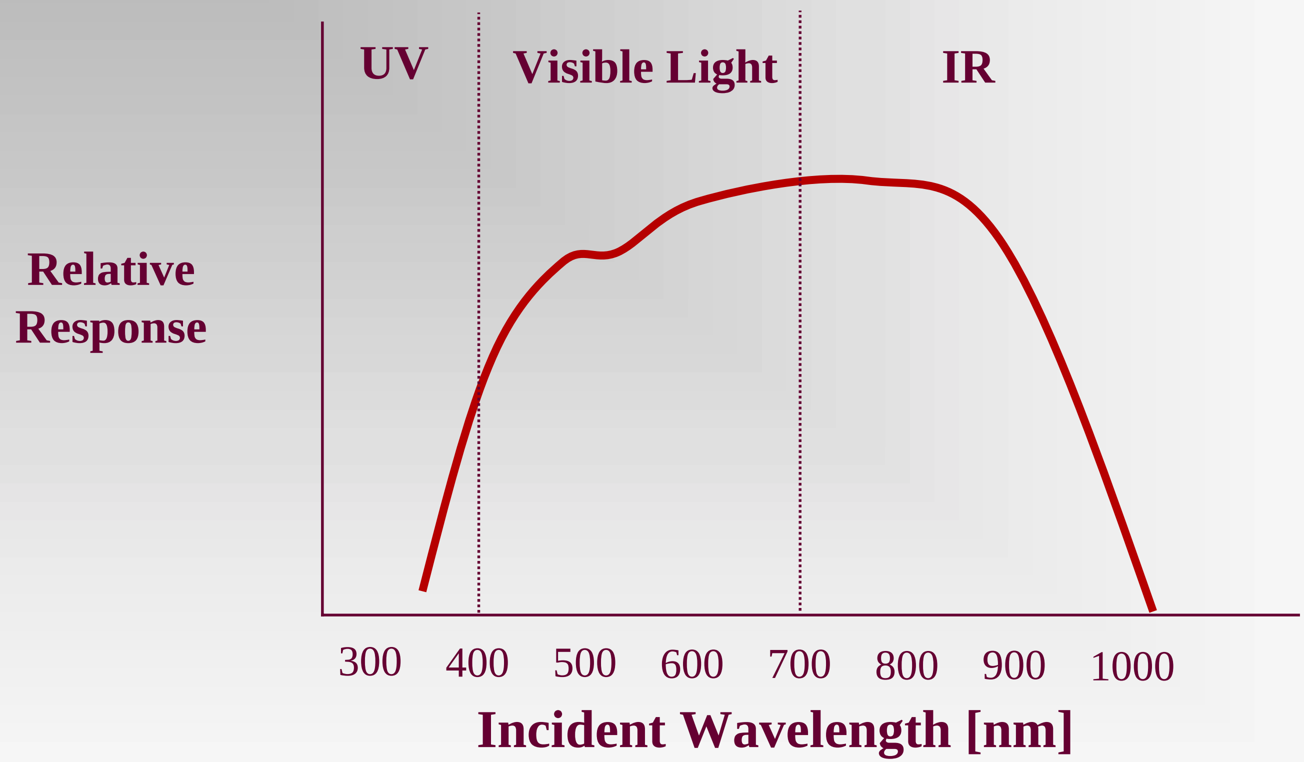

- Silicon is a semiconductor.
- Oxide layer is an insulator.
- Metal gates are conductors.
- Made with microlithographic process.
- One pixel may be made up of two or more metal gates.

# Photon/Silicon Interaction



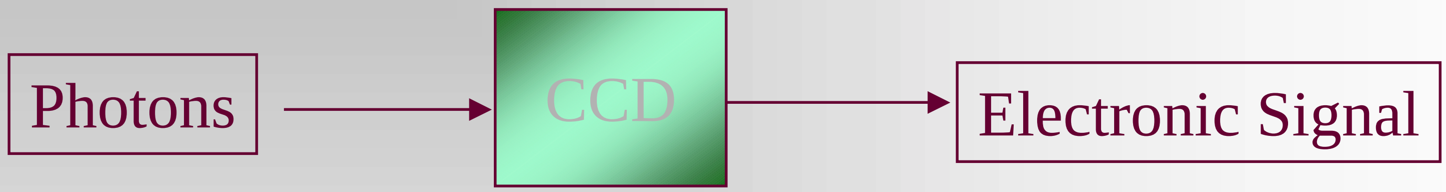

- Photon knocks off one of the *electrons* from the silicon matrix.
- Electron "wanders around" randomly through the matrix.
- Electron gets *absorbed* into the silicon matrix after some period.

# Spectral Response (sensitivity) of a typical CCD



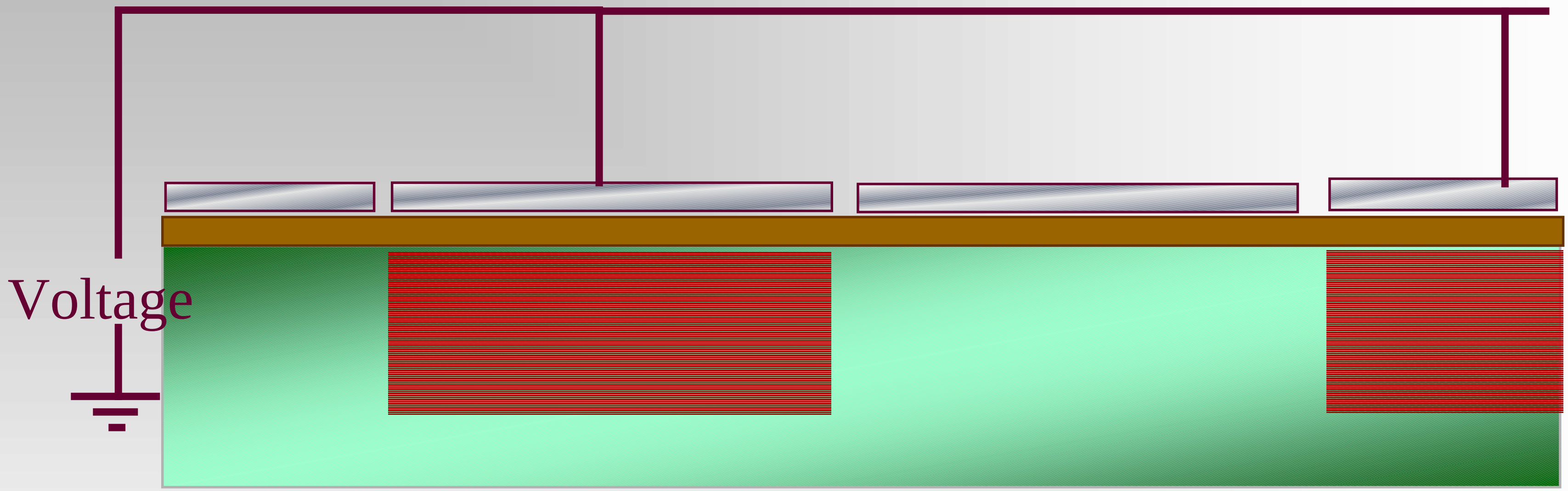

- Response is large in visible region, falls off for ultraviolet (UV) and infrared (IR)

# Goal of CCD



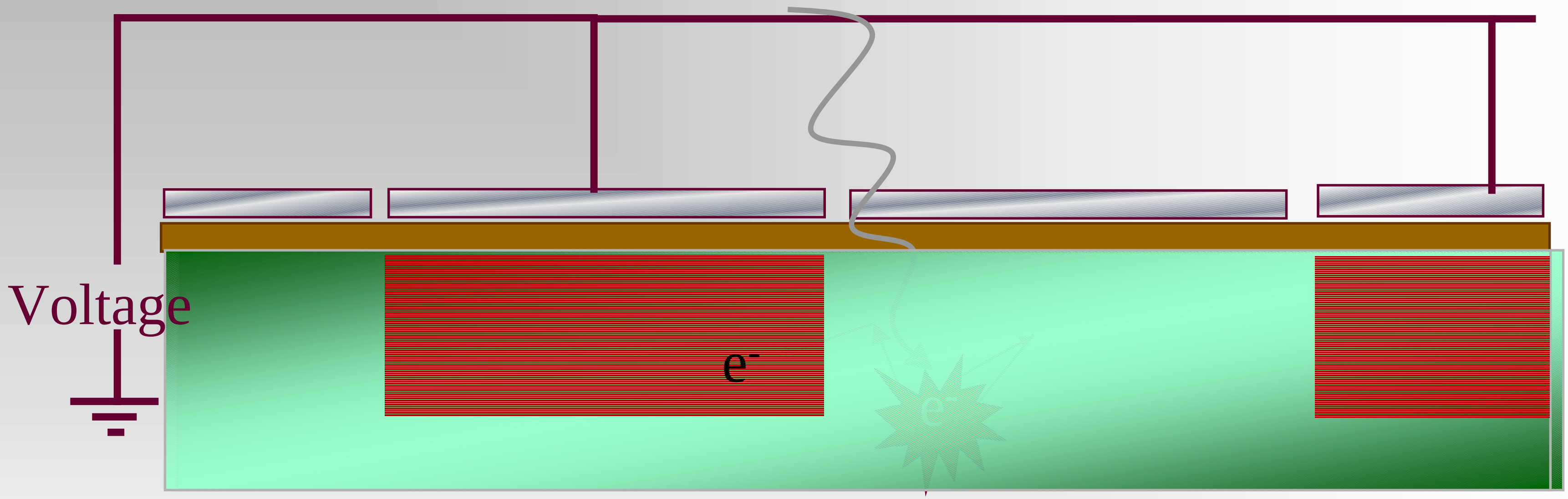

- Capture electrons formed by interaction of photons with the silicon
- Measure the electrons from each picture element as a voltage

# Collection stage



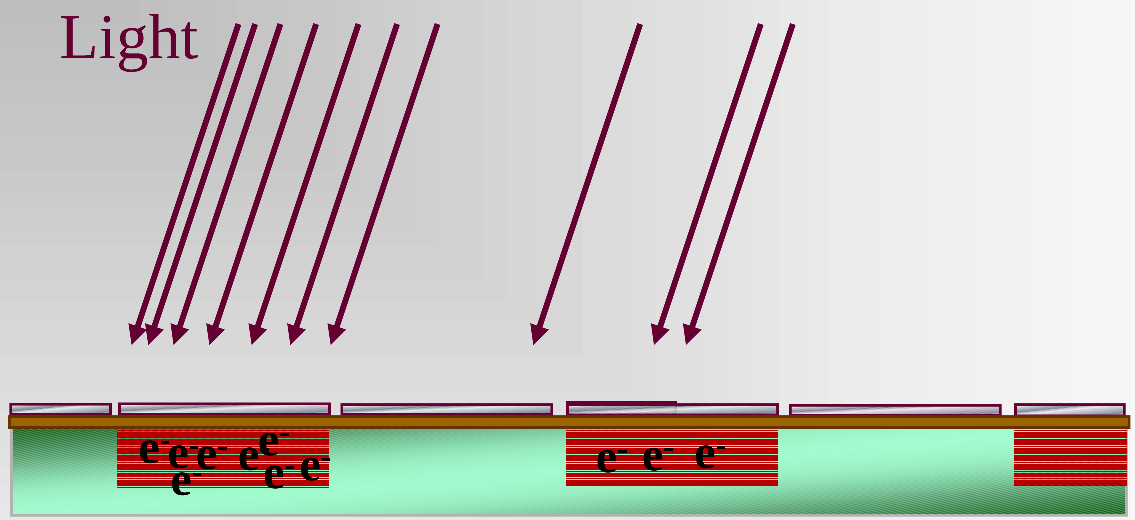

- Voltage applied to the metal gates produces a *depletion region* in the silicon. (depleted of electrons)
- Depletion region is the "light sensitive" area where electrons formed from the photon interacting with the silicon base are collected.

# Collection stage



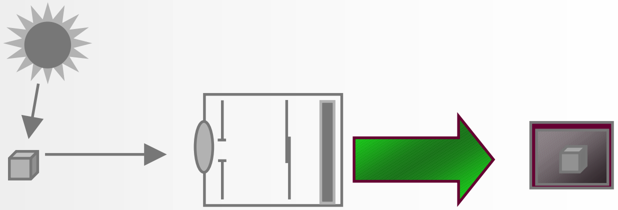

- Electron formed in the silicon matrix by a photon.
  - Electron wanders around the matrix.
  - If the electron wanders into the depletion region, the electron is captured, never recombining with the silicon matrix.

# Collection

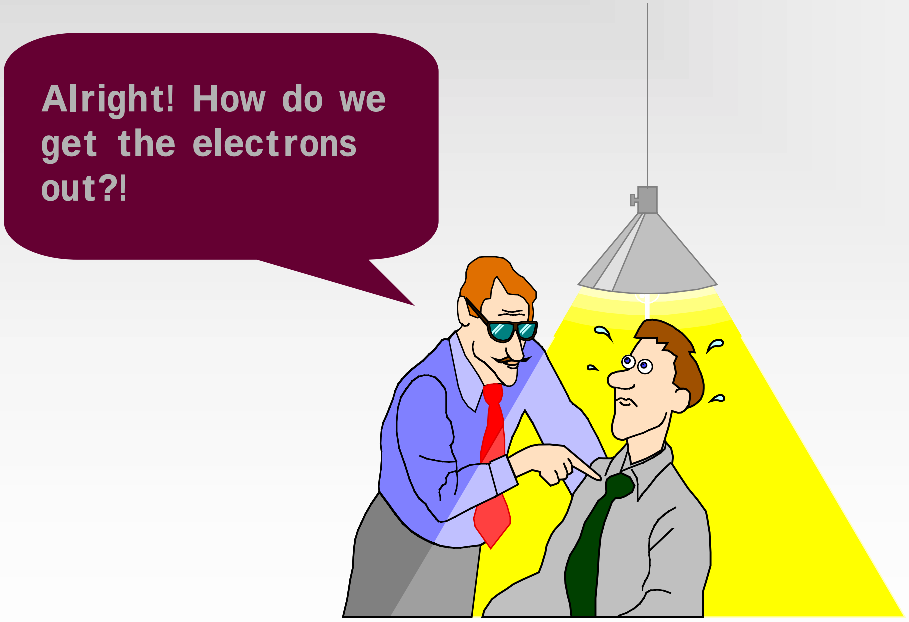



- The number of electrons accumulated is proportional to the amount of light that hit the pixel.
- There is a maximum number of electron that these “wells” can hold.

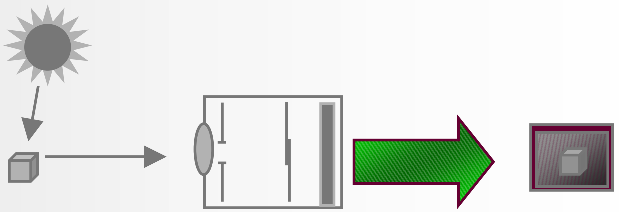
# Readout



- Now that the electrons are collected in the individual pixels, how do we get the information out?

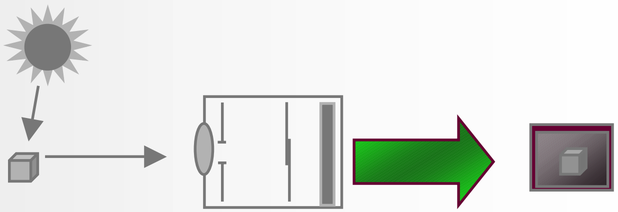


# Readout

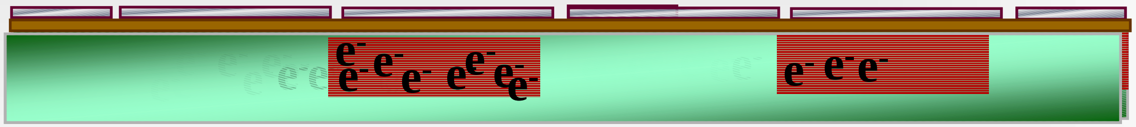


- How do you access so much data efficiently? (i.e. a 1024 x 1024 CCD has 1,048,576 pixels!)
  - Possible solutions:
    - *1. Have output for individual pixels.*
      - Too many “wires”
    - *2. Somehow move the charges across the CCD array and read out one by one.*
      - Bucket Brigade

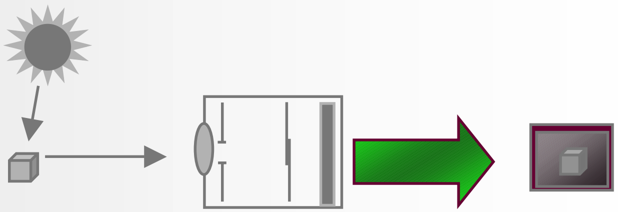
# Bucket Brigade



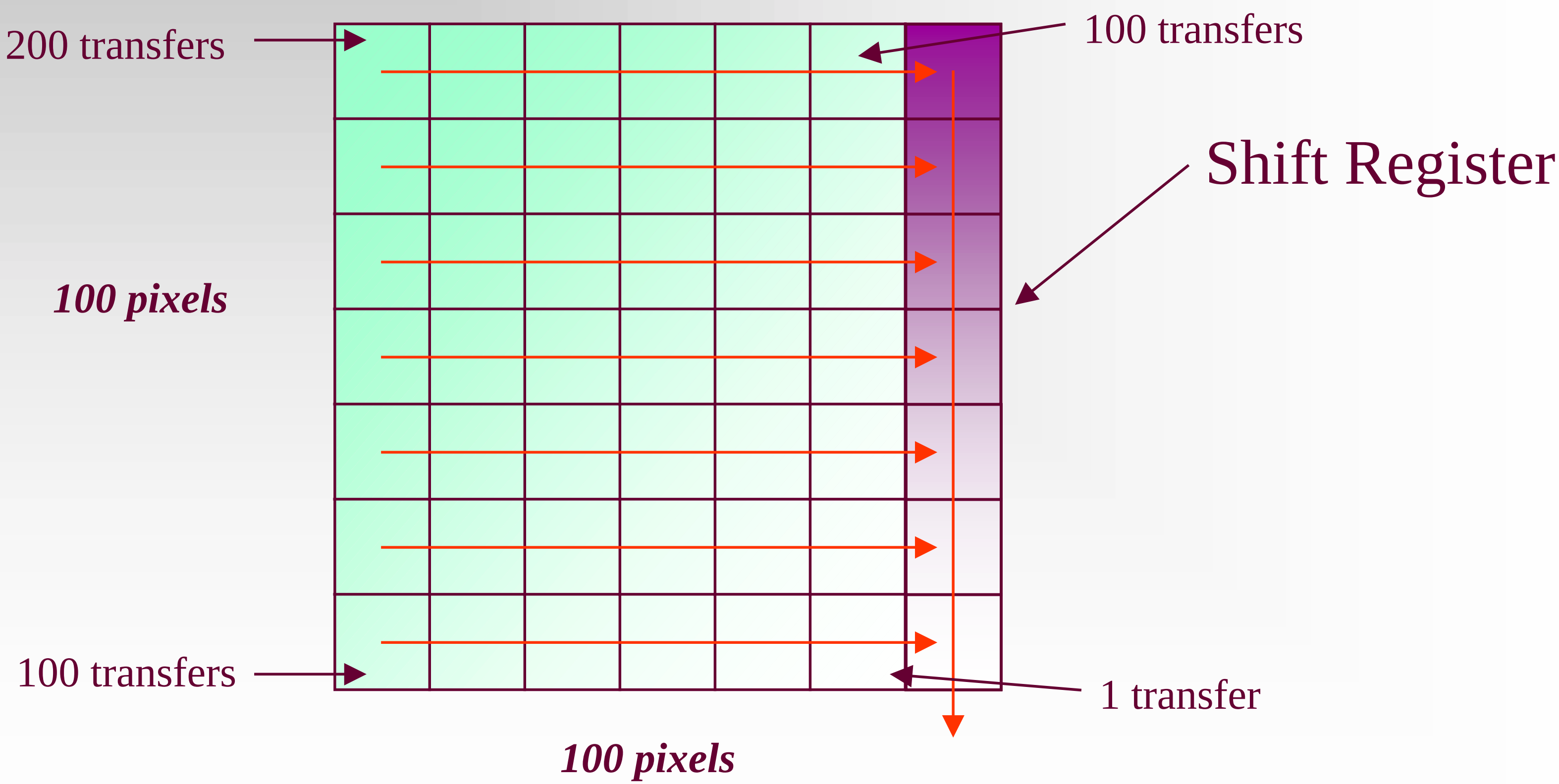
- By alternating the voltage applied to the metal gates, collected electrons may be moved across the columns.



# Bucket Brigade

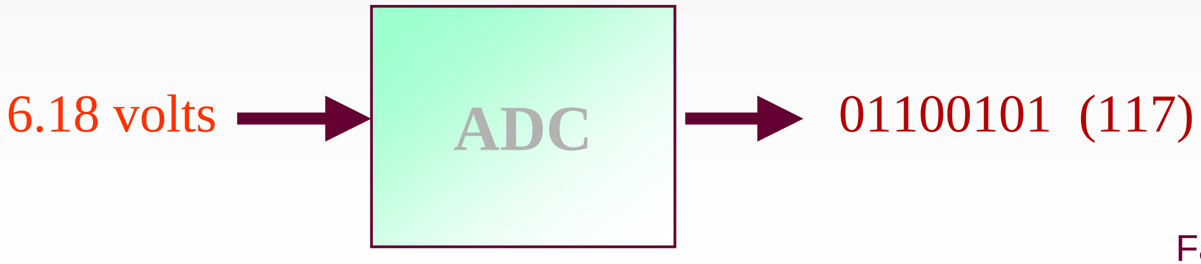


- Charge is marched across the columns into the shift register, then read out 1 pixel at a time.

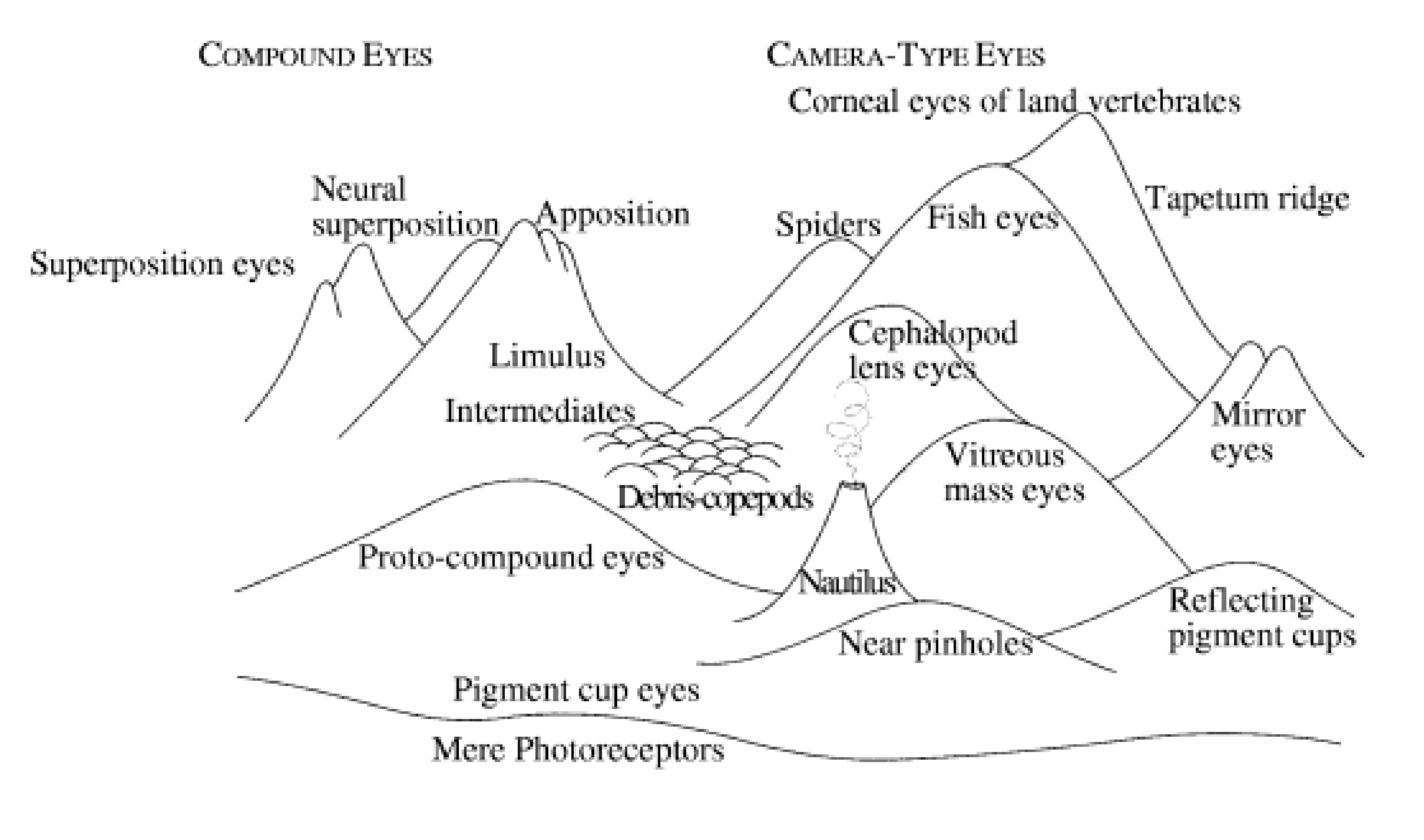


# Converting Analog Voltages to Digital

- Analog voltage is converted to a *digital count* using an ***Analog-to-Digital Converter*** (ADC)
  - Also called a ***digitizer***
- The input voltage is ***quantized***:
  - Assigned to one of a set of discrete steps
    - Steps are labeled by integers
    - Number of steps determined by the number of available bits
  - Decimal Integer is converted to a binary number for computation

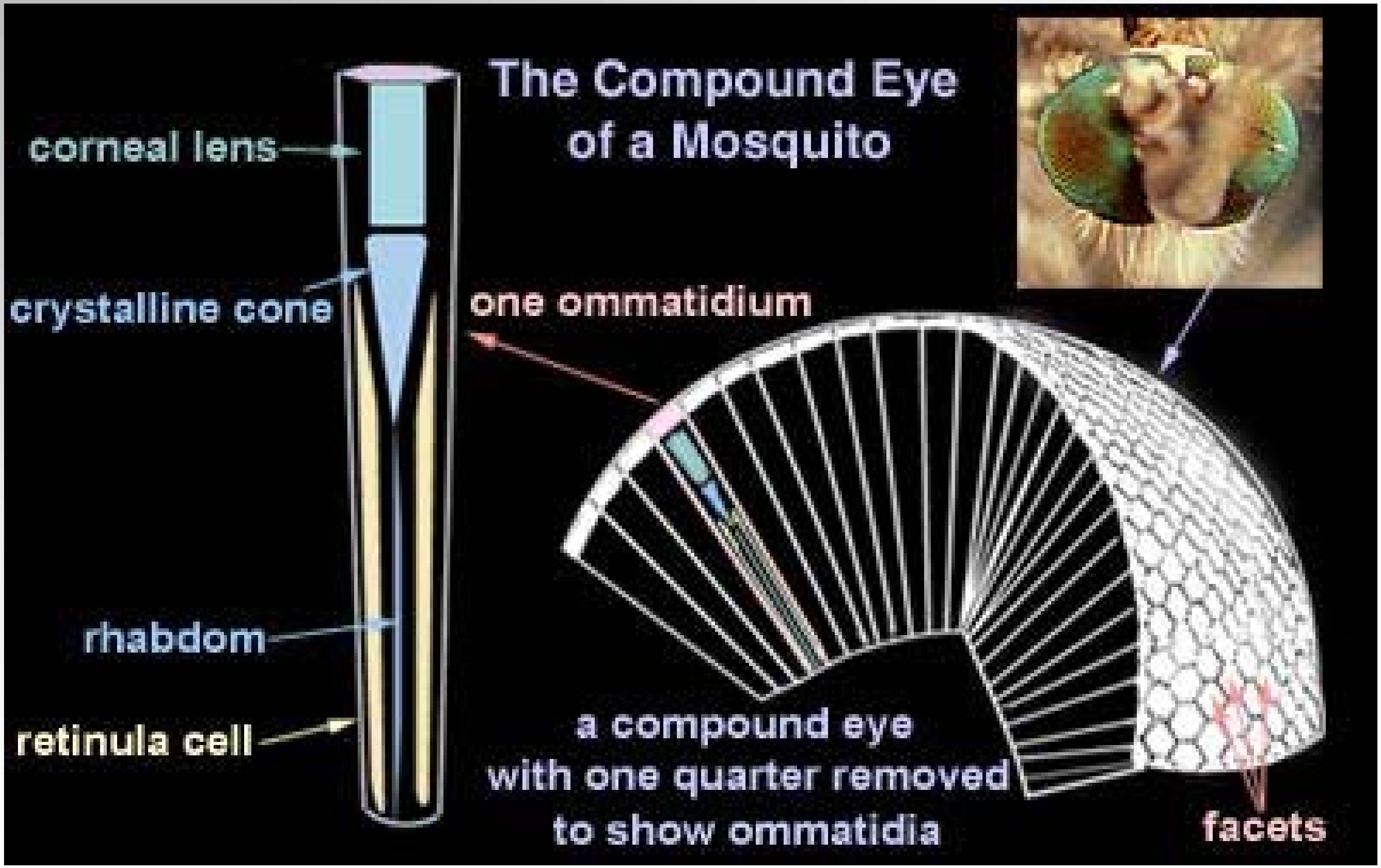


# Biological camera design is purposive camera design

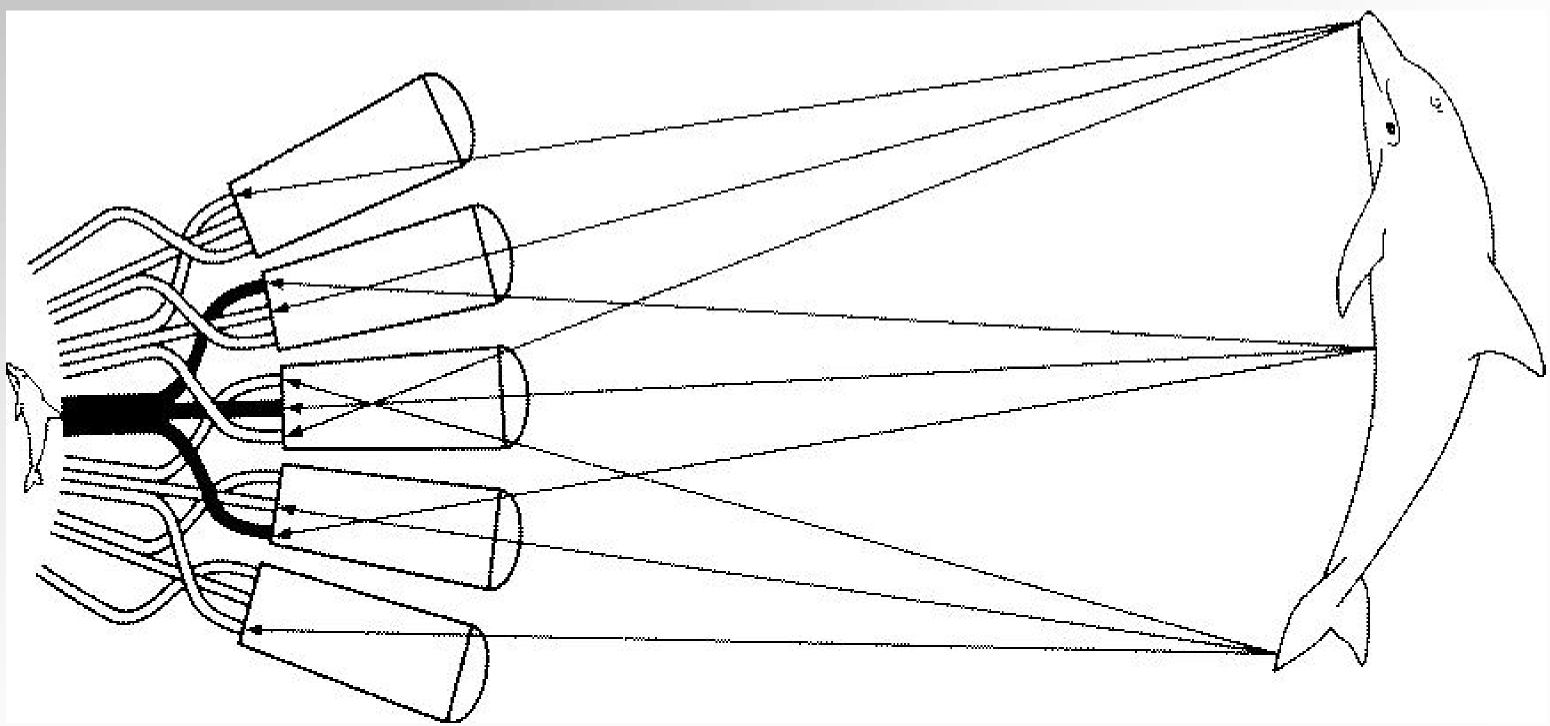



"Landscape of Eye Evolution"
from R. Dawkins' "Climbing Mount Improbable"

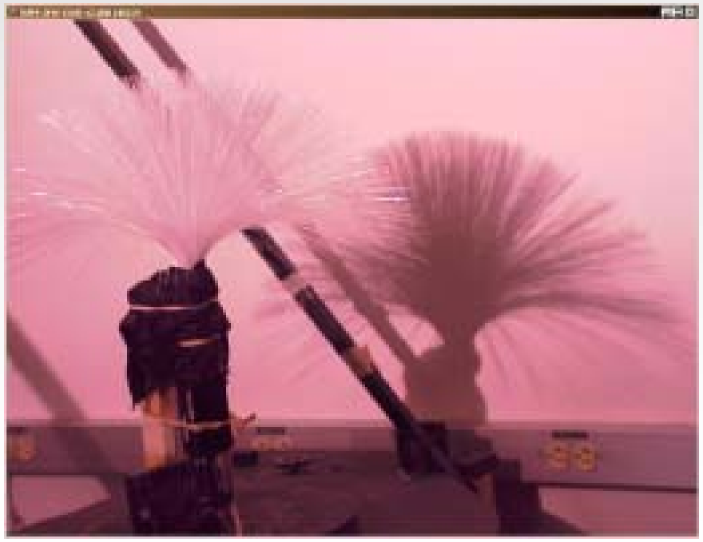
# A biological camera



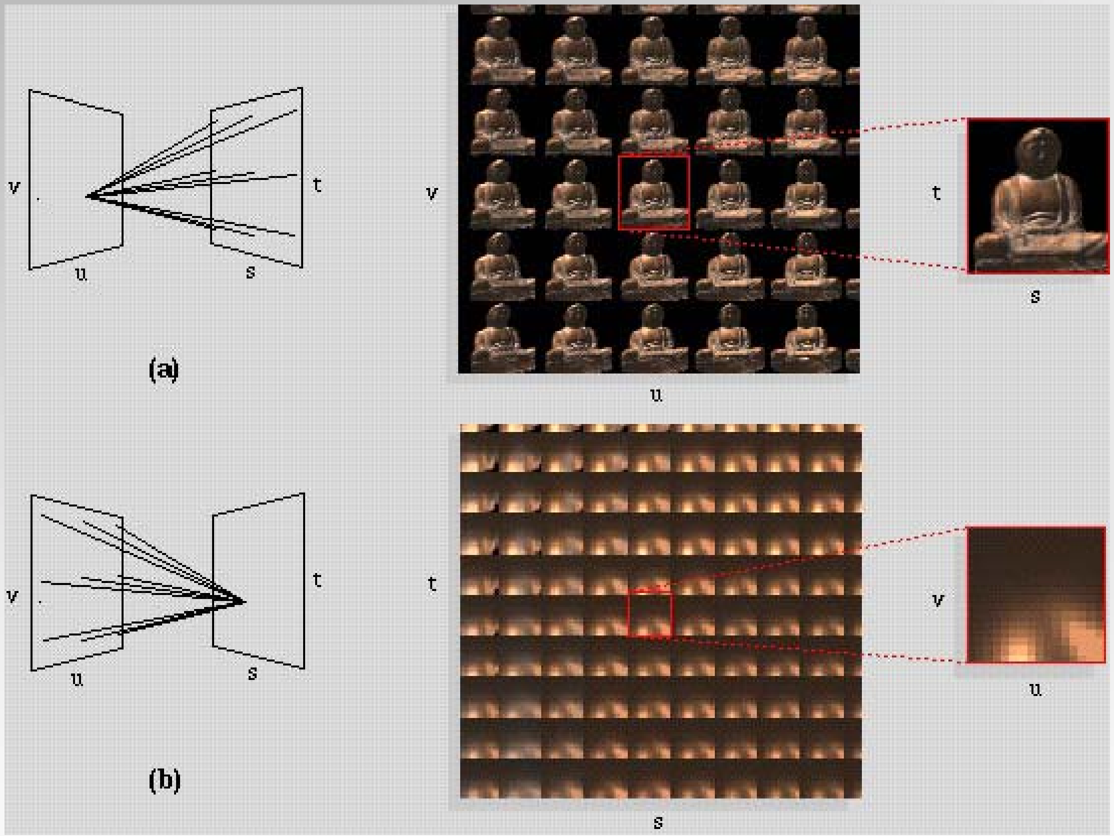
# A biological camera

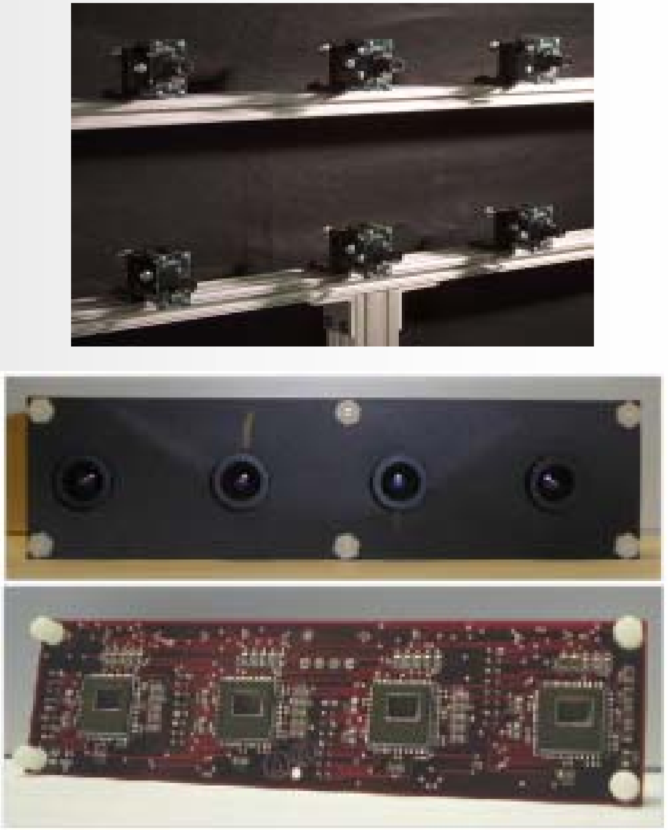


# Omni-directional vision

# Light field cameras

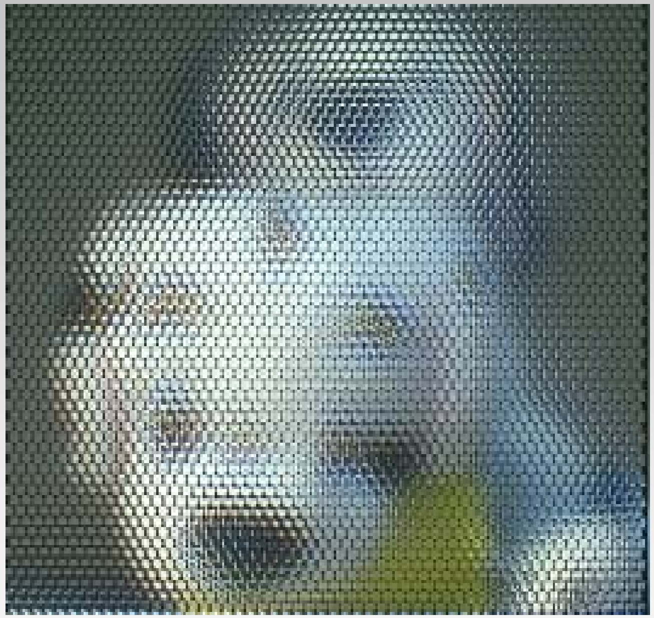


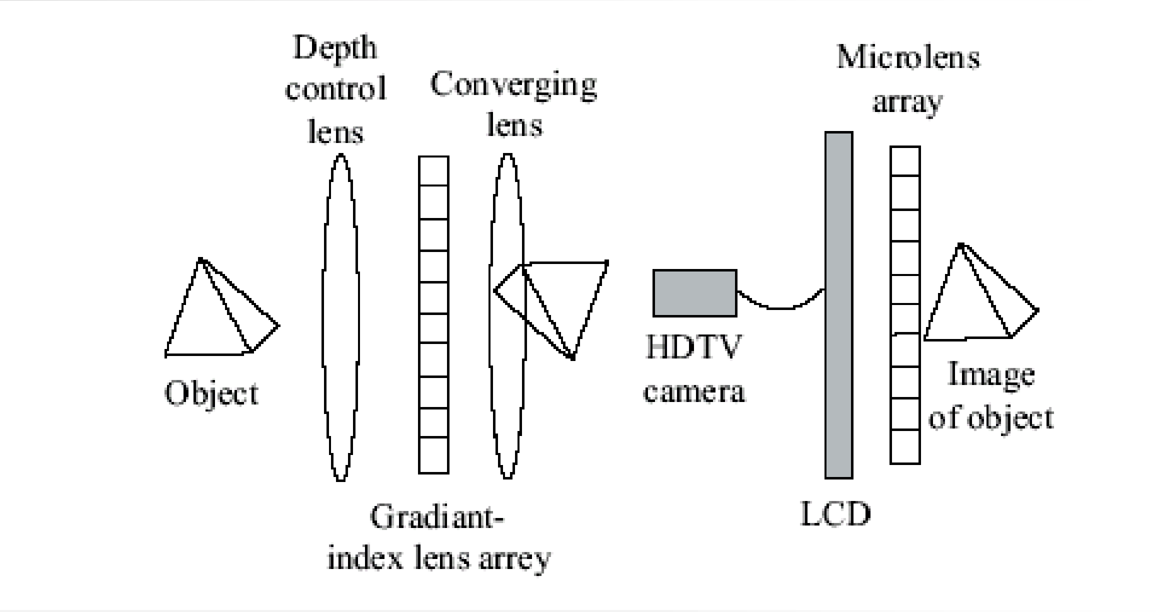



From Levoy & Hanrahan “Light Field Rendering”, SIGGRAPH96

Wilburn et al., ‘02

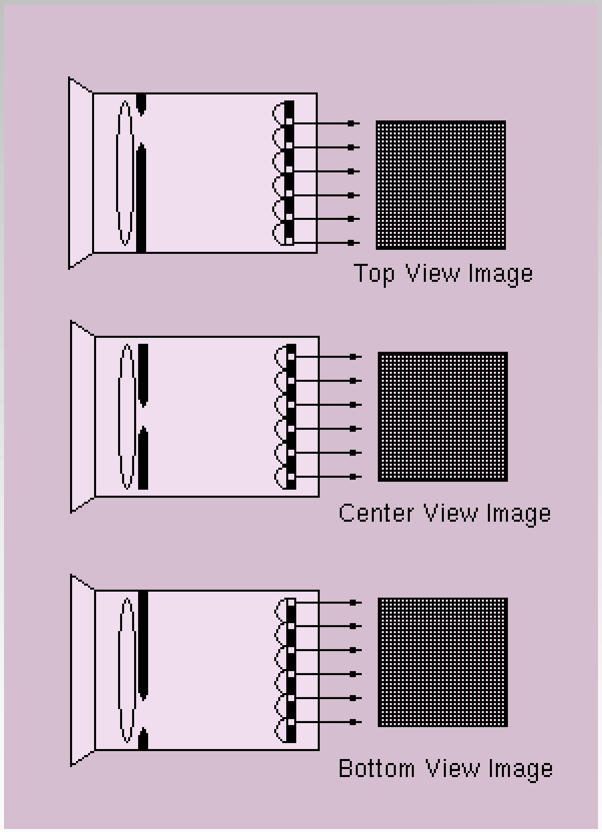
# Integral photography

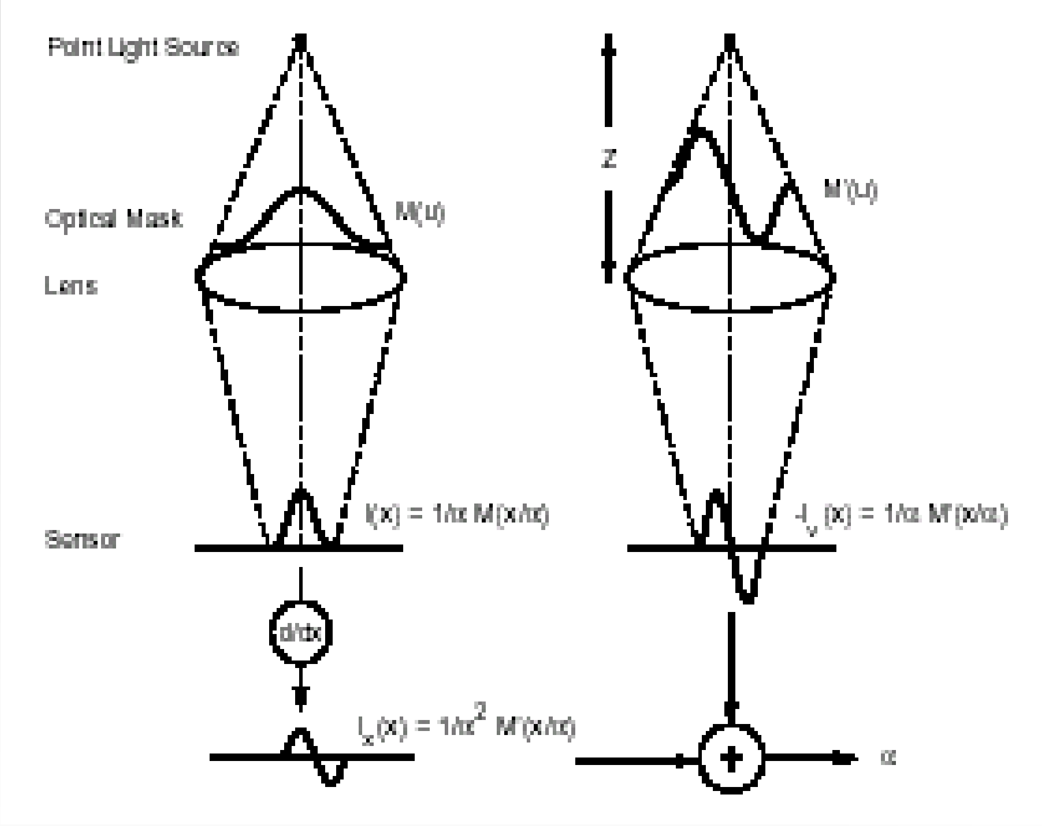





- Integral photography
(Lippman '08, Ives '30, Naemura `01)

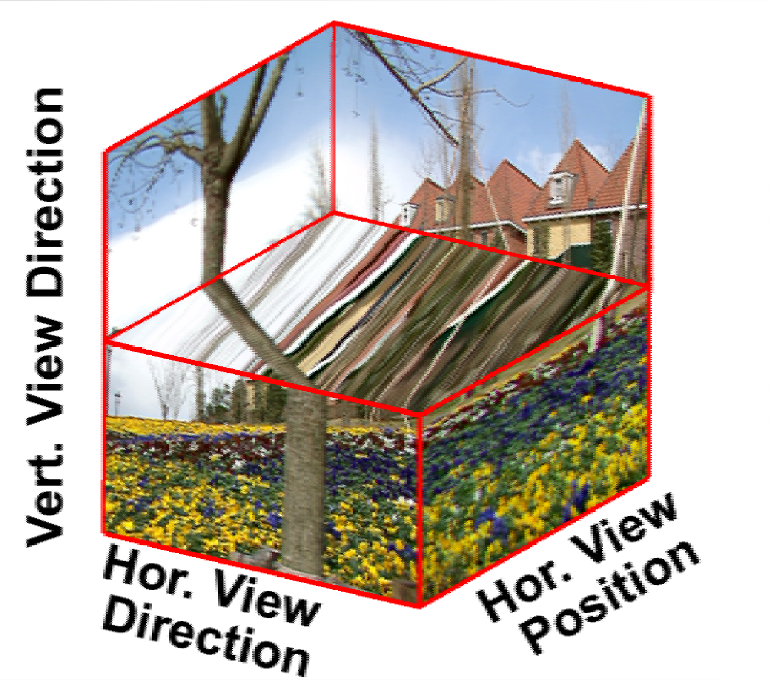
# Plenoptic cameras



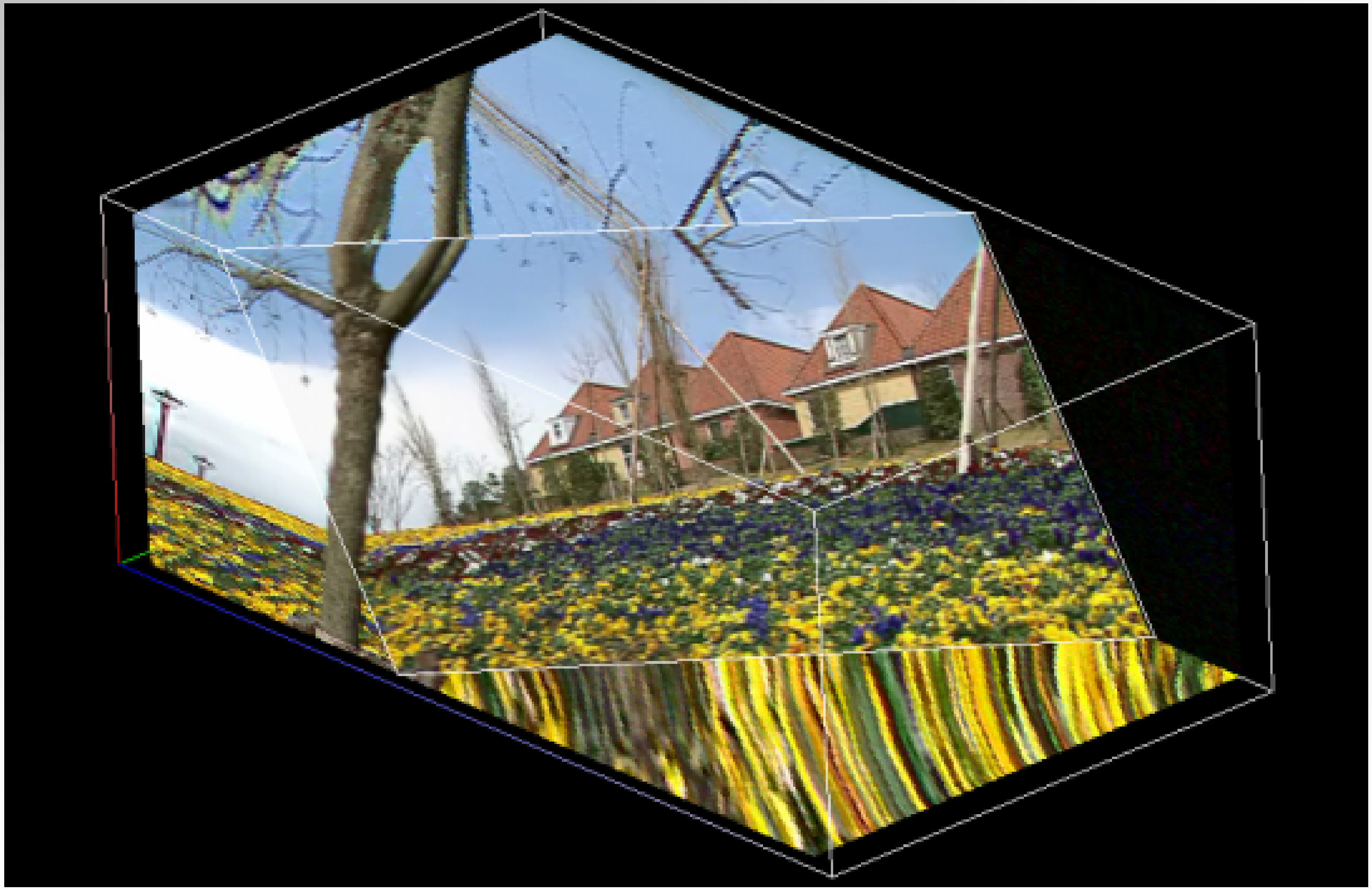

Adelson & Wang '92 Farid & Simoncelli '97

# Does a computer need to see the world with our eyes?




Example: Epipolar Image Volume

# Does a computer need to see the world with our eyes?



Picture made using the VideoCube program by M. Cohen