Announcements

- Send email to the TA for the mailing list
  aster@umiacs.umd.edu
- For problem set 1: turn in written answers to problems 2 and 3.
- Everything printed, plus source code emailed to the TA.
- "What kind of geometric object?", eg., circle, line, ….
- It is possible that the first problem set is kind of hard. Start early.

Fourier Transform

- Analytic geometry gives a coordinate system for describing geometric objects.
- Fourier transform gives a coordinate system for functions.

Basis

- P=(x,y) means P = x(1,0)+y(0,1)
- Similarly:

$$f(\theta) = a_{11}\cos(\theta) + a_{12}\sin(\theta) + a_{21}\cos(2\theta) + a_{22}\sin(2\theta) + \ldots$$

- *Matlab*

Note, I'm showing non-standard basis, these are from basis using complex functions.

Example

$$\forall c, \exists a_1, a_2 \text{ such that}:$$

$$\cos(\theta + c) = a_1\cos\theta + a_2\sin\theta$$

Matlab

Orthonormal Basis

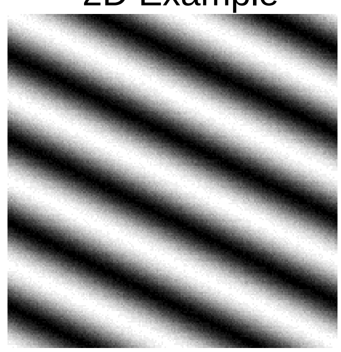
- ||(1,0)||=||(0,1)||=1
- (1,0).(0,1)=0
- Similarly we use normal basis elements eg:

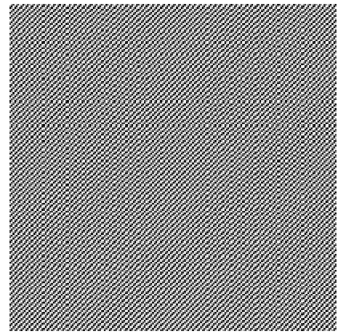
$$\frac{\cos(\theta)}{\|\cos(\theta)\|} \qquad \|\cos(\theta)\| = \sqrt{\int_0^{2\pi}\cos^2\theta\, d\theta}$$

- While, eg:

$$\int_0^{2\pi}\cos\theta\sin\theta\; d\theta = 0$$

2D Example





Remember Convolution

I

10	11	10	0	0	1
9	$\boxed{10}$	11	1	0	1
10	9	10	0	2	1
11	10	9	10	9	11
9	10	11	9	99	11
10	9	9	11	10	10

F

1/9

1	1	1
1	1	1
1	1	1

O

X	X	X	X	X	X
X	$\boxed{10}$				X
X					X
X					X
X					X
X	X	X	X	X	X

1/9.(10x1 + 11x1 + 10x1 + 9x1 + 10x1 + 11x1 + 10x1 + 9x1 + 10x1) =
1/9.(90) = 10

Convolution Theorem

$$f \otimes g = T^{-1} F * G$$

- *F,G* are transform of *f,g*

That is, *F* contains coefficients, when we write *f* as linear combinations of harmonic basis.

Examples

$$\cos\theta \otimes \cos\theta = ?$$

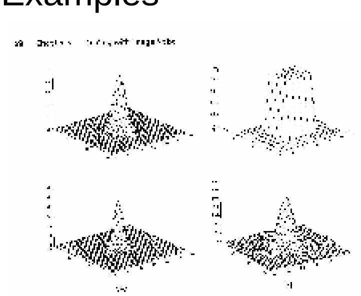
$$\cos\theta \otimes \cos 2\theta = ?$$

$$\cos\theta \otimes f = ?$$

$$(\cos\theta + .2\cos 2\theta + .1\cos 3\theta) \otimes f = ?$$

Examples

- Transform of box filter is sinc.
- Transform of Gaussian is Gaussian.



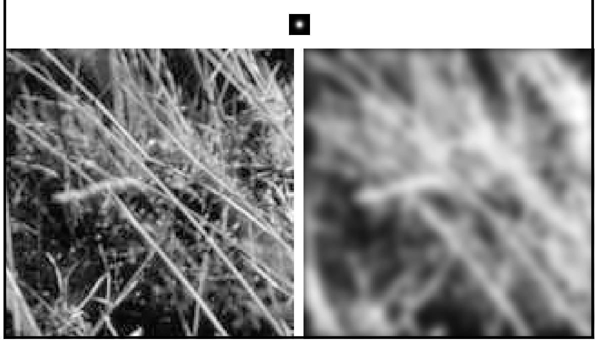
(Trucco and Verri)

Implications

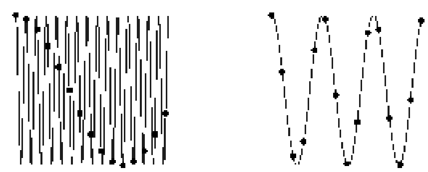
- Smoothing means removing high frequencies. This is one definition of scale.
- Sinc function explains artifacts.
- Need smoothing before subsampling to avoid aliasing.

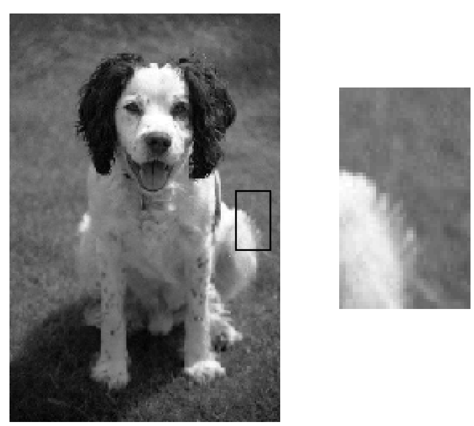
Example: Smoothing by Averaging

Smoothing with a Gaussian



Sampling

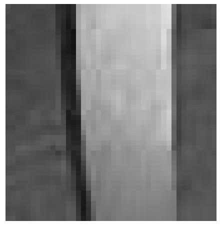




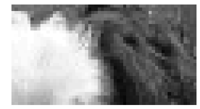
Boundary Detection - Edges

- Boundaries of objects
  - Usually different materials/orientations, intensity changes.

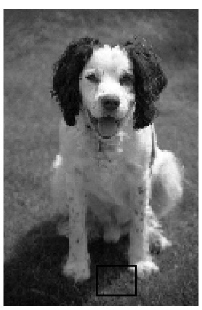
We also get: Boundaries of surfaces

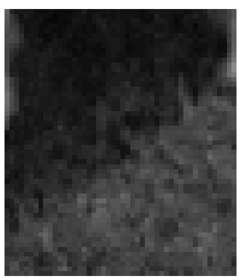


Boundaries of materials properties



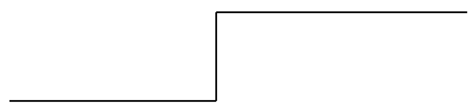
Boundaries of lighting





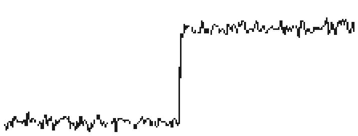
Edge is Where Change Occurs

- Change is measured by derivative in 1D
- Biggest change, derivative has maximum magnitude
- Or 2nd derivative is zero.



Noisy Step Edge

- Gradient is high everywhere.
- Must smooth before taking gradient.



Optimal Edge Detection: Canny

- Assume:
  - Linear filtering
  - Additive iid Gaussian noise
- Edge detector should have:
  - Good Detection. Filter responds to edge, not noise.
  - Good Localization: detected edge near true edge.
  - Single Response: one per edge.

Optimal Edge Detection: Canny (continued)

- Optimal Detector is approximately Derivative of Gaussian.
- Detection/Localization trade-off
  - More smoothing improves detection
  - And hurts localization.
- This is what you might guess from (detect change) + (remove noise)

So, 1D Edge Detection has steps:

1. Filter out noise: convolve with Gaussian
2. Take a derivative: convolve with [-1 0 1]
3. Find the peak.

- *Matlab*
- We can combine 1 and 2.
- *Matlab*