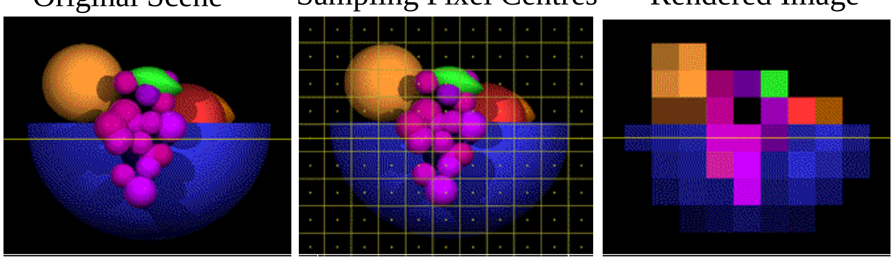
Sampling and Aliasing

- Sects. 4.17-18 in H&B
- Display is discrete and world is continuous
  (at least at the level we perceive)
- *Sampling:* Convert continuous to discrete
- *Reconstruction:* Converting from discrete to continuous
- *Aliasing:* Artifacts arising from sampling and consequent loss of information
- *Anti-aliasing:* Attempts to overcome aliasing

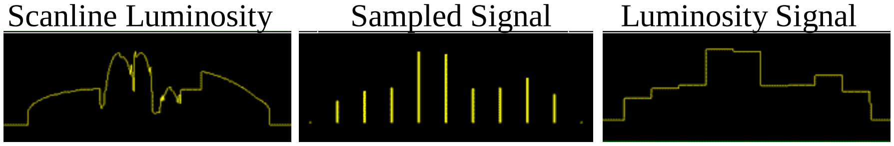
Sampling

Occurs when the sampling inherent in rendering does not contain enough information for an accurate image.

Original Scene | Sampling Pixel Centres | Rendered Image



Scanline Luminosity | Sampled Signal | Luminosity Signal



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Some Aliasing Artifacts

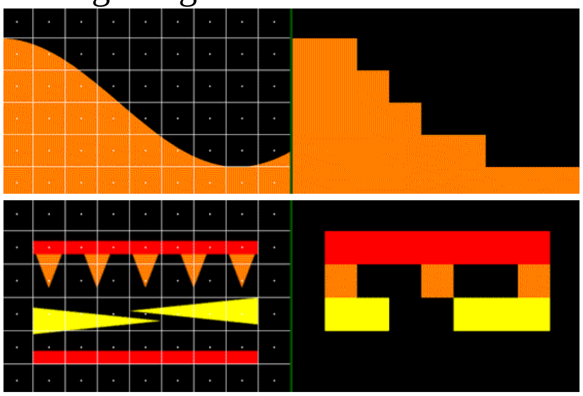
- *Spatial:* Jaggies, Moire
- *Temporal:* Strobe lights, “Wrong” wheel rotations
- *Spatio-Temporal:* Small objects appearing and disappearing

Effects of Aliasing

Common aliasing errors (called artefacts) are: jagged profiles, disappearing or improper fine detail, and disintegrating textures.

Jagged Profiles

Loss of Detail



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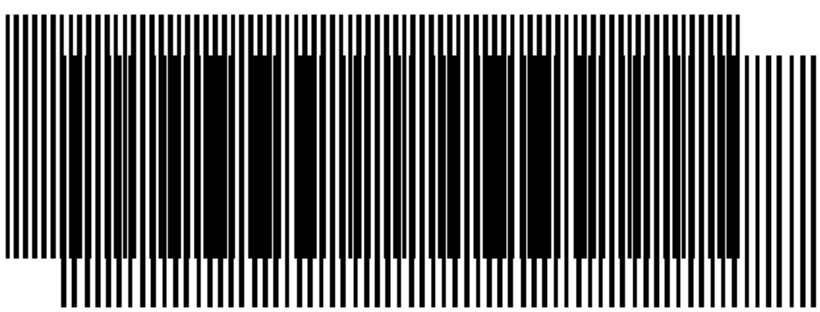
Jaggies

Moire Patterns



Moire Patterns

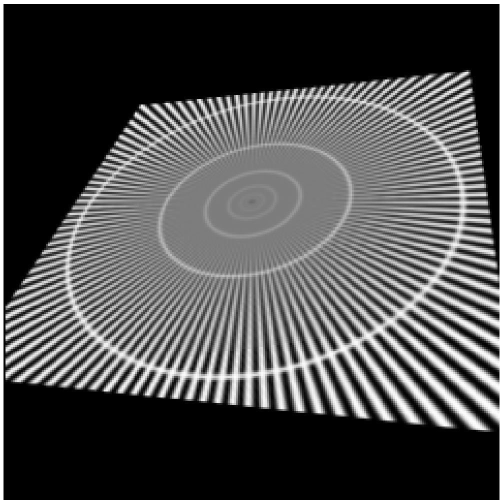


Image Courtesy:
Chen, Dachille, Kaufman

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Moire Patterns

Image from *http://www.daube.ch/docu/glossary/moiree.html*

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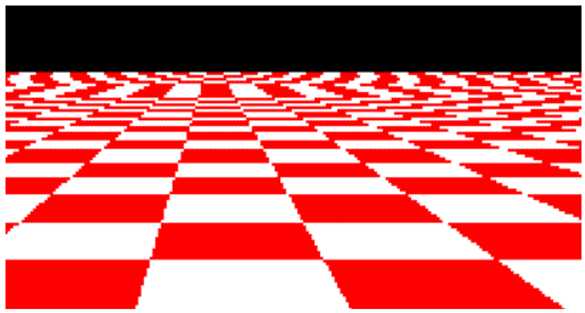
Disintegrating Texture

- The checkers on a plane should become smaller with distance.
- But aliasing causes them to become larger and/or irregular.
- Increasing resolution only moves the artefact closer to the horizon.



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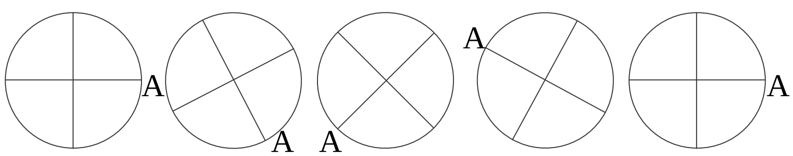
Temporal Aliasing



The wheel appears to be moving backwards at about ¼ angular frequency:

http://lite.bu.edu/lite1/perception/anamorphic/wade.html

Image as a 2D Signal

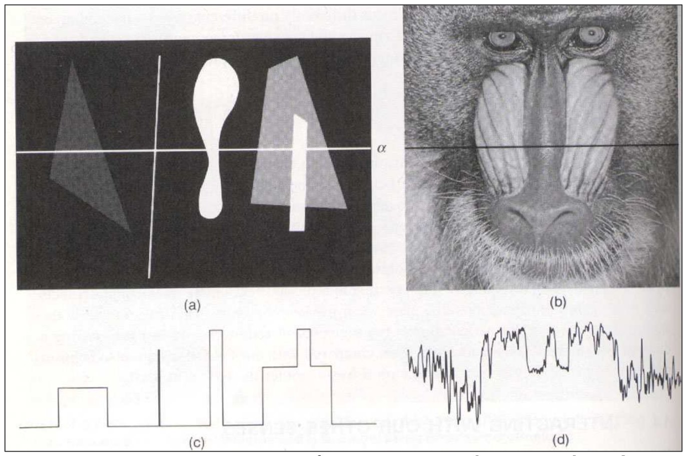


Image from *Computer Graphics: Principles and Practice*
by Foley, van Dam, Feiner, and Hughes

Fourier Series

- Analytic geometry gives a coordinate system for describing geometric objects.
- Fourier series gives a coordinate system for functions.

Basis

- P=(x,y) means P = x(1,0)+y(0,1)
- Similarly:

$$f(\theta) = a_{00} + a_{11}\cos(\theta) + a_{12}\sin(\theta) + a_{21}\cos(2\theta) + a_{22}\sin(2\theta) + \dots$$

Note, I'm showing non-standard basis, these are from basis using complex functions.

Fourier Analysis

Represent a function as a weighted sum of sines

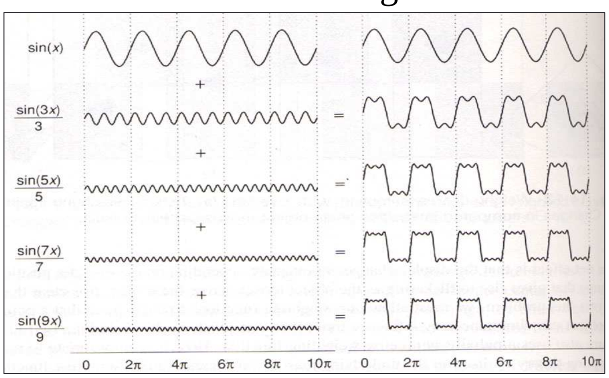


Image from *Computer Graphics: Principles and Practice* by Foley, van Dam, Feiner, and Hughes

Example

$$\forall c, \exists a_1, a_2 \text{ such that}:$$

$$\cos(\theta + c) = a_1 \cos\theta + a_2 \sin\theta$$

Orthonormal Basis

- ||(1,0)||=||(0,1)||=1
- (1,0).(0,1)=0
- Similarly we use normal basis elements eg:

$$\frac{\cos(\theta)}{\left\|\cos(\theta)\right\|} \qquad \left\|\cos(\theta)\right\| = \sqrt{\int_0^{2\pi} \cos^2\theta \, d\theta}$$

- While, eg:

$$\int_0^{2\pi} \cos\theta \sin\theta \, d\theta = 0 \qquad \int_0^{2\pi} \cos\theta \cos 2\theta \, d\theta = 0$$

Coordinates with Inner Products

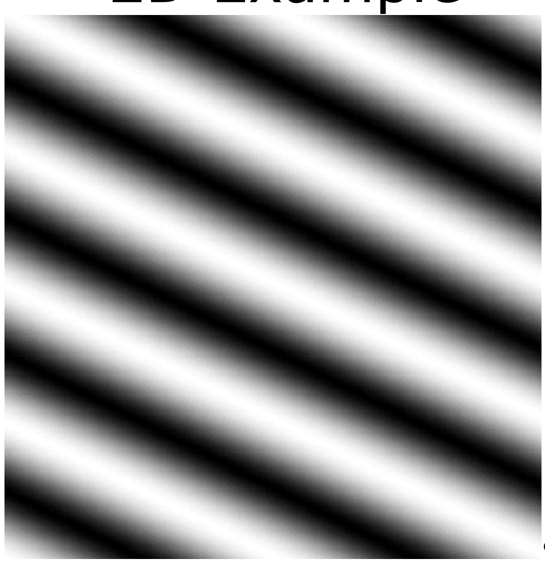
$$x = (x, y) \bullet (1,0) \quad y = (x, y) \bullet (0,1)$$

$$(x, y) = x(1,0) + y(0,1)$$

$$a_{i,1} = \int_0^{2\pi} f * \frac{\cos i\theta}{\|\cos i\theta\|} d\theta \quad a_{i,2} = \int_0^{2\pi} f * \frac{\sin i\theta}{\|\sin i\theta\|} d\theta$$

$$f = \sum a_{i,1} \frac{\cos i\theta}{\|\cos i\theta\|} + \sum a_{i,2} \frac{\sin i\theta}{\|\sin i\theta\|} + a_{0,0}$$

2D Example



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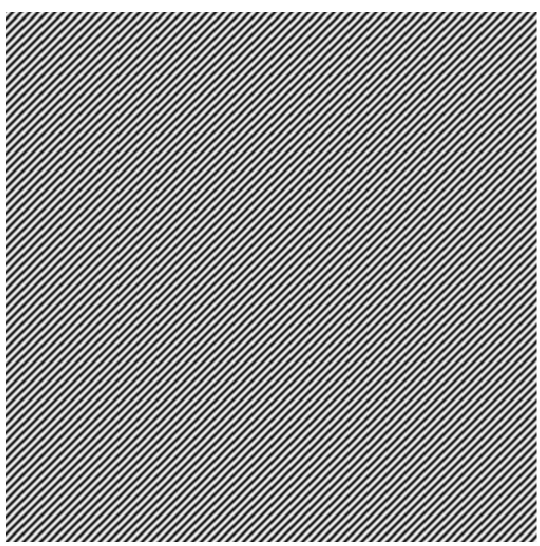

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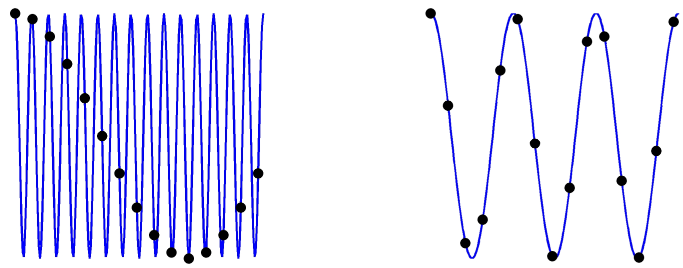
Aliasing is caused by sampling frequencies that are too high



• The Fourier basis allows us to see which part of the image is causing trouble. The high frequency harmonics.

• We'll get rid of these.

Convolution

- This is a very general concept
- But in this class, we'll just talk about convolutions that perform smoothing.
- Averaging: replace each pixel by the average in its neighborhood.
  - This can be done for continuous images with an integral
- Can take a weighted average.
  - The most common uses a normal (Gaussian) function for weights.

Weighted Average

- We define a set of weights, w_{-k}, w_{-k+1}, …,

w_{-1},w_0, w_1,…, w_k. Then, smooth *g* by: $\sum_{i=-k}^{k} w_i g(x+i)$

- This set of weights is a *filter*.
- For simple average, all $w = 1/(2k+1)$
- Other common alternative is to pick *w* from Normal (Gaussian) distribution.

Convolution Theorem

$$f \otimes g = T^{-1} F * G$$

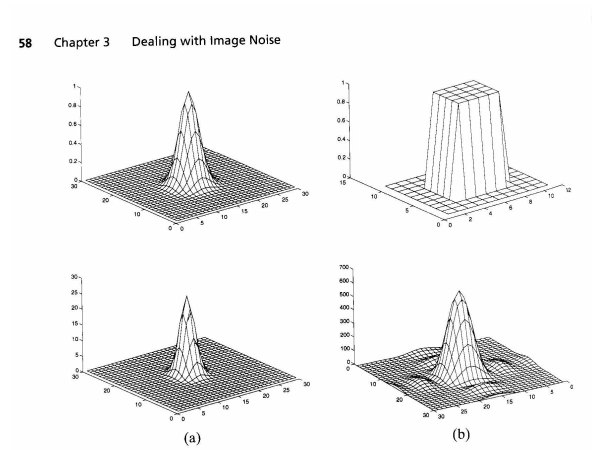
- f is a description of the weights in the weighted average (the *filter)*, g is the image.
- *F,G* are transform of *f,g*

That is, *F* contains coefficients, when we write *f* as linear combinations of harmonic basis.

This says convolution is equivalent to multiplication in the transform domain.

Examples

- Transform of box filter is sinc.
- Transform of Gaussian is Gaussian.



(Trucco and Verri)

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Implications

- Smoothing a sine or cosine just scales it (reduces amplitude).
- Smoothing a high frequency sine or cosine reduces its amplitude more than smooth a low frequency one.
- Smoothing an image reduces high frequencies more than low ones.
- Sinc function explains artifacts.
- Need smoothing before subsampling to avoid aliasing.

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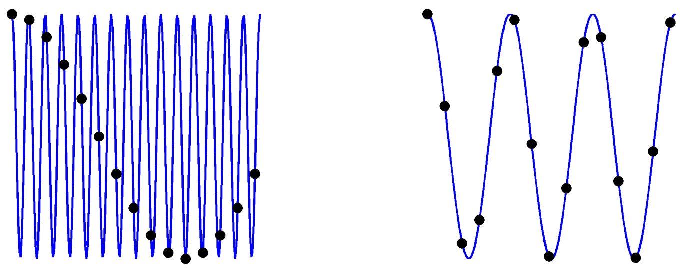
Sampling



Every sample gives a linear equation in $a_{i,j}$.

Need two samples for every frequency.

Previous examples

- Disintegrating Texture
  – As squares get small, frequency grows.
  – At some point, sampling is slower than frequency of changes.
- Wagon wheel.
  – As wheel moves fast, we cannot sample world that fast.

Anti-aliasing: Fixing Aliasing

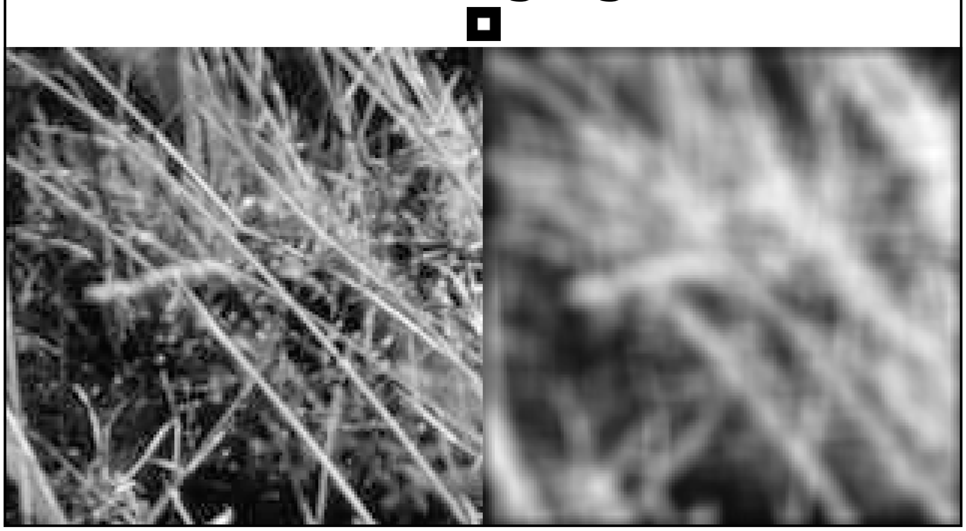
- *Nyquist Frequency:* Need at least twice the highest frequency in the signal to correctly reconstruct
- Example
  - Phone: 700 Hz at 8 bits per sample
  - CD Player: 44.1 kHz at 16 bits per sample
- Solutions:
  - **Prefiltering:** Lower the maximum frequency (filter out high frequencies)
  - **Super Sampling:** Raise the sampling frequency
  - **Stochastic Sampling:** Sample randomly instead of uniformly

Strategy for anti-aliasing

- Remove high frequencies
  - Intuitively, we do this by averaging.
  - High frequency = rapid change
  - Averaging slows down the changes
  - This can be made more rigorous
- Then sample

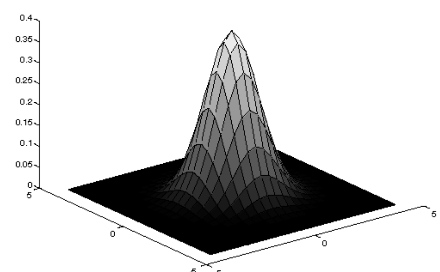
Demo

Example: Smoothing by Averaging



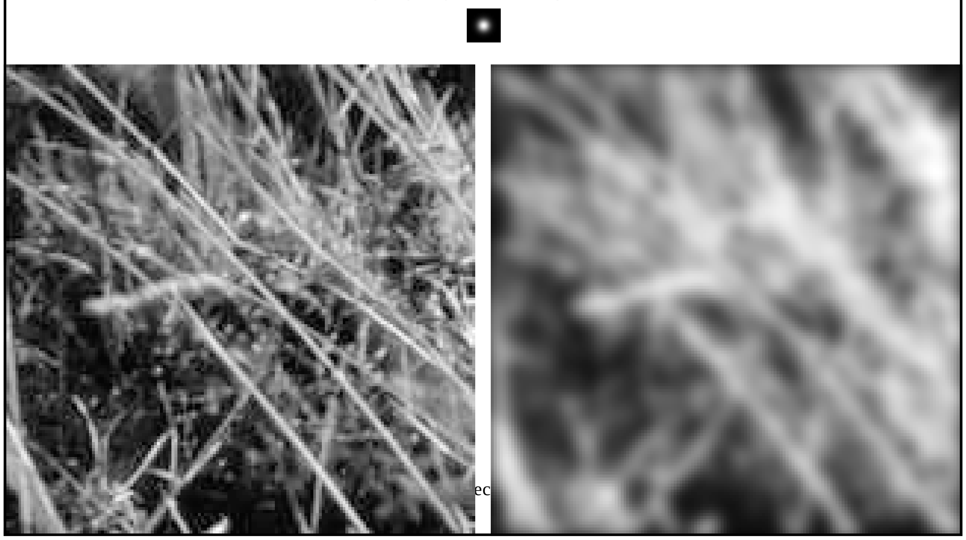
Gaussian Averaging

- Rotationally symmetric.
- Weights nearby pixels more than distant ones.
  - This makes sense as probabalistic inference.



- A Gaussian gives a good model of a fuzzy blob

Smoothing with a Gaussian



Lowering the Max Frequency

Image from *http://www.lunaloca.com/tutorials/antialiasing/*

Lowering the Max Frequency

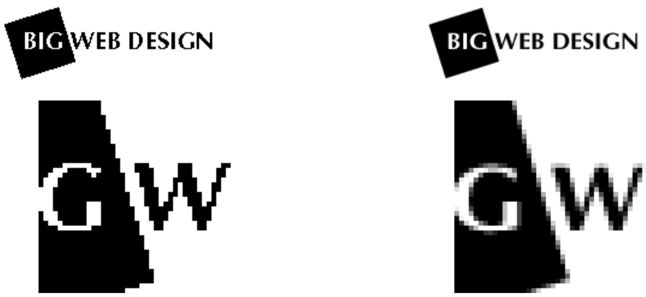
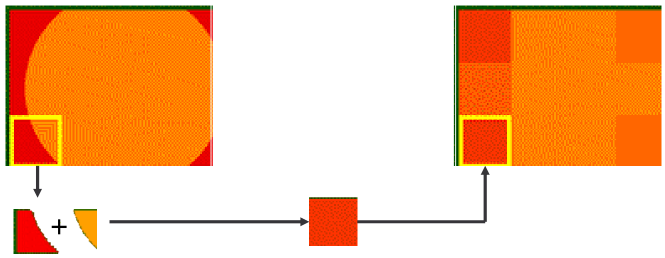


Image from *http://www.lunaloca.com/tutorials/antialiasing/*

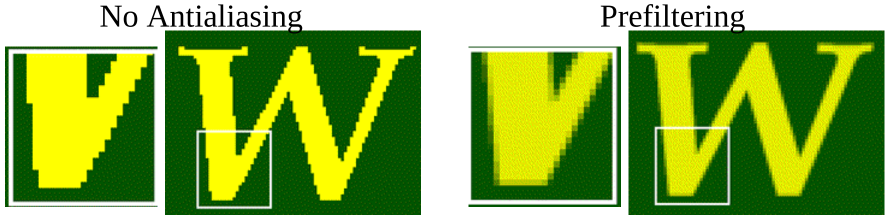
Prefiltering

- Blur the images by some low-pass filters
- Easy for 2D text, images, and 2D graphics
- Difficult for 3D graphics (visibility is an issue since translucency is expensive)

Prefiltering



No Antialiasing | Prefiltering



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Increasing Resolution

- Physically increase the display resolution
  - Not always possible
- Perform *Postfiltering* / *Supersampling*
  - Actual display is $m \times n$, sample on a $sm \times sn$ virtual display
  - Then "average" several (typically $s \times s$) virtual sub-pixels to yield one actual pixel
  - Averaging
    - unweighted (simple average)
    - weighted (more importance to virtual sub-pixels closer to the center)

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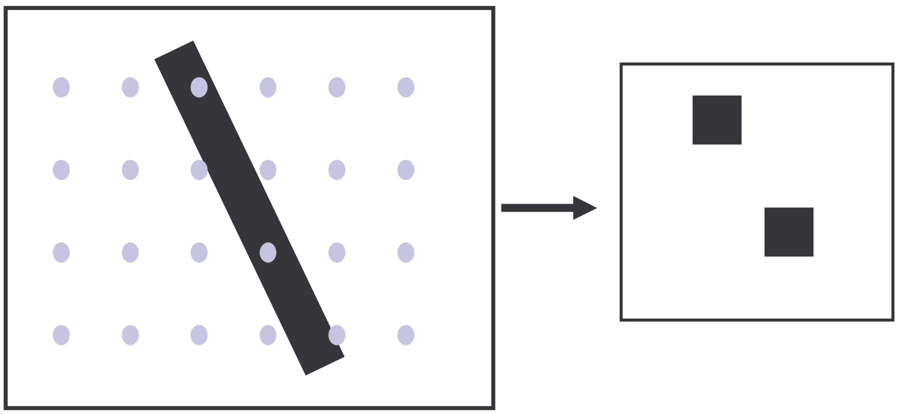
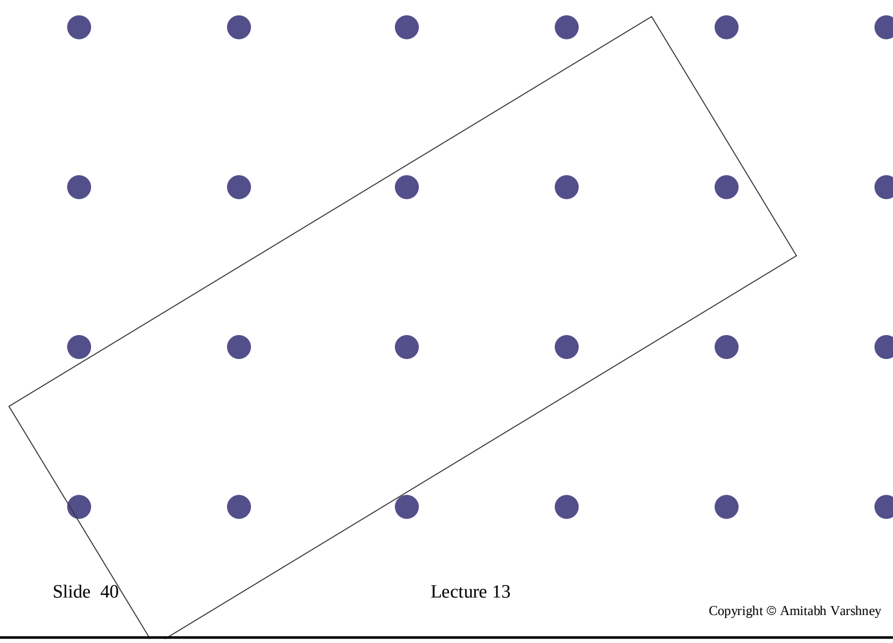
Averaging with supersampling



Matlab



Stochastic Sampling

- Sample at a random place within the pixel to determine its color (**Jittering**)
- Pick a random point and keep it if its nearest neighbor is > r units away (**Poisson Disk Sampling**)
- Often use stochastic sampling with supersampling (eg: 16 stochastic samples/pixel)
- Reduces aliasing at the cost of added noise.

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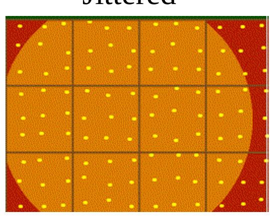
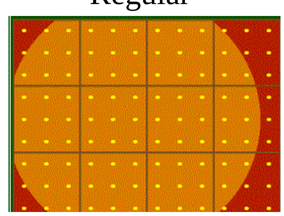
- Sampling: take *n x n* jittered, regular or random samples.

Jittered



Regular



- Averaging: use an *n x n* filter to assign weights to each sample. Sum all the weighted samples to obtain a pixel.

$$\begin{bmatrix} \frac{1}{16} & \frac{1}{8} & \frac{1}{16} \\ \frac{1}{8} & \frac{1}{4} & \frac{1}{8} \\ \frac{1}{16} & \frac{1}{8} & \frac{1}{16} \end{bmatrix}$$

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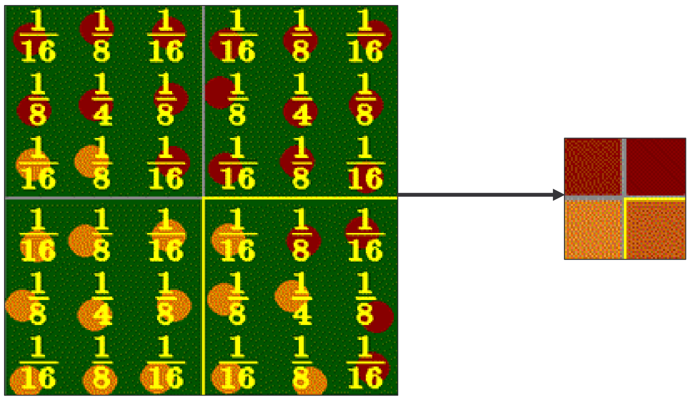
Filtering Example



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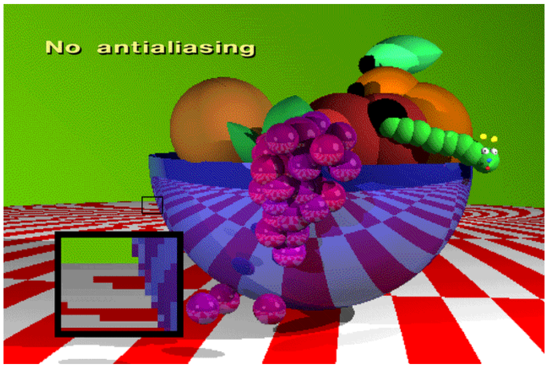
Anti-aliasing for Geometric Objects

- Use gray value proportional to area of intersection
  - Overcomes jagged edges
  - Gives equal orientation lines same brightness
  - Equivalent to supersampling with averaging in the limit of infinite supersampling.

Antialiasing Examples



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Antialiasing Examples

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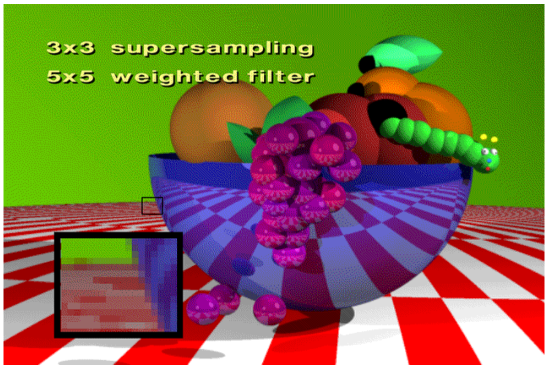
Antialiasing Examples



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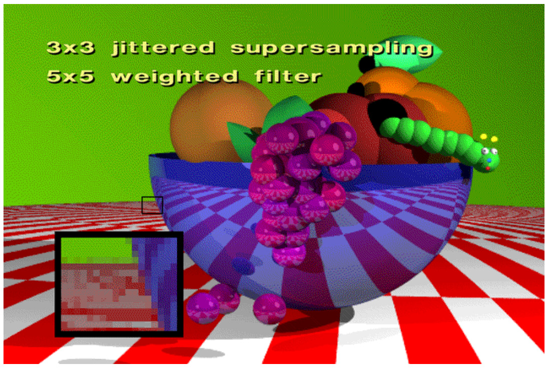
Antialiasing Examples



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