Practice Midterm

CMSC 426

The midterm will cover material up to and including human perceptual grouping. Here are some topics it will be helpful to master for the midterm:

- Correlation
  - How do you perform correlation in 1D or 2D? What is a box filter? How do you create a Gaussian filter? How do you create filters to compute derivatives? Fourier transforms (only for challenge problems).
- Edge detection
  - Image gradients
    - How to compute the image gradient for a continuous or discrete image.
    - What is the direction and magnitude of a gradient?
    - How to compute the directional derivative of an image.
    - What does the gradient tell you about the direction in which the image changes most rapidly, and how rapidly it changes in that direction.
  - Non-maximum suppression
- Human perceptual grouping
  - Basic gestalt grouping cues.
- There will not be questions about Matlab coding.

In particular, you should understand a few equations:

1D and 2D correlation

$$F \circ I(x) = \sum_{i=-N}^{N} F(i) I(x+i) \qquad F \circ I(x,y) = \sum_{j=-N}^{N} \sum_{i=-N}^{N} F(i,j) I(x+i, y+j)$$

1D and 2D Convolution

$$F * I(x) = \sum_{i=-N}^{N} F(i) I(x-i) \qquad F * I(x,y) = \sum_{j=-N}^{N} \sum_{i=-N}^{N} F(i,j) I(x-i, y-j)$$

Definition of a gradient:

$$\nabla I = \left(\frac{\partial I}{\partial x}, \frac{\partial I}{\partial y} \right)$$

Formula for using the gradient to determine how the image changes as you move in a particular direction

$$I(x+\Delta\cos\theta, y+\Delta\sin\theta) - I(x,y) \approx \langle v, \nabla I \rangle \text{ where } v \equiv (\Delta\cos\theta, \Delta\sin\theta)$$

Definition of the partial derivative.

$$\frac{\partial I(x,y)}{\partial x} = \lim_{\Delta x \to 0} \frac{I(x+\Delta x, y) - I(x,y)}{\Delta x}$$

I'm asking you to "memorize" these formulas because I feel that if you really understand what is in the formula, you will have it memorized.

Below are some practice problems. These are not guaranteed to cover everything you need to know, but should be helpful.

Practice Problems

1. Show the result of convolving the kernel, *k*, with the image, *I*.

1/4	1/2	1/4

k

0	0	...	0	1	2	...	10	0	0	...

I

2. Give an example that shows that correlation is not associative
3. Suppose we have an image whose intensities are described by the equation $x^3-2xy+7$.
   a) What is the magnitude of the image gradient at the point (7,4)?
   b) Suppose we move in the image to the point $(7+\delta, 4+2\delta)$. Use the image gradient to predict what the intensity will be at this point.
   c) **Challenge problem:** What is the actual intensity at this point. What does this tell you about the accuracy of your solution using the gradient?
4. Consider the following, small image:

4	6	9	12	15
3	4	6	9	12
3	3	**4**	6	9
2	3	3	4	6
2	1	3	3	4

   a) Compute the magnitude and the direction of the gradient for the central point (**4**).

b) Would the Canny edge detector consider this point to be an edge? Explain why or why not in detail, describing what conditions are needed to decide this.

Workshop Questions

1. Consider an image in which the intensities are given by the equation $I(x,y) = x^2+2y$.
    a. What is the gradient of the image?
    b. What is the gradient at the location (x,y) = (2,3)?
    c. At the location (x,y) = (2,3), what is the directional derivative in the direction of (3,2)? That is, if you were to move a small amount in this direction, how much would you expect the image to change?
2. Make part of a discrete image with intensities chosen according to the equation in 1. That is, for example, make an image so that the pixel at location (7,3) has the value $7^2+2*3 = 55$.
    a. If you were to compute the discrete image gradient at each pixel using derivative filters, what values would you get for the image gradient everywhere?
    b. How do these compare to the gradients you get in problem 1? If there is a difference, why is this?
3. Suppose you have an image in which the intensities are given by the equation $I(x,y) = 100 - x^2 - y^2 + xy + x + 4y$. What is the intensity of the brightest point in the image?
4. Suppose g is a Gaussian filter with a standard deviation of 1.
    a. If I filter an image twice with g, show why this is equivalent to filtering it once with a larger Gaussian.
    b. What should the standard deviation of the larger Gaussian filter be?
5. Suppose g is a Gaussian filter with a standard deviation of 1.
    a. If I filter an image twice with g, show why this is equivalent to filtering it once with a larger Gaussian.
    b. What should the standard deviation of the larger Gaussian filter be?