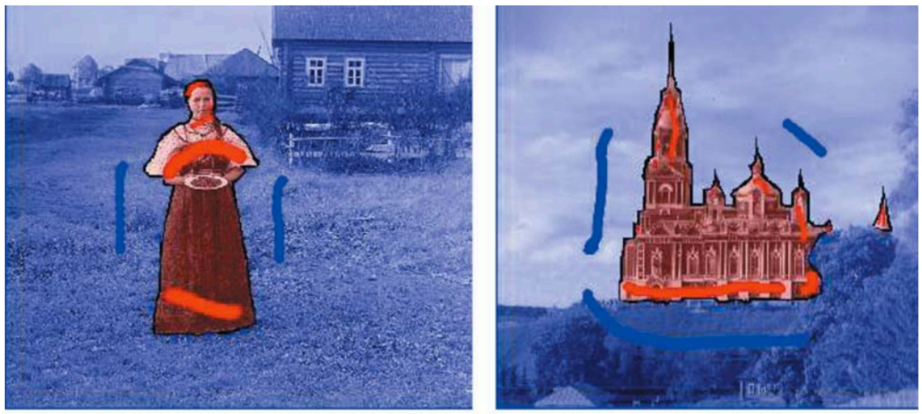
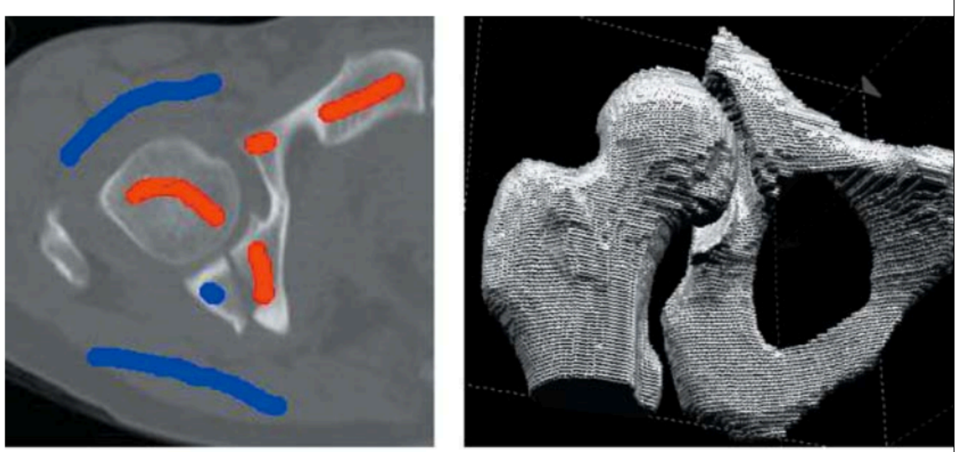


(a) A woman from a village (b) A church in Mozhaisk (near Moscow)



(a) A slice with seeds (b) 3D object

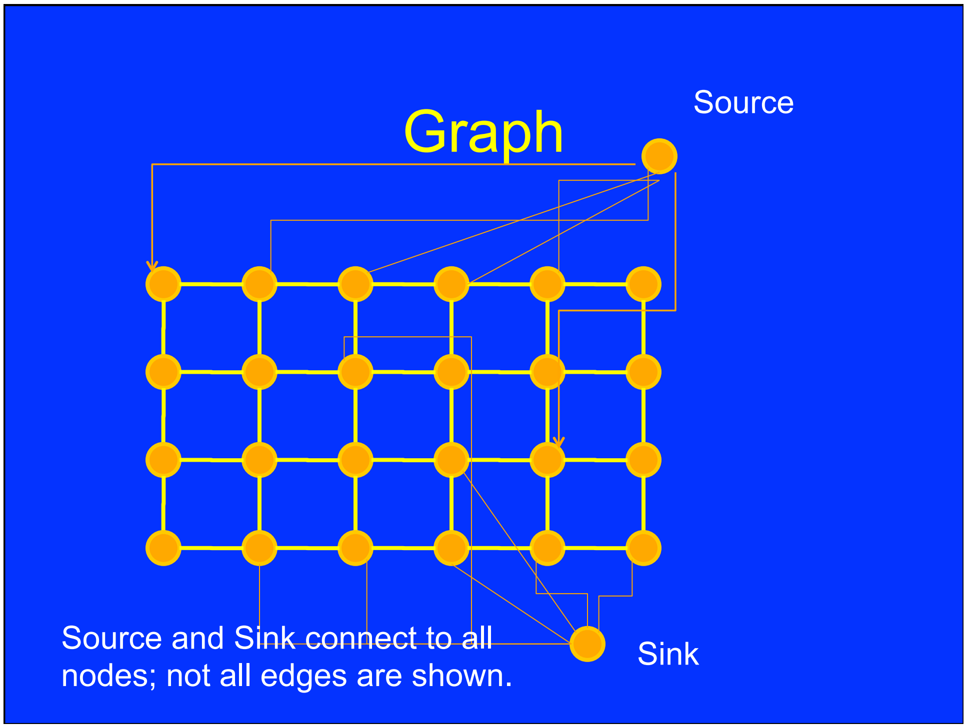
ntation of bones in a CT volume [256x256x119].

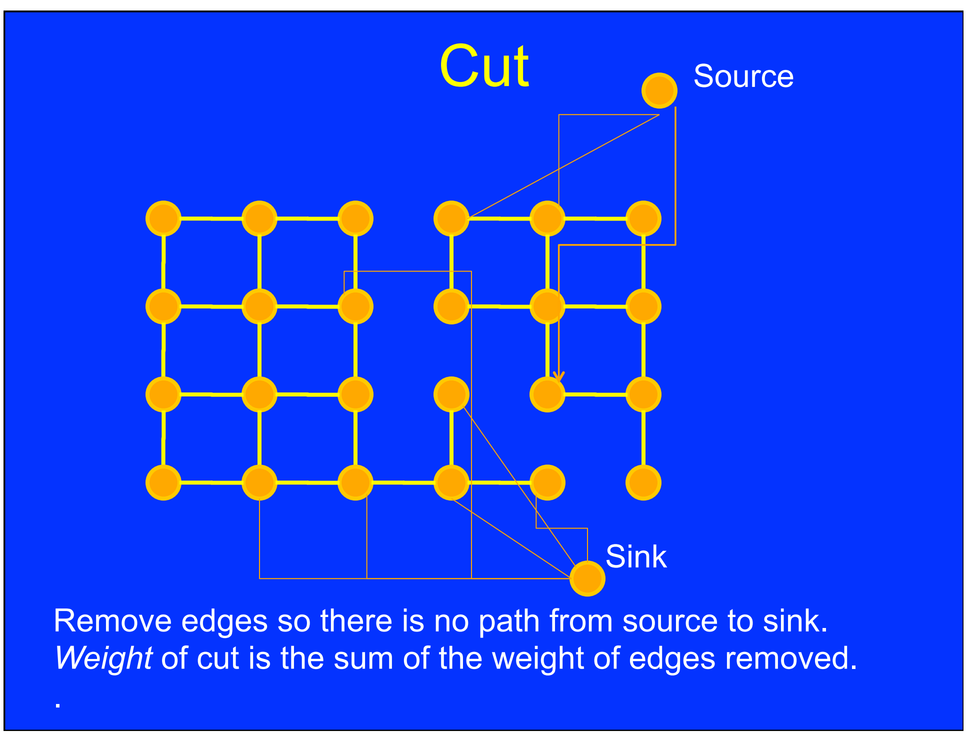
Concepts for Graph Cuts

- Segmentation by estimating *probability* that each pixel is foreground (fg) or background (bg).
  - User input: w/ probability 1 pixels are fg or bg.
  - These provides information about *color* of pixel.
    - Using histograms to estimate probabilities.
  - Maximizing probabilities by sum of weights.
  - Pairwise Probabilities
- Maximizing probabilities using graph cuts.

Graph Cuts for Segmentation

- Seek division of image into *foreground* and *background.*
- Turn image into graph, each pixel connected to neighbors and special source (foreground) and sink (background nodes).
- A *cut* of the graph divides it into foreground and background.
- Edge weights determine:
  - Is a pixel likely to be foreground or background?
  - Is a pixel likely to have same label as neighbors?


Graph
Source
Sink
Source and Sink connect to all nodes; not all edges are shown.


Cut
Source
Sink
Remove edges so there is no path from source to sink.
Weight of cut is the sum of the weight of edges removed.
.

Min Cut

- *Min Cut* is the cut with the lowest weight
- Well studied problem with many practical applications.

Min Cut for Interactive Segmentation

- Assume user has specified some pixels as foreground/background.
- Identify a cut as a segmentation:
  - Pixels connected to source are foreground.
  - Pixels connected to sink are background.
  - The weight of edges in the cut should reflect knowledge of foreground and background.

Hard constraints

- Let S be source, T be sink, w(p,q) is weight of edge between nodes p & q.
- If pixel p definitely is foreground, make:

  w(p,S) very big, w(p,T) = 0.
  - Edge from p to S (E(p,S)) will never be cut.
  - E(q,T) must therefore always be cut so there's no path from S to T

Data Term

- Let F be set of pixels known to be foreground. Let B be background pixels.
- What about $\boxed{p \notin F \cup B}$?
- Compare properties of p to foreground and background pixels.

Color Histogram Comparison

1. Compute color histograms for foreground and background, $h_{f,}$ h_b
2. *Smooth* histograms by adding a constant to each bin.
3. Normalize histograms so they sum to 1 (like probabilities).
4. Find Pr(p|Foreground), Pr(p|Background) by finding bin p belongs to, and looking up values in normalized histograms.
5. w(p,S) = -log(Pr(p|Background))
6. w(p,T) = -log(Pr(p|Foreground))

Histograms with Graph Cut

- Why –log?
  - We are adding weights. We multiply probabilities, so add logs.
  - We maximize probabilities, so minimize –log.
- Example: if p has a color that rarely appears in foreground, edge to source will have low weight.
- Why smooth? We only have a small sample.

Graph Cut with Data Term

- Suppose we compute mincut with just these edges to source and sink.
- Segmentation respects user input.
- Other pixels classified based on whether they resemble foreground or background.
- Results can be quite spotty.

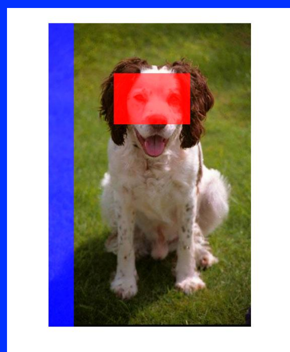
Smoothness term

- If a pixel, p, is foreground, its neighbor, q, is likely to be foreground.
  - Especially if p and q are similar.

$$w(p,q) = e^{-\frac{(I(p)-I(q))^2}{2\sigma^2}}$$

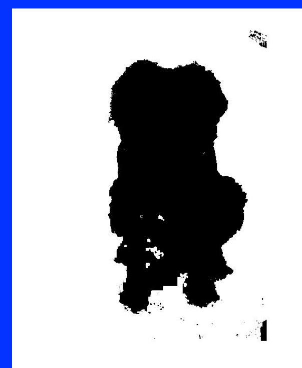
  - This is gradient, normalized in ad-hoc way.
  - Note, gradient is taken between pixels, not on one pixel.

Results



User Input

No Edge Weights
Just Data Term



Full segmentation