This problem set is designed to provide practice for the midterm. We will go over it in class on Tuesday, March 31st. **This assignment will not be accepted after the start of class on Tuesday, March 31st.** It cannot be turned in late.

1. Suppose there is a camera with a focal point at (0,0,0) and an image plane at z = 1. There is an image pixel whose lower left corner is at (.49, .49, 1) and whose upper right corner is at (.51, .51, 1). In the scene is a triangle with vertices: v1 = (5,1,8), v2 = (0,1, 10) and v3 = (5, 8, 8). **50 points, 10 points each.**
   a. Give an equation for the plane that the triangle lies in.
   b. If we cast a ray from the focal point through the center of the pixel, what is an equation for the ray?
   c. Where will the ray intersect the plane of the triangle?
   d. Show that this point is inside the triangle.
   e. Suppose v1 is black, with an intensity of 0, v2 is dark gray, with an intensity of 100, and v3 is white, with an intensity of 255. If we use bilinear interpolation to determine the intensity of other points in the triangle, what will be the intensity of the pixel in the image?

2. Show the result of convolving the kernel, k, with the image, I. **10 points**

3. Suppose there is a white sphere centered at the point (0,0,B) with unit radius, illuminated by a directional light source from the direction (1,0,0). B is some arbitrary, very big number. We are making B big only so that you can assume that all vectors from the viewer to any point on the sphere are in the same direction. We view the sphere from the position (0,0,0). **30 points, 10 points each.**
   a. Suppose the sphere is Lambertian. If the point on the sphere (1,0,B) appears in an image with intensity 1, what would be the intensity of the point (cos(π/20), 0, B – sin(π/20))?  
   b. Suppose the sphere reflects light according to the Phong model. If the point on the sphere (cos(π/4),0,B-sin(π/4)) appears in an image with intensity 1, and a point on the sphere at (cos(5π/16),0,B-sin(5π/16)) appears with intensity .4602, what will be the intensity of a point at (cos(9π/32),0,B-sin(9π/32))?
c. In (b), if we were using Gouraud shading, and interpolated the value of the intensity at \((\cos(9\pi/32),0,\sin(9\pi/32))\) from the intensities at \((\cos(5\pi/16),0,\sin(5\pi/16))\) and \((\cos(\pi/4),0,\sin(\pi/4))\) how much error would be caused by this interpolation?

4. Consider 2D perspective projection. We have a focal point at the origin, \((0,0)\), and an image plane of \(y = 1\). So in this case, the formula for perspective would tell us that a 2D point at \((x,y)\) will appear in the 1D image at the location \(x/y\). **10 points**

As shown in the figure (A) above, suppose we can see the part of the world that is contained in the truncated triangle given by the polygon with vertices at \((-1,1), (-2,2), (1,1), (2,2)\). Find a 2D projective transformation that will transform all points in this polygon so that (a) they lie in the rectangle shown in (B); (b) every point will appear in the same location in the image before and after the transformation. So, for example, the point \((x,y)\) will appear at \((x/y)\). If after transforming this point it appears at \((x',y')\), then \(x/y = x'/y'\). (c) relative depth is preserved. So if one point is further from the focal point than a second point, after we apply this projective transformation to the points, the second point will still be further from the origin than the first point.