Practice Midterm 2

CMSC 426

The midterm will cover material up to and including stereo geometry. The midterm will be cumulative, covering everything up to this point, including material covered in the first midterm. However, there may be an extra emphasis on material not covered in the midterm. The previous practice midterm can serve as a review for that material; here I'll only discuss new topics. But you should think of both practice midterms as providing preparation for the second midterm.

Here are some topics it will be helpful to master for the midterm:

- Interactive Segmentation with Min Cut
  - How you can represent image as a graph.
  - Understand how to create edge weights.
- Color and Texture
- Optical Flow
- Corner Detection
- Perspective projection
  - Using the formula for the standard configuration
    - That is, $(x,y) = f(X/Z, Y/Z)$
  - Using geometry for perspective with an arbitrary camera.
  - Given an image, where is the object in the world?
  - Vanishing points and the horizon
- Epipolar Geometry
  - Given two cameras and one image point, where does point appear in second image
    - For simple case.
    - For more generic case
  - Given two cameras and a 3D point, what are the conjugate epipolar lines?
  - Given two cameras, provide some epipolar lines.
- Disparity equation
  - Given two image points with standard cameras, find 3D point.
  - Do this for non-standard cameras
- Image rectification

Practice: The goal of this is to give you samples of the sorts of questions and topics that will come up in the midterm. Some of these questions may be a bit more involved or more vague than those that I would ask in a real midterm. It is very likely that there will be at least one question on the midterm that is quite similar to a practice question.

1. Suppose we have a camera with a focal point at (10,0,10). It is looking in the *x* direction with a focal length of 1/2. Suppose there is a point in the scene with 3D coordinates (20, 10, 8).
    a. Where is the image plane?

x = 10 ½.

    b. Give the 3D coordinates of the point in the image plane where the point will appear.

A line connecting the focal point and scene point is given by (x,y,z) = (10,0,10) + t(10,10,-2). If we want a point on this line that intersects the image plane, we will have t = 1/20. The corresponding point is (10 ½, ½, 9 9/10).

2. Suppose we have a camera with a focal point at (10,4,8). The image plane is at *y = 5.*
    a. What is the focal length of the camera?

1.

    b. Suppose we observe a point in the world at the location (12,5, 6) in the camera. Give an equation for a line that describes the location of that point in the world.

This is just the line connecting the focal point to the image point. (x,y,z) = (10,4,8) + t(2,1,-2).

3. Suppose we have a camera with a focal point at (0,0,0), pointing in the *z* direction, with a focal length of 1. We have a second camera with a focal point of (10,0,0) and an image plane at *x+2z = 9.* Give two conjugate epipolar lines.

    We can get these by taking any plane that includes the two focal points. Such a plane is y = 0. One way to describe a line in 3D is as the intersection of two planes. So we can just say that the first line is: y = 0, z = 1, and the second is y=0, x+2z = 9.

4. Suppose we have a left camera with focal point (0,0,0) and a right camera with focal point (5,0,0). Both cameras point in the *z* direction with a focal length of 1. Choose a point in 3D. Calculate where it will appear in both cameras. Then demonstrate that you can use the formula Z = fT/d to recover the *z* coordinate of the point.

Let's pick the point at (2,5,10). This will appear in the left image at (1/5, ½, 1). It will appear in the right image at (4.7, ½, 1). So the disparity is ½. f = 1 and T = 5. So we have Z = 5/(1/2) = 10.

5. Suppose we have two cameras with focal point of (0,0,0) and (10,0,0). The image plane for the first camera is $z = 1$, and for the second camera is $z = 2$. Are the epipolar lines still horizontal (in class we showed they'd be horizontal if both cameras had image planes of $z = 1$)? Show that they are or explain why they aren't.

Any plane that goes through the two focal points has an equation of Ay = z, for some A. If we intersect this with the plane z = c, where c is some constant, we get Ay = c, or y = A/c. So this plane intersects the image planes in lines of a constant height.

6. Suppose I look at the world with a camera with a focal point of (0,0,0) and an image plane of $z = 1$. I see a distinctive red point at (5,5,1). Then I rotate the camera so that it still has a focal point at (0,0,0), but now the image plane is at x = 1. I see the red dot in this image at the location (1,1,1/5). Can I apply stereo to the two images and locate the point in 3D? Explain why not, or determine the point's location.

The first image tells me that the point lies on the line (x,y,z) = t(5,5,1). The second image tells me it lies on the line (x,y,z) = t(1,1,1/5). These are the same line. So the second image gives me no extra information.

7. Why do we represent a colored image using three values for each pixel (red, green, and blue). Why don't we use four colors, or two, or six?

Because the human visual system has three types of cones, sensitive to red, blue and green. So we can reproduce the sensation of most colors by just mixing three colors together.

8. Show that given any two cameras, all the epipolar lines in one image are either parallel or intersect in a single, common point.

Consider the line that connects two focal points. This is in every plane that connects the focal points. The point where this intersects an image plane is called the epipole. The epipole is in the intersection of the image plane and any plane containing the two focal points, so that it is on every epipolar line. If the line between the two focal points is parallel to the image plane then it is like our standard setup, and all epipolar lines are parallel.

9. Suppose we have a 5x5 window, in which 24 of the image gradients are pointing in the x direction, and 1 image gradient is pointing in the y direction. All the gradients have a magnitude of one.
   a. If we use this window to build the matrix C:

$$C = \begin{bmatrix} \sum I_x^2 & \sum I_x I_y \\ \sum I_x I_y & \sum I_y^2 \end{bmatrix} \overset{\text{Using SVD}}{=} R_2 \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} R_1$$

what will be the smallest eigenvalue of C?

C will be [24, 0; 0, 1]. So the smallest eigenvalue will be 1.

b. What will happen if we rotate the gradients by 45 degrees. So we still have 24 gradients in one direction and one gradient in the orthogonal direction? What will be the smallest eigenvalue then? You can either answer this mathematically, or using Matlab.

Rotating the gradients will have no effect.