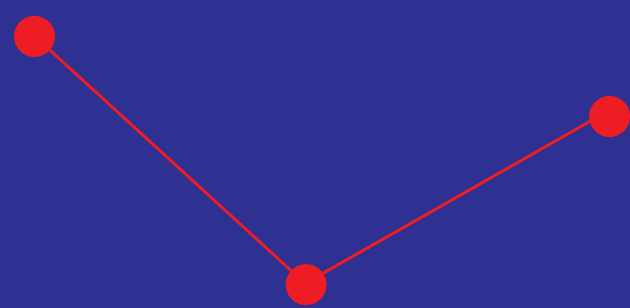
Interpolation

Interpolation: Discrete to Continuous

- Given some values at some points, determine continuous range of values.
- Uses:
  - Synthesis
    - Morph between two images
    - Interpolate a curve between points
  - Continuous range of values between vertices.
  - Blowing up an image.

Linear Interpolation in 1D

- Example: fading.



Linear Interpolation

- Given a function defined at two points, $f(0)$, $f(1)$, we want to find values for intermediate points, eg., $f(x)$, $0 < x < 1$.
- Can take weighted average:

$$f(x) = (1-x)*f(0) + x*f(1) = f(0) + x(f(1)-f(0))$$

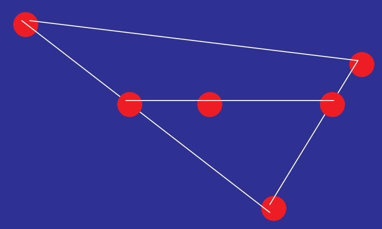
- This is equation for line with slope $f(1)-f(0)$.

Linear Interpolation of 2D Points

- Interpolate between p1, p2.
- Intermediate points are weighted averages.
- p = (1-t)*p1 + t*p2 for 0 < t < 1.
  - p goes from p1 to p2 as t goes from 0 to 1.
- p = p1 + t(p2-p1), ie. equation for a line, restricted to 0 < t < 1.
- This is *convex combination* of p1 and p2.
- Application: Interpolate location for morphing (move position of nose from face 1 to face 2) or motion synthesis.

Bilinear Interpolation - triangle

- Given value of function at vertices of triangle, interpolate values inside.
- Example, given z values at corners, determine z values for whole triangle.



Interpolate between corners to get two points on sides collinear with middle point. Then interpolate between these.

Or, represent p as weighted average of p1, p2, p3, and take same weighted average of f(p1), f(p2), f(p3). Ie, find a, b, c so that

p = a*p1 + b*p2 + c*p3, a+b+c=1 and find

f(p) = a*f(p1) + b*f(p2) + c*f(p3).

These produce the same result.

Bilinear Interpolation – 4 points

- Given values at (0,0), (1,0), (0,1), (1,1) find value at (x,y).
- Linearly interpolate (x,0), (x,1), then interpolate (x,y).
- Or, find (0,y) and (1,y) and interpolate.
- These produce same results.

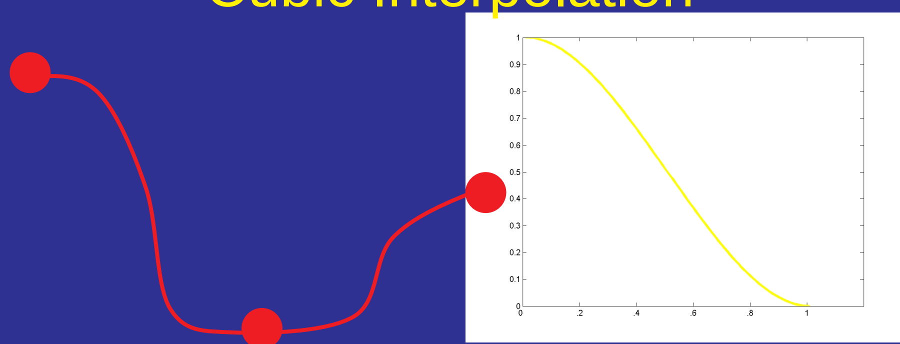
If we interpolate to get f(x,0) = (1-x)f(0,0)+xf(1,0), f(x,1) = (1-x)f(0,1) + xf(1,1). Then f(x,y) = ((1-x)f(0,0)+xf(1,0))(1-y) + (f(x,1) = (1-x)f(0,1) + xf(1,1))y.

If we interpolate to get f(0,y) = (1-y)f(0,0) + yf(0,1), f(1,y) = (1-y)f(1,0) + y(f(1,1). Then

f(x,y) = ((1-y)f(0,0) + yf(0,1))(1-x) + ((1-y)f(1,0)+y(f(1,1)))x

These are the same.

Cubic Interpolation



Instead of weighting by distance, d, weight by:

$1 – 3d^2 + 2|d|^3$

- Smooth
- Symmetric

Suppose 0<= x <= 1, and a function f is defined on f(0), f(1). We want to define it for f(x) so that f(x) is smooth.
If we do this by averaging neighbors, we have:
f(x) = g(x)*f(0) + g(1-x)f(1). Then we want a function g that is smooth, and in which g(0) = 1 and g(1) = 0, and in which g is symmetric so that g(x) + g(1-x) = 1.

With linear interpolation g(x) = 1-x. This fits the second two criteria, but this g is not smooth. There is a discontinuity at f(0), since we suddenly switch between averaging f(0) and f(1) and averaging f(0) and f(-1)

So instead, we want f(x) near f(0) to be based mostly on the value of f(0), and only to gradually average in f(1) as we get closer to it.

A nice function that does this is 1 – 3*d*d +2*|d*d*d|

Note that g(1-x) = 1 – 3*(1-x)(1-x) +2(1-x)(1-x)(1-x)

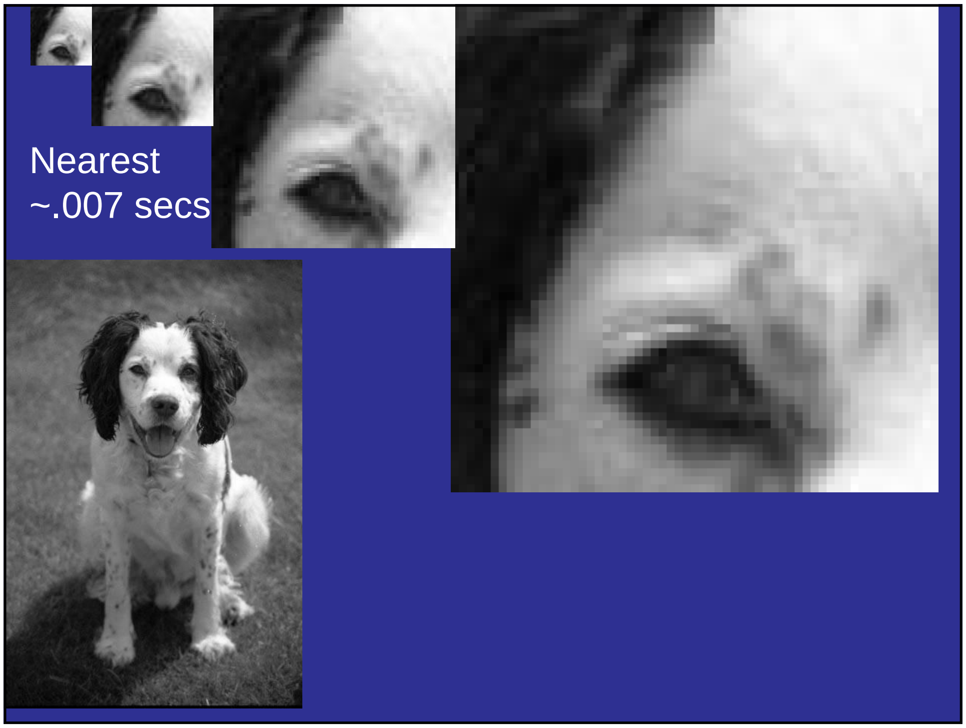
= 1 – 3 + 6x –3x*x + 2 – 6x + 6x*x – 2x*x*x = 3x*x – 2*x*x*x

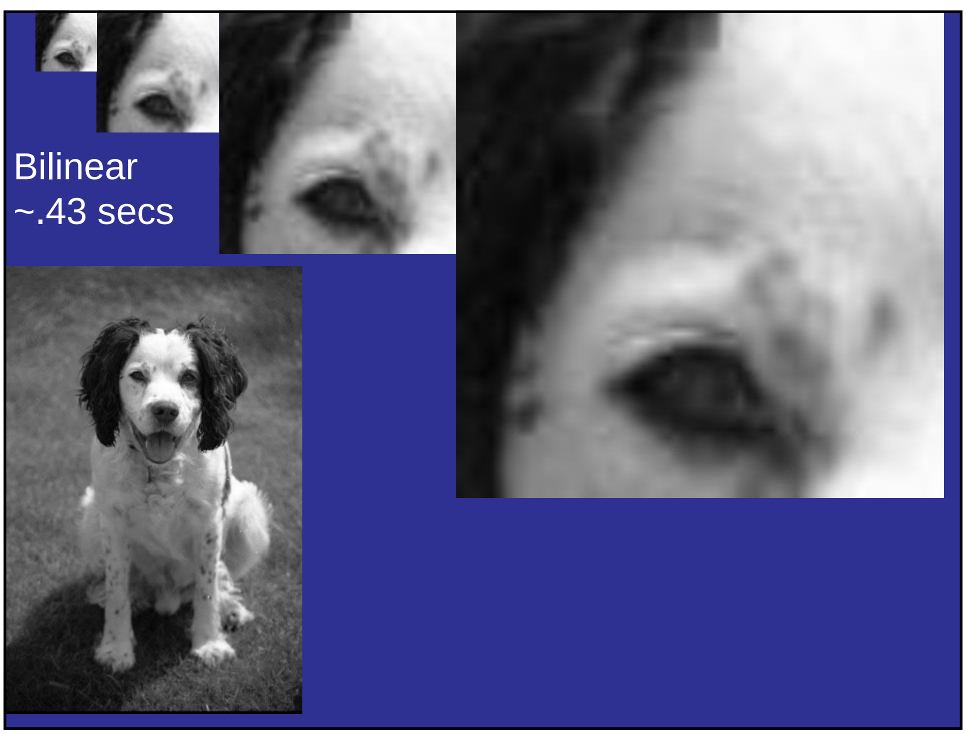
= 1 – (1 – 3x*x + 2x*x*x)

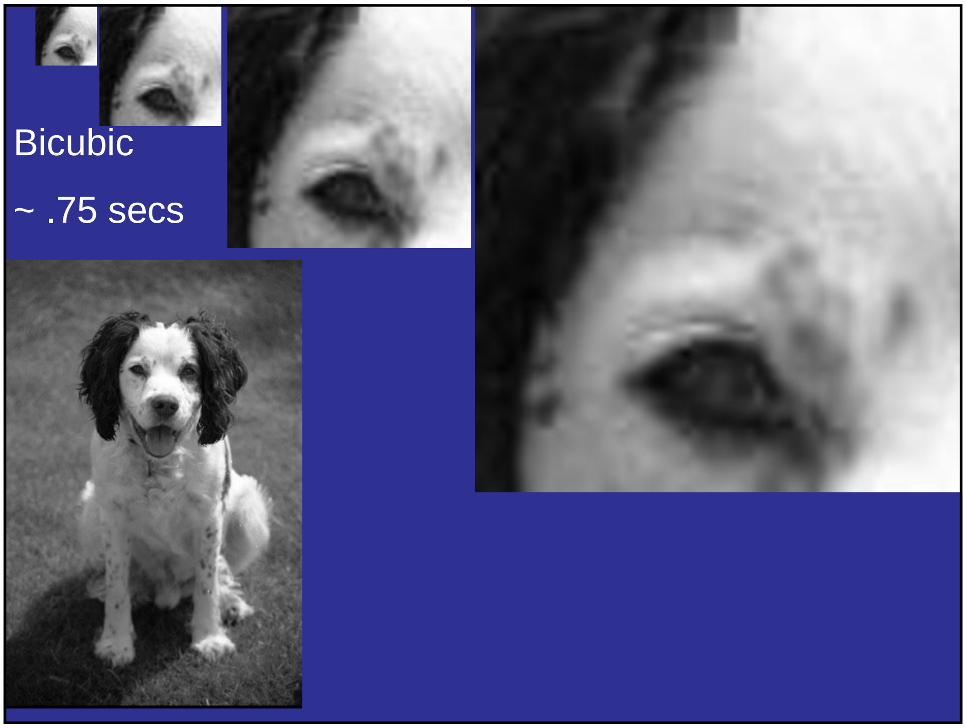
Also, we can see that when x -> 0, g(x) -> 1 – 3x*x + 2*x*x*x -> 1, and that g(1-x) similarly goes to 0. This means that (g(x)f(0) + g(1-x)f(1))-f(0) / x -> 0, which shows that the tangent at f(0) on the right side of the curve is 0. Similarly, the tangent on the other side is also zero, so two interpolating curves meet at x=0 with the same tangent, ie., smoothly.

Application: Image Resizing

- When we enlarge an image, we need values for the new pixels.
- Common methods:
  - Nearest neighbor
  - Bilinear interpolation
  - Bicubic interpolation

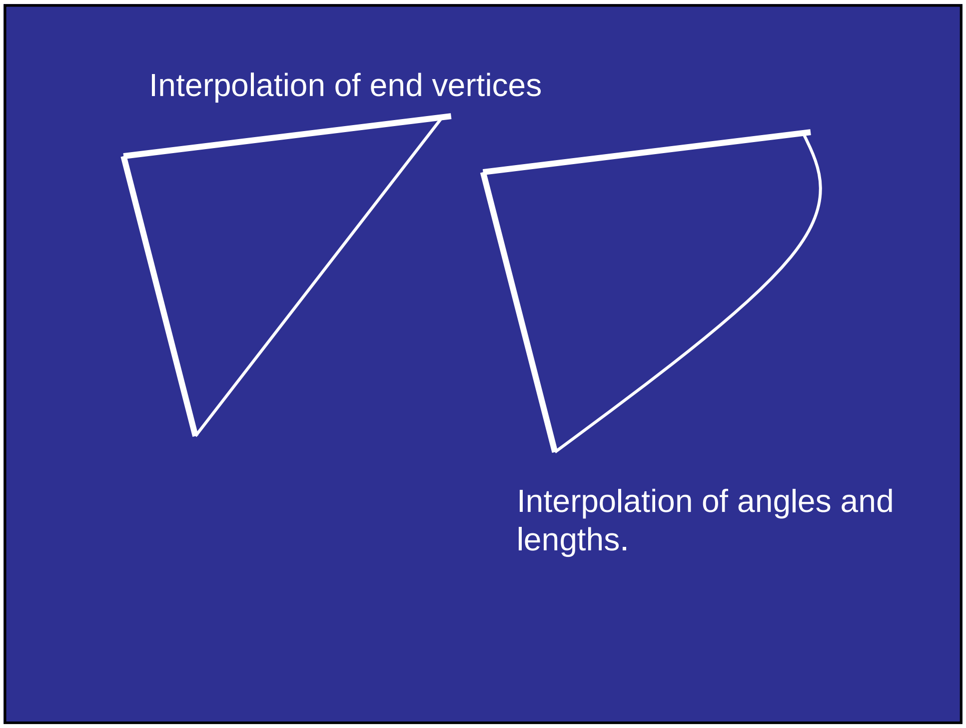

Nearest
~.007 secs


Bilinear
~.43 secs



Interpolation of Angles

- Linear interpolation of angles, in 2D.
- In 3D, find the plane that contains two vectors, and interpolate angle in that plane.
- May interpolate lines by interpolating angles and lengths, instead of end points.


Interpolation of end vertices
Interpolation of angles and lengths.