Announcements

- Final Exam May 16th, 8 am (not my idea).
- Practice quiz handout 5/8.
- Review session: think about good times.
- PS5: For challenge problems, use built in functions as you like. Be careful that you use them properly. Useful ones: conv2, fspecial, imresize.
- Questions on problem set?
- Readings for today and Tuesday:

Forsyth and Ponce 18.1,18.2,18.5,18.6.

Movies we missed last time:

Recognizing Objects: Feature Matching

- Problem: Match when viewing conditions change a lot.
  - Lighting changes: brightness constancy false.
  - Viewpoint changes: appearance changes, many viewpoints.
- One Solution: Match using edge-based features.
  - Edges less variable to lighting, viewpoint.
  - More compact representation can lead to efficiency.
- Match image or *object* to image
  - If object, matching may be asymmetric
  - Object may be 3D.
- Line finding was an example: line=object; points=image.

Lighting affects appearance

Problem Definition

- An Image is a set of 2D geometric features, along with positions.
- An Object is a set of 2D/3D geometric features, along with positions.
- A pose positions the object relative to the image.
  - 2D Translation; 2D translation + rotation; 2D translation, rotation and scale; planar or 3D object positioned in 3D with perspective or scaled orth.
- The best pose places the object features nearest the image features

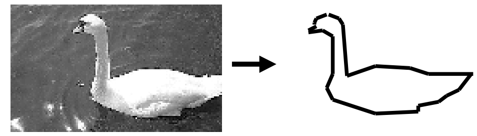
Strategy

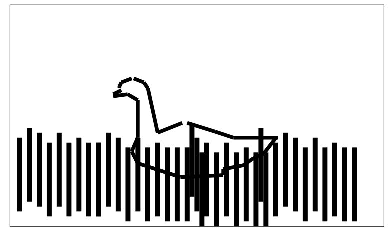
- Build feature descriptions
- Search possible poses.
  - Can search space of poses
  - Or search feature matches, which produce pose
- Transform model by pose.
- Compare transformed model and image.
- Pick best pose.

Presentation Strategy

- Already discussed finding features.
- First discuss picking best pose since this defines the problem.
- Second discuss search methods appropriate for 2D.
- Third discuss transforming model in 2D and 3D.
- Fourth discuss search for 3D objects.

Example

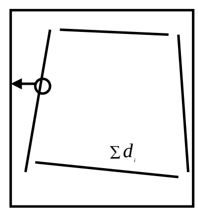




Evaluate Pose

- We look at this first, since it defines the problem.
- Again, no perfect measure;
  - Trade-offs between veracity of measure and computational considerations.

Chamfer Matching

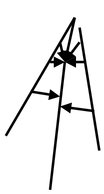


For every edge point in the transformed object, compute the distance to the nearest image edge point. Sum distances.

$$\sum_{i=1}^{n} \min(\| p_i, q_1 \|, \| p_i, q_2 \|, \ldots \| p_i, q_m \|)$$

Main Feature:

- Every model point matches an image point.
- An image point can match 0, 1, or more model points.



Variations

- Sum a different distance
  - *f(d)* = d^2
  - or *Manhattan distance*.
  - *f(d) = 1* if *d* < threshold, 0 otherwise.
    - This is called *bounded error*.
- Use maximum distance instead of sum.
  - This is called: *directed Hausdorff distance*.
- Use other features
  - Corners.
  - Lines. Then position and angles of lines must be similar.
    - Model line may be subset of image line.

Other comparisons

- Enforce each image feature can match only one model feature.
- Enforce continuity, ordering along curves.
- These are more complex to optimize.

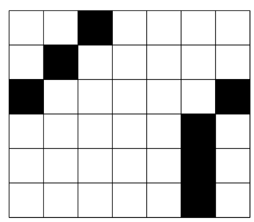
Presentation Strategy

- Already discussed finding features.
- First discuss picking best pose since this defines the problem.
- Second discuss search methods appropriate for 2D.
- Third discuss transforming model in 2D and 3D.
- Fourth discuss search for 3D objects.

Pose Search

- Simplest approach is to try every pose.
- Two problems: many poses, costly to evaluate each.
- We can reduce the second problem with:

Pose: Chamfer Matching with the Distance Transform



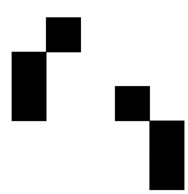
2	1	0	1	2	3	2
1	0	1	2	3	2	1
0	1	2	3	2	1	0
1	2	3	2	1	0	1
2	3	3	2	1	0	1
3	4	3	2	1	0	1

Example: Each pixel has (Manhattan) distance to nearest edge pixel.

D.T. Adds Efficiency

- Compute once.
- Fast algorithms to compute it.
- Makes Chamfer Matching simple.

Then, try all translations of model edges. Add distances under each edge pixel.



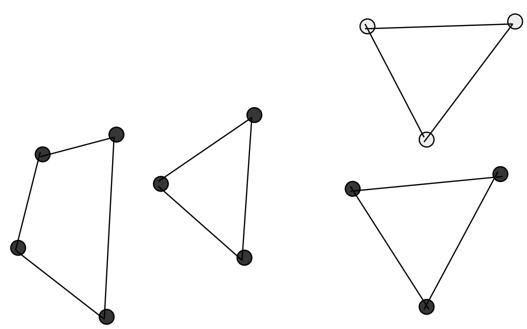
2	1	0	1	2	3	2
1	0	1	2	3	2	1
0	1	2	3	2	1	0
1	2	3	2	1	0	1
2	3	3	2	1	0	1
3	4	3	2	1	0	1

Computing Distance Transform

- It’s only done once, per problem, not once per pose.
- Basically a shortest path problem.
- Simple solution passing through image once for each distance.
  - First pass mark edges 0.
  - Second, mark 1 anything next to 0, unless it’s already marked. Etc….
- Actually, a more clever method requires 2 passes.

Pose: Ransac

- Match enough features in model to features in image to determine pose.
- Examples:
  - match a point and determine translation.
  - match a corner and determine translation and rotation.
  - Points and translation, rotation, scaling?
  - Lines and rotation and translation?



Pose: Generalized Hough Transform

- Like Hough Transform, but for general shapes.
- Example: match one point to one point, and for every rotation of the object its translation is determined.

Presentation Strategy

- Already discussed finding features.
- First discuss picking best pose since this defines the problem.
- Second discuss search methods appropriate for 2D.
- Third discuss transforming model in 2D and 3D.
- Fourth discuss search for 3D objects.

Computing Pose: Points

- Solve *I* = *S*P.*
  - In Structure-from-Motion, we knew *I*.
  - In Recognition, we know *P.*
- This is just set of linear equations
  - Ok, maybe with some non-linear constraints.

Linear Pose: 2D Translation

$$\begin{pmatrix} u_1 & u_2 & . & . & . & u_n \\ v_1 & v_2 & & & & v_n \end{pmatrix} = \begin{pmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \end{pmatrix} \begin{pmatrix} x_1 & x_2 & . & . & . & x_n \\ y_1 & y_2 & & & & y_n \\ 1 & 1 & & & & 1 \end{pmatrix}$$

We know *x,y,u,v*, need to find translation.

For one point, $u_1 - x_1 = t_x$; $v_1 - x_1 = t_y$

For more points we can solve a least squares problem.

Linear Pose: 2D rotation, translation and scale

$$\begin{pmatrix} u_1 & u_2 & . & . & . & u_n \\ v_1 & v_2 & & & & v_n \end{pmatrix} = s\begin{pmatrix} \cos\theta & \sin\theta & t_x \\ -\sin\theta & \cos\theta & t_y \end{pmatrix} \begin{pmatrix} x_1 & x_2 & . & . & . & x_n \\ y_1 & y_2 & & & & y_n \\ 1 & 1 & & & & 1 \end{pmatrix}$$

$$\begin{pmatrix} a & b & t_x \\ -b & a & t_y \end{pmatrix} \begin{pmatrix} x_1 & x_2 & . & . & . & x_n \\ y_1 & y_2 & & & & y_n \\ 1 & 1 & & & & 1 \end{pmatrix}$$

with $a = s\cos\theta, \quad b = s\sin\theta$

- Notice *a* and *b* can take on any values.
- Equations linear in *a, b*, translation.
- Solve exactly with 2 points, or overconstrained system with more.

$$s = \sqrt{a^2 + b^2} \qquad \cos\theta = a/s$$

Linear Pose: 3D Affine

$$\begin{pmatrix} u_1 & u_2 & . & . & . & u_n \\ v_1 & v_2 & & & & v_n \end{pmatrix} = \begin{pmatrix} s_{1,1} & s_{1,2} & s_{1,3} & t_x \\ s_{2,1} & s_{2,2} & s_{2,3} & t_y \end{pmatrix} \begin{pmatrix} x_1 & x_2 & . & . & . & x_n \\ y_1 & y_2 & & & & y_n \\ z_1 & z_2 & & & & z_n \\ 1 & 1 & & & & 1 \end{pmatrix}$$

Pose: Scaled Orthographic Projection of Planar points

$$\begin{pmatrix} u_1 & u_2 & . & . & . & u_n \\ v_1 & v_2 & & & & v_n \end{pmatrix} = \begin{pmatrix} s_{1,1} & s_{1,2} & s_{1,3} & t_x \\ s_{2,1} & s_{2,2} & s_{2,3} & t_y \end{pmatrix} \begin{pmatrix} x_1 & x_2 & . & . & . & x_n \\ y_1 & y_2 & & & & y_n \\ 0 & 0 & & & & 0 \\ 1 & 1 & & & & 1 \end{pmatrix}$$

$s_{1,3}$, $s_{2,3}$ disappear. Non-linear constraints disappear with them.

Non-linear pose

- A bit trickier. Some results:
- 2D rotation and translation. Need 2 points.
- 3D scaled orthographic. Need 3 points, give 2 solutions.
- 3D perspective, camera known. Need 3 points. Solve 4th degree polynomial. 4 solutions.

Transforming the Object

We don't really want to know pose, we want to know what the object looks like in that pose.

We start with:

$$\begin{pmatrix} u_1 & u_2 & u_3 & u_4 & ? & ? & ? \\ v_1 & v_2 & v_3 & v_4 & ? & ? & ? \end{pmatrix} = \begin{pmatrix} ? & ? & ? & ? \\ ? & ? & ? & ? \end{pmatrix} \begin{pmatrix} x_1 & x_2 & . & . & . & x_n \\ y_1 & y_2 & & & & y_n \\ z_1 & z_2 & & & & z_n \\ 1 & 1 & & & & 1 \end{pmatrix}$$

Solve for pose:

$$\begin{pmatrix} u_1 & u_2 & u_3 & u_4 & ? & ? & ? \\ v_1 & v_2 & v_3 & v_4 & ? & ? & ? \end{pmatrix} = \begin{pmatrix} s_{1,1} & s_{1,2} & s_{1,3} & t_x \\ s_{2,1} & s_{2,2} & s_{2,3} & t_y \end{pmatrix} \begin{pmatrix} x_1 & x_2 & . & . & . & x_n \\ y_1 & y_2 & & & & y_n \\ z_1 & z_2 & & & & z_n \\ 1 & 1 & & & & 1 \end{pmatrix}$$

Project rest of points:

$$\begin{pmatrix} u_1 & u_2 & . & . & . & u_n \\ v_1 & v_2 & & & & v_n \end{pmatrix} = \begin{pmatrix} s_{1,1} & s_{1,2} & s_{1,3} & t_x \\ s_{2,1} & s_{2,2} & s_{2,3} & t_y \end{pmatrix} \begin{pmatrix} x_1 & x_2 & . & . & . & x_n \\ y_1 & y_2 & & & & y_n \\ z_1 & z_2 & & & & z_n \\ 1 & 1 & & & & 1 \end{pmatrix}$$

Transforming object with Linear Combinations

No 3D model, but we've seen object twice before.

$$\begin{pmatrix} u_1^1 & u_2^1 & . & . & . & u_n^1 \\ v_1^1 & v_2^1 & & & & v_n^1 \\ u_1^2 & u_2^2 & & & & u_n^2 \\ v_1^2 & v_2^2 & & & & v_n^2 \end{pmatrix}$$

See four points in third image, need to fill in location of other points.

Just use rank theorem.

$$\begin{pmatrix} u_1^1 & u_2^1 & u_3^1 & u_4^1 & . & u_n^1 \\ v_1^1 & v_2^1 & v_3^1 & v_4^1 & . & v_n^1 \\ u_1^2 & u_2^2 & u_3^2 & u_4^2 & . & u_n^2 \\ v_1^2 & v_2^2 & v_3^2 & v_4^2 & . & v_n^2 \\ u_1^3 & u_2^3 & u_3^3 & u_4^3 & ? & ? \\ v_1^3 & v_2^3 & v_3^3 & v_4^3 & ? & ? \end{pmatrix}$$

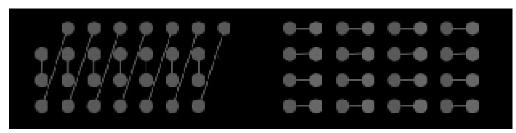
Recap: Recognition w/ RANSAC

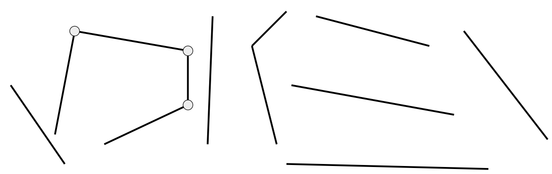
1. Find features in model and image.
   – Such as corners.
2. Match enough to determine pose.
   – Such as 3 points for planar object, scaled orthographic projection.
3. Determine pose.
4. Project rest of object features into image.
5. Look to see how many image features they match.
   – Example: with bounded error, count how many object features project near an image feature.
6. Repeat steps 2-5 a bunch of times.
7. Pick pose that matches most features.

Recognizing 3D Objects

- Previous approach will work.
- But slow. RANSAC considers n^3m^3 possible matches. About m^3 correct.
- Solutions:
  – Grouping. Find features coming from single object.
  – Viewpoint invariance. Match to small set of model features that could produce them.

Grouping: Continuity





Connectivity

- Connected lines likely to come from boundary of same object.
  – Boundary of object often produces connected contours.
  – Different objects more rarely do; only when overlapping.
- Connected image lines match connected model lines.
  – Disconnected model lines generally don't appear connected.

Other Viewpoint Invariants

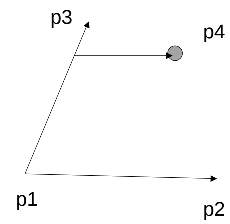
- Parallelism
- Convexity
- Common region
- ….

Planar Invariants

$$\begin{pmatrix} u_1 & u_2 & . & . & . & u_n \\ v_1 & v_2 & & & & v_n \end{pmatrix} = \begin{pmatrix} s_{1,1} & s_{1,2} & s_{1,3} & t_x \\ s_{2,1} & s_{2,2} & s_{2,3} & t_y \end{pmatrix} \begin{pmatrix} x_1 & x_2 & . & . & . & x_n \\ y_1 & y_2 & & & & y_n \\ 0 & 0 & & & & 0 \\ 1 & 1 & & & & 1 \end{pmatrix}$$

$$= \begin{pmatrix} s_{1,1} & s_{1,2} & t_x \\ s_{2,1} & s_{2,2} & t_y \end{pmatrix} \begin{pmatrix} x_1 & x_2 & . & . & . & x_n \\ y_1 & y_2 & & & & y_n \\ 1 & 1 & & & & 1 \end{pmatrix}$$

A t



p4 = p1 + a(p2-p1) + b(p3-p1)

A(p4)+ t = A(p1+a(p2-p1) + b(p3-p1)) + t

= A(p1)+t + a(A(p2)+t – A(p1)-t) + b(A(p3)+t – A(p1)-t)

p4 is linear combination of p1,p2,p3. Transformed p4 is same linear combination of transformed p1, p2, p3.

What we didn't talk about

- Smooth 3D objects.
- Can we find the guaranteed optimal solution?
- Indexing with invariants.
- Error propagation.