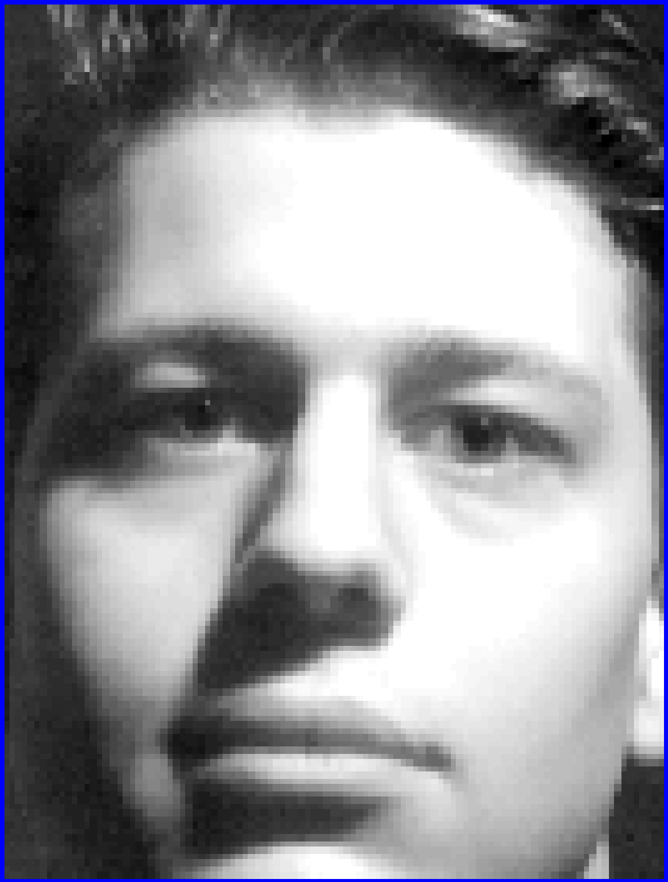


Lighting affects appearance


How do we represent light? (1)

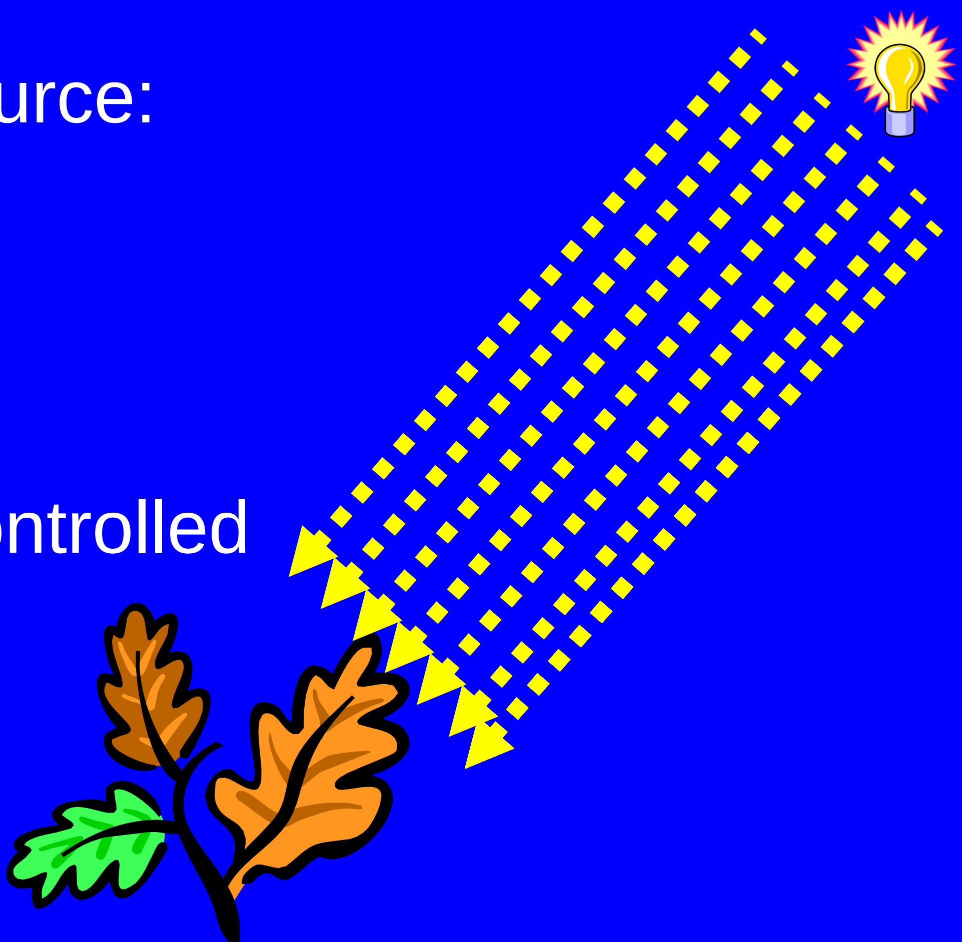
- Ideal distant point source:

  - No cast shadows

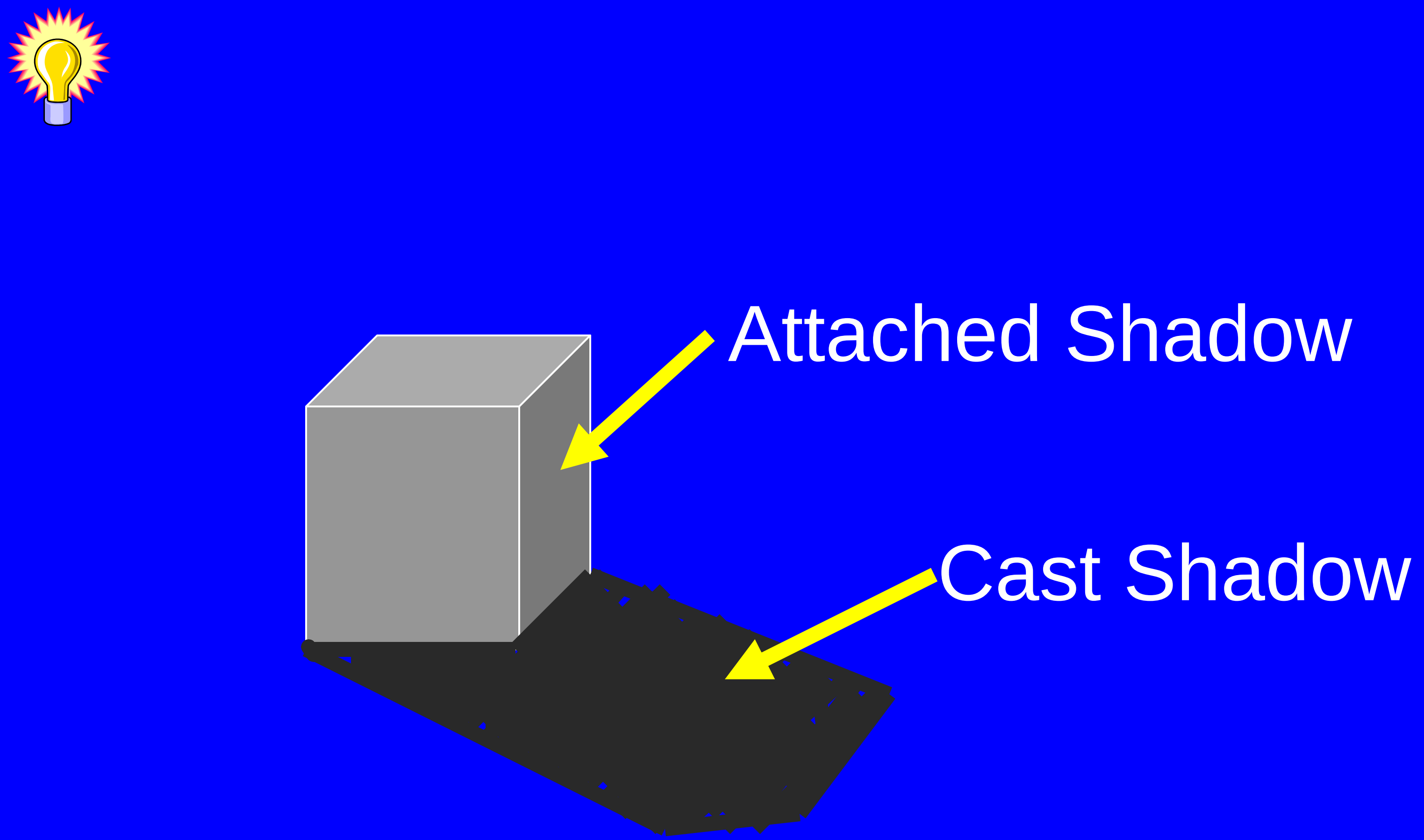
  - Light distant

  - Three parameters

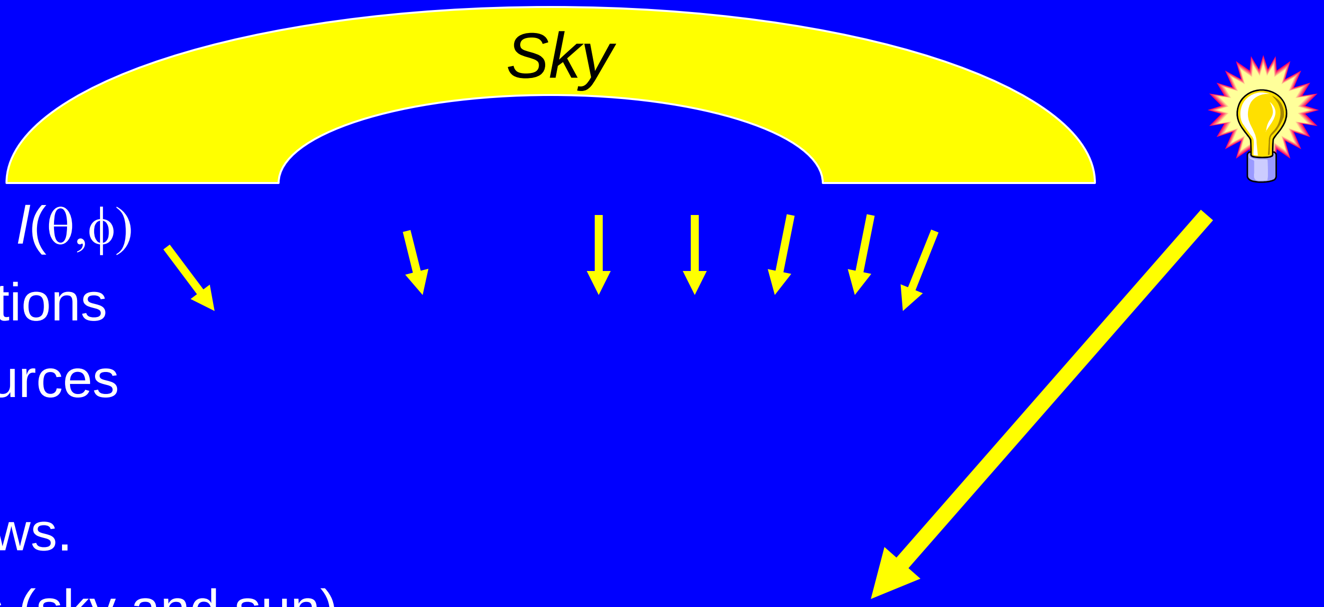
  - Example: lab with controlled light



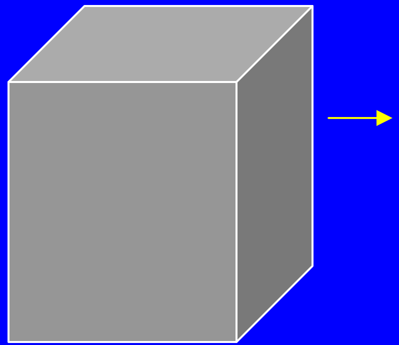
Shadows

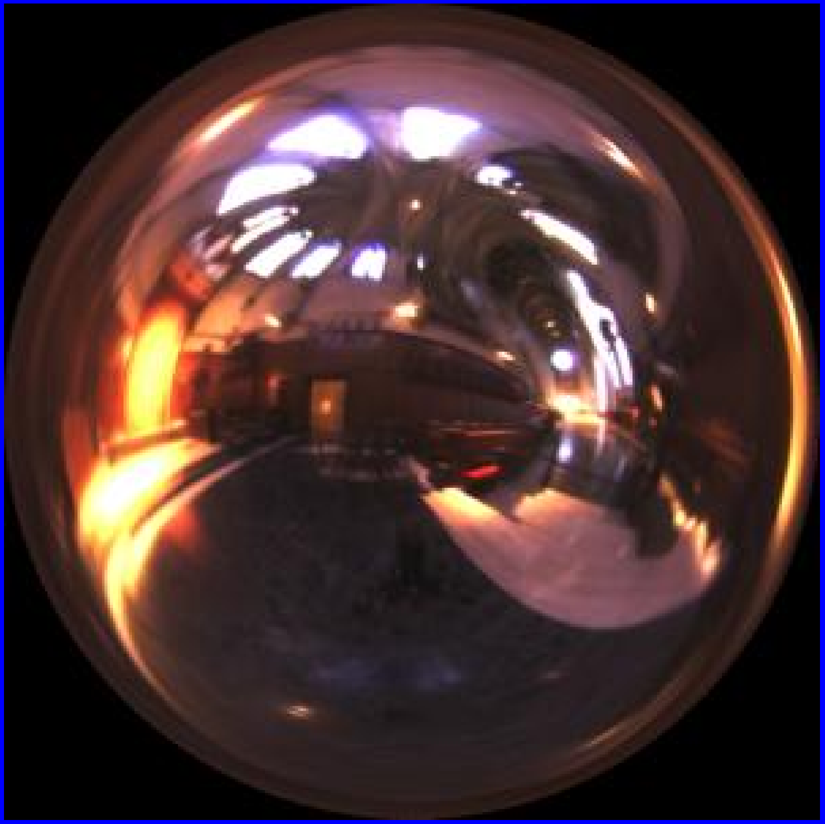


How do we represent light? (2)



- Environment map: $I(\theta,\phi)$
  - Light from all directions
  - Diffuse or point sources
  - Still distant
  - Still no cast shadows.
  - Example: outdoors (sky and sun)





Lambertian + Point Source

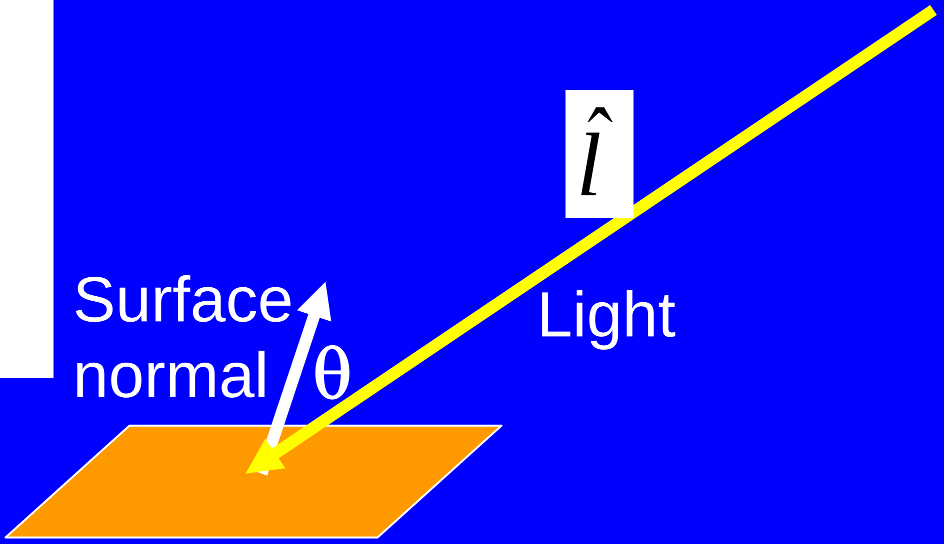
$$\vec{l} = l \bullet \vec{l} \qquad \begin{cases} \vec{l} \text{ is direction of light} \\ l \text{ is intensity of light} \end{cases}$$

$$i = \max(0, \lambda(\vec{l} \bullet \hat{n}))$$

i is radiance

λ is *albedo*

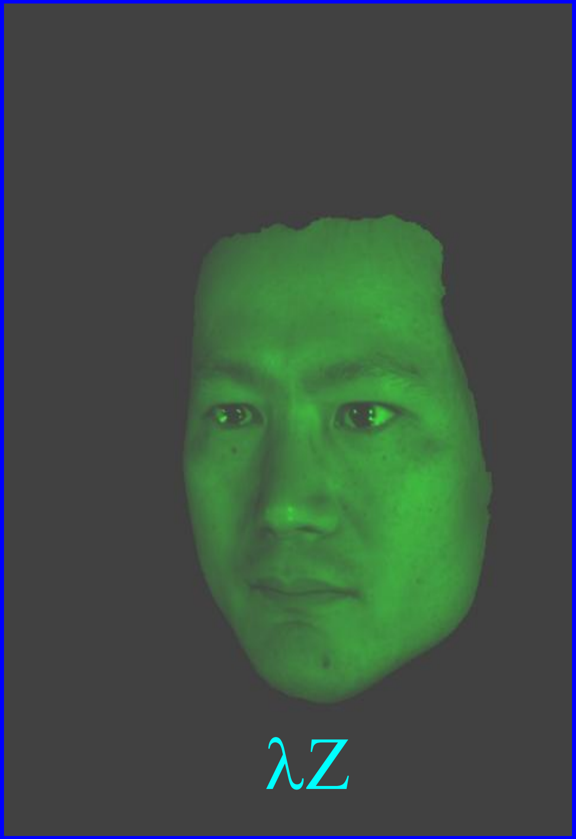
$\hat{n}$ is surface normal



Lambertian, point sources, no shadows. (Shashua, Moses)

- *Whiteboard*
- Solution linear
- Linear ambiguity in recovering scaled normals
- Lighting not known.
- Recognition by linear combinations.

Linear basis for lighting



With Shadows: PCA

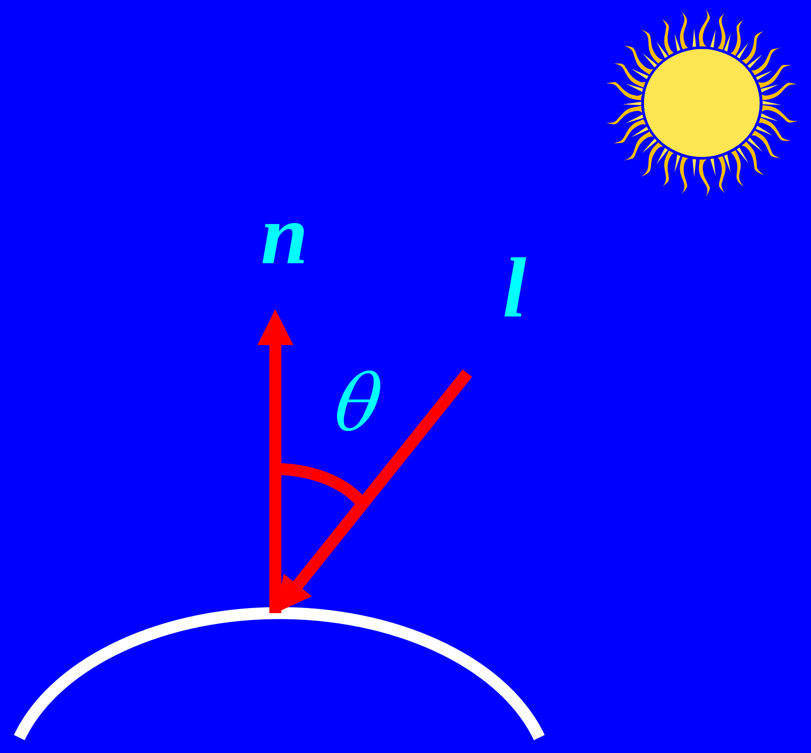
(Epstein, Hallinan and Yuille;
see also Hallinan; Belhumeur and Kriegman)

	Ball	Face	Phone	Parrot
#1	48.2	53.7	67.9	42.8
#3	94.4	90.2	88.2	76.3
#5	97.9	93.5	94.1	84.7
#7	99.1	95.3	96.3	88.5
#9	99.5	96.3	97.2	90.7

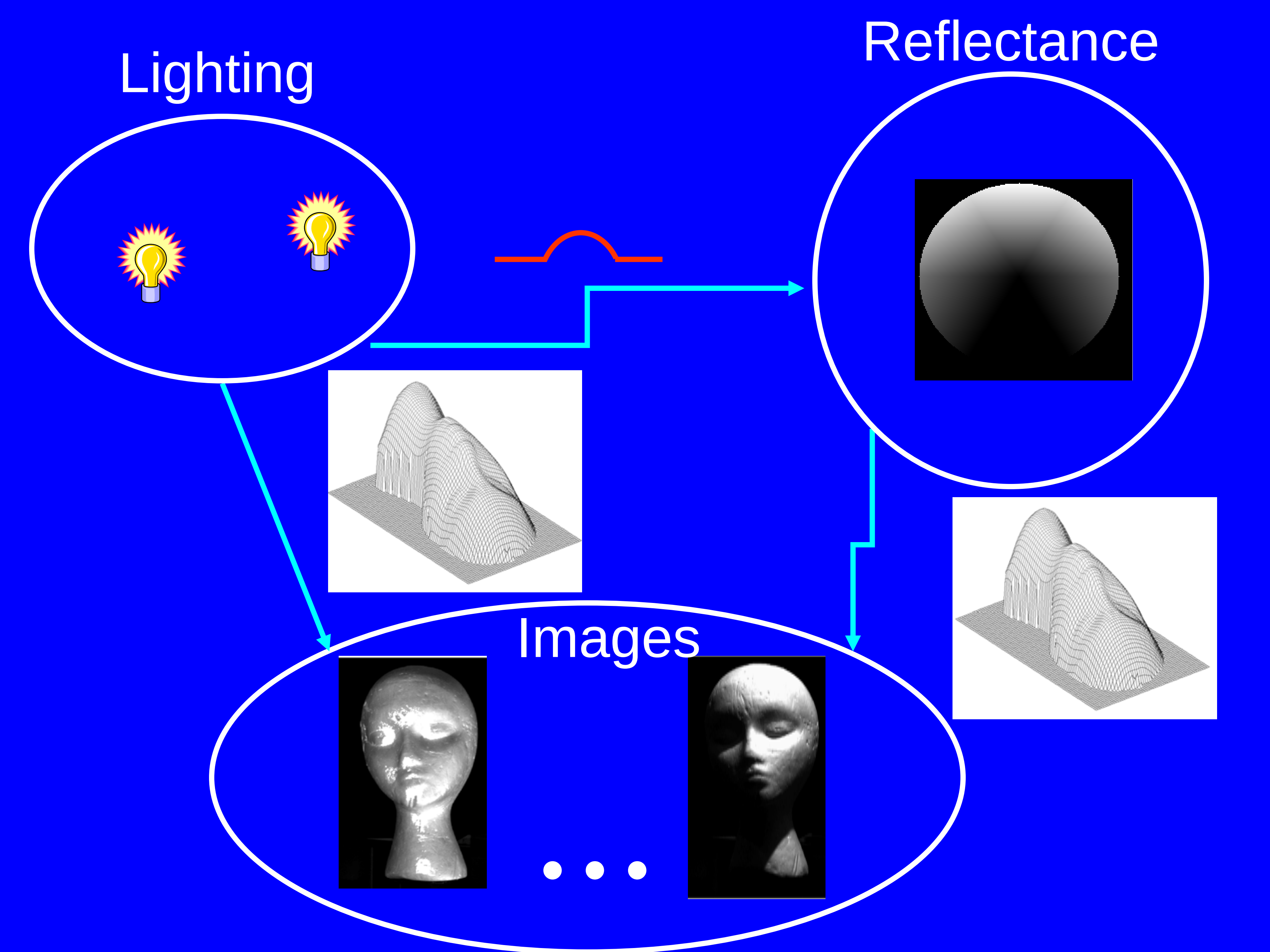
Dimension: $5 \pm 2\mathrm{D}$

Domain

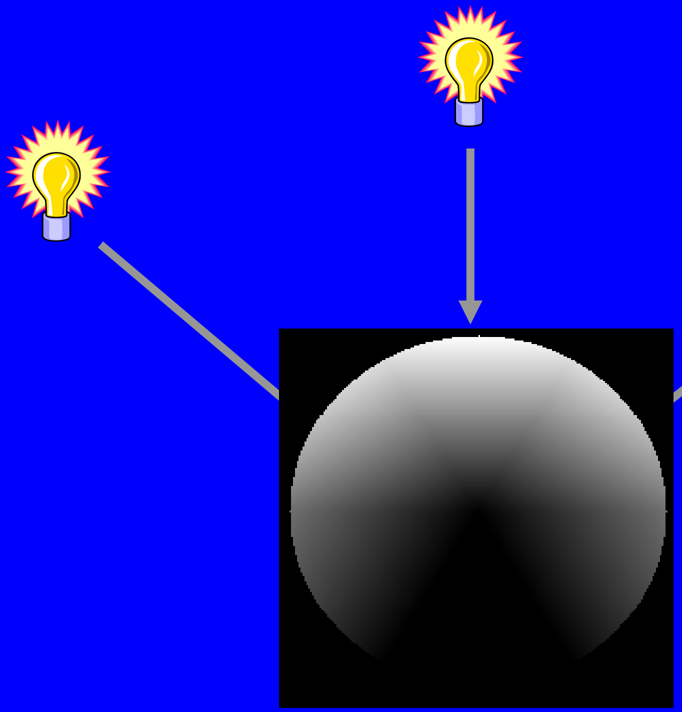
- Lambertian
- Environment map

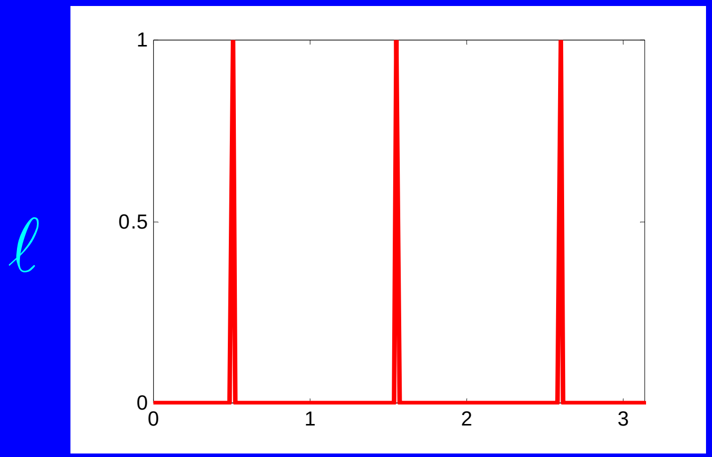


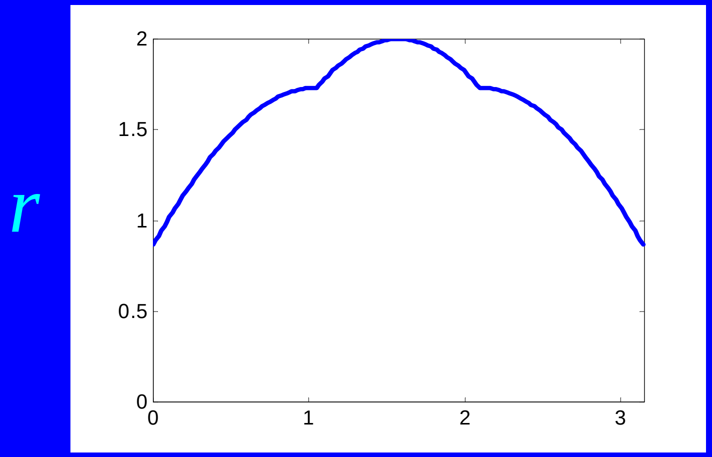
$l\lambda \max(\cos\theta, 0)$

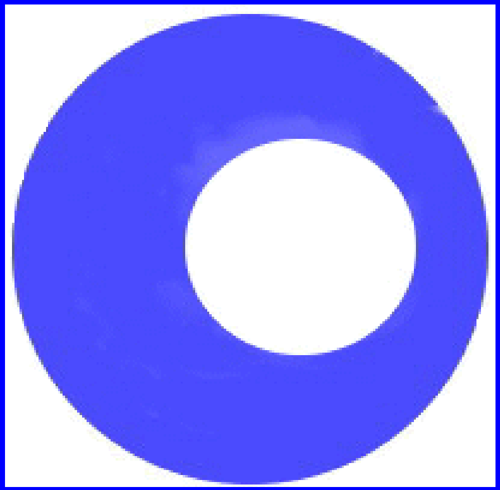
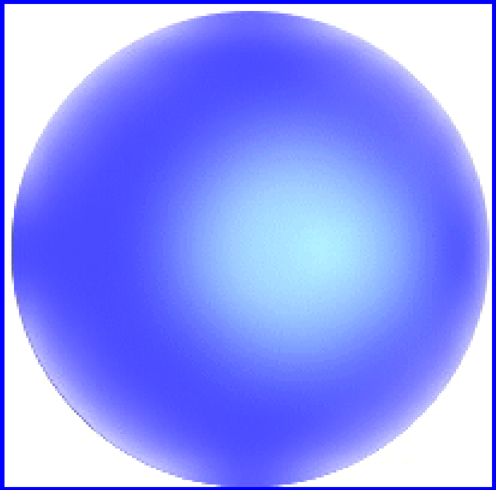

Lighting
Reflectance
Images
• • •

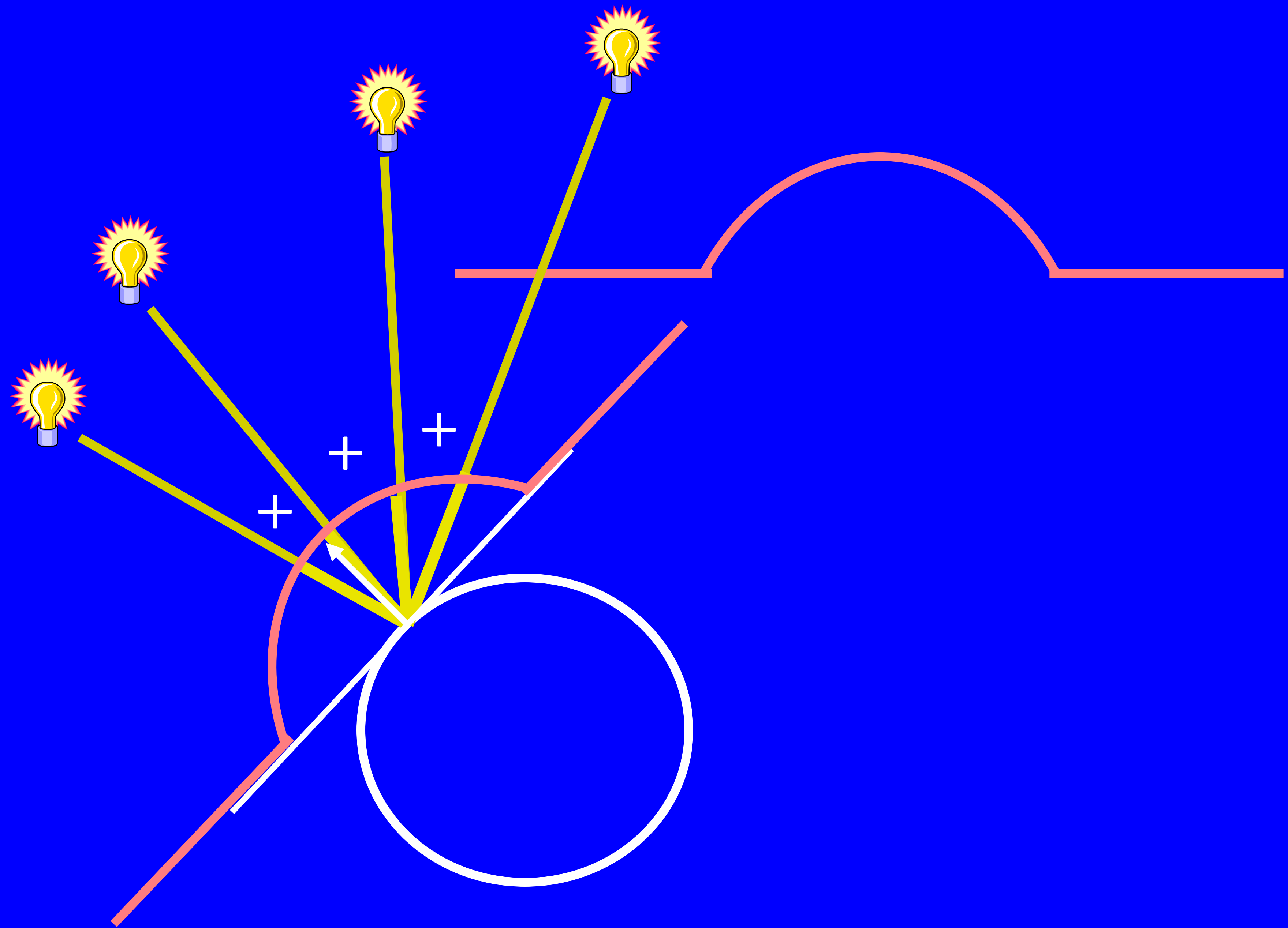
Lighting to Reflectance: Intuition









Spherical Harmonics

- Orthonormal basis, h_{nm}, for functions on the sphere.
- n'th order harmonics have $2n+1$ components.
- Rotation = phase shift (same n, different m).
- In space coordinates: polynomials of degree n.
- S.H. used for BRDFs (Cabral et al.; Westin et al;). (See also Koenderink and van Doorn.)

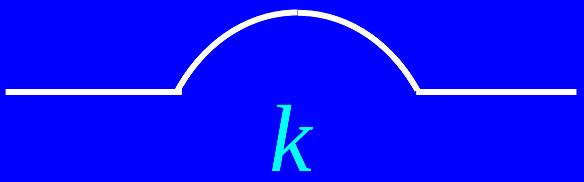
$$h_{nm}(\theta,\phi) = \sqrt{\frac{(2n+1)}{4\pi}\frac{(n-m)!}{(n+m)!}}P_{nm}(\cos\theta)e^{im\phi}$$

$$P_{nm}(z) = \frac{(1-z^2)^{m/2}}{2^n n!}\frac{d^{n+m}}{dz^{n+m}}(z^2-1)^n$$

S.H. analog to convolution theorem

- Funk-Hecke theorem: “Convolution” in function domain is multiplication in spherical harmonic domain.

- *k* is low-pass filter.

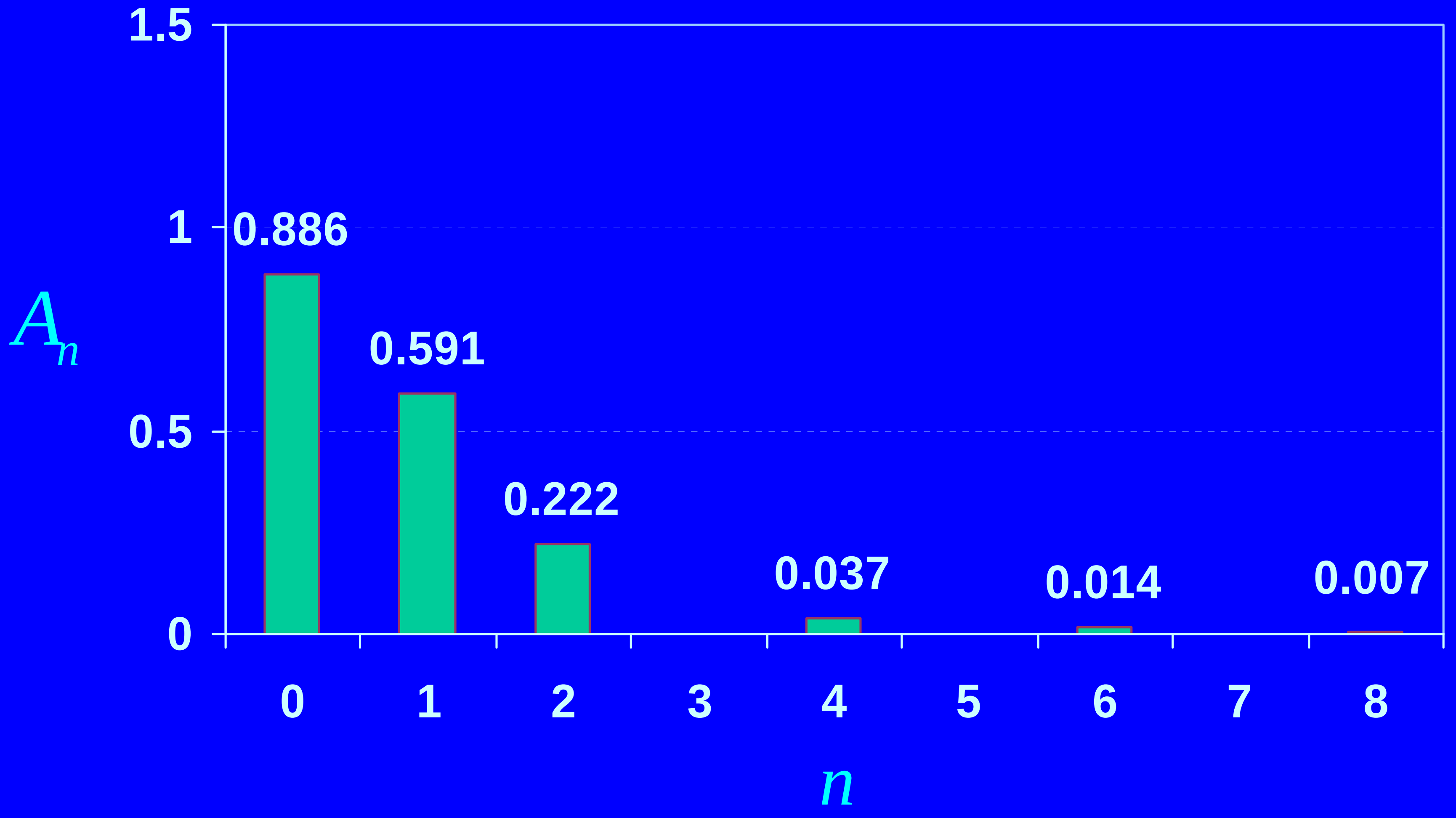


Harmonic Transform of Kernel

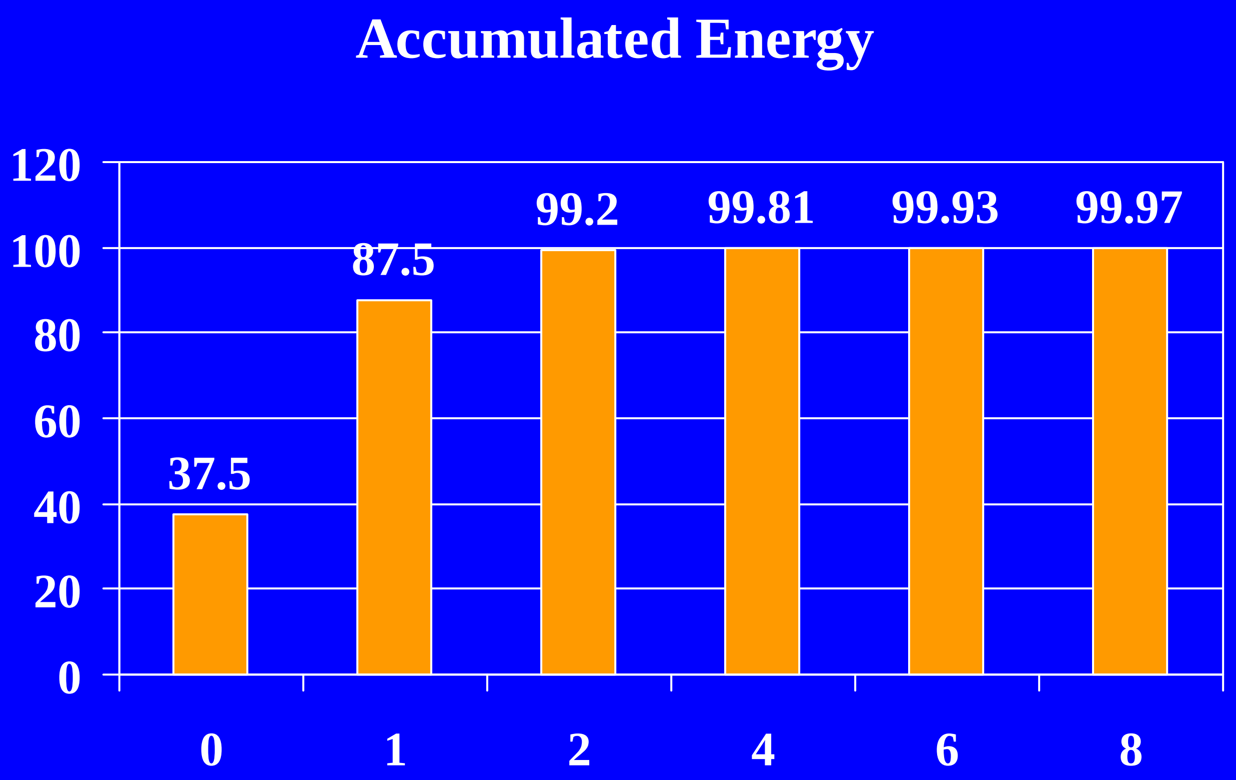
$$k(\theta) = \max(\cos\theta, 0) = \sum_{n=0}^{\infty} k_n h_{n0}$$

$$k_n = \begin{cases} \dfrac{\sqrt{\pi}}{2} & n = 0 \\ \sqrt{\dfrac{\pi}{3}} & n = 1 \\ (-1)^{\frac{n}{2}+1} \dfrac{(n-2)!\sqrt{(2n+1)\pi}}{2^n(\frac{n}{2}-1)!(\frac{n}{2}+1)!} & n \geq 2, \text{even} \\ 0 & n \geq 2, \text{odd} \end{cases}$$

Amplitudes of Kernel



Energy of Lambertian Kernel in low order harmonics



Reflectance Functions Near Low-dimensional Linear Subspace

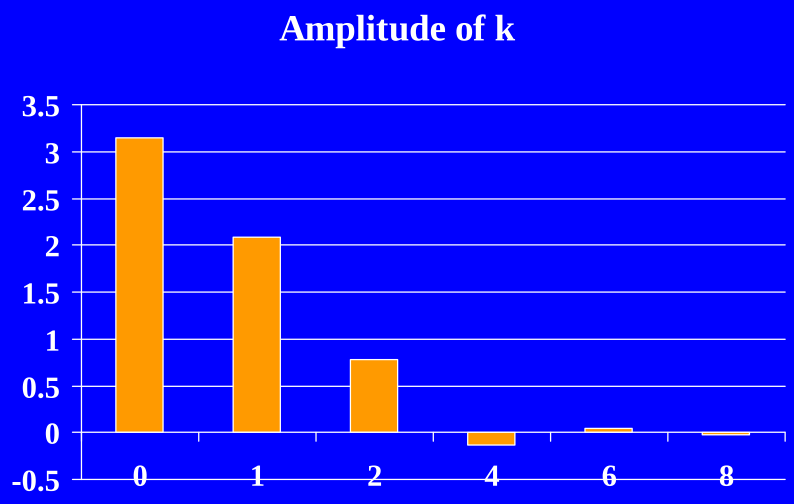
$$r = k * l = \sum_{n=0}^{\cancel{\infty}} \sum_{m=-n}^{n} (K_{nm} L_{nm}) h_{nm}$$

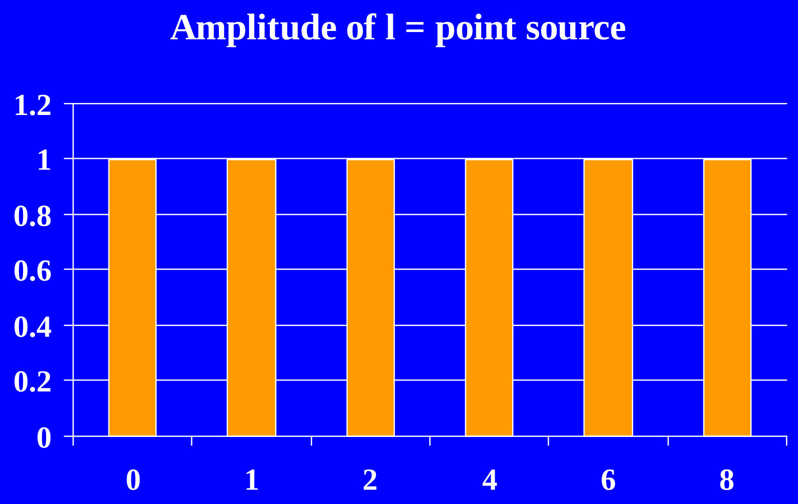
$$\approx \sum_{n=0}^{2} \sum_{m=-n}^{n} (K_{nm} L_{nm}) h_{nm}$$

Yields 9D linear subspace.

How accurate is approximation? Point light source

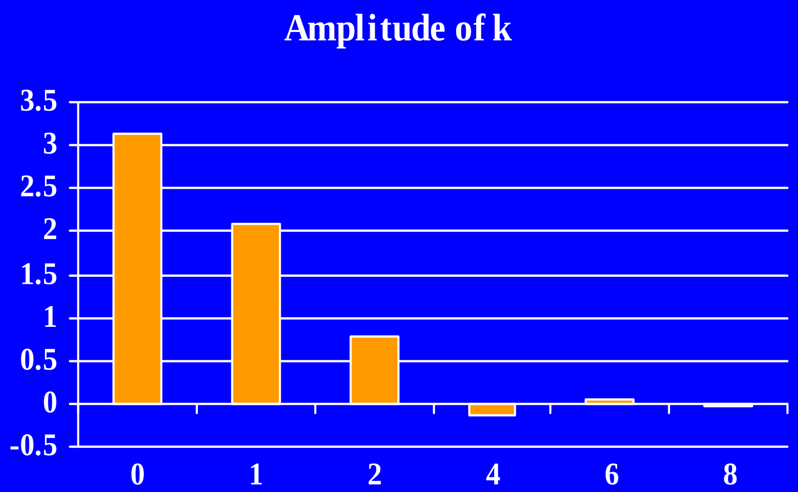
$$r = k * l = \sum_{n=0}^{\infty}\sum_{m=-n}^{n}(K_{nm}L_{nm})h_{nm} \approx \sum_{n=0}^{2}\sum_{m=-n}^{n}(K_{nm}L_{nm})h_{nm}$$

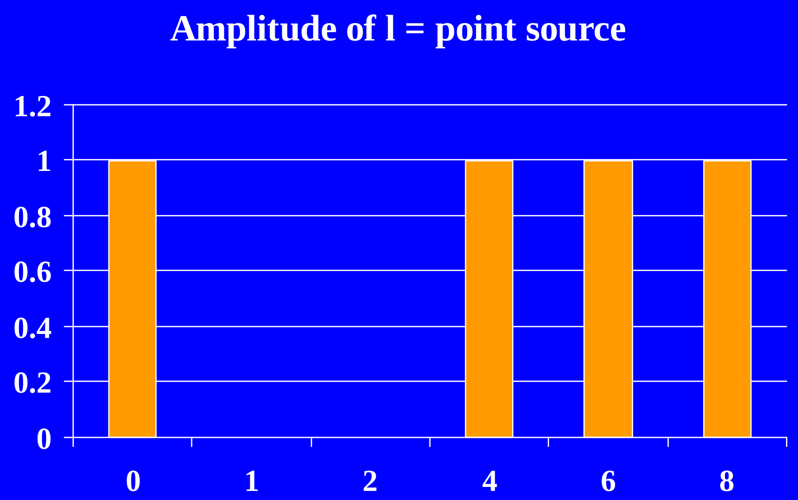




9D space captures 99.2% of energy

How accurate is approximation? (2)
Worst case.





- DC component as big as any other.
- 1st and 2nd harmonics of light could have zero energy

9D space captures 98% of energy

Forming Harmonic Images

$$b_{nm}(p)=\lambda r_{nm}(X,Y,Z)$$

λ

λZ

λX

λY

$2\lambda(Z^2-X^2-Y^2)$

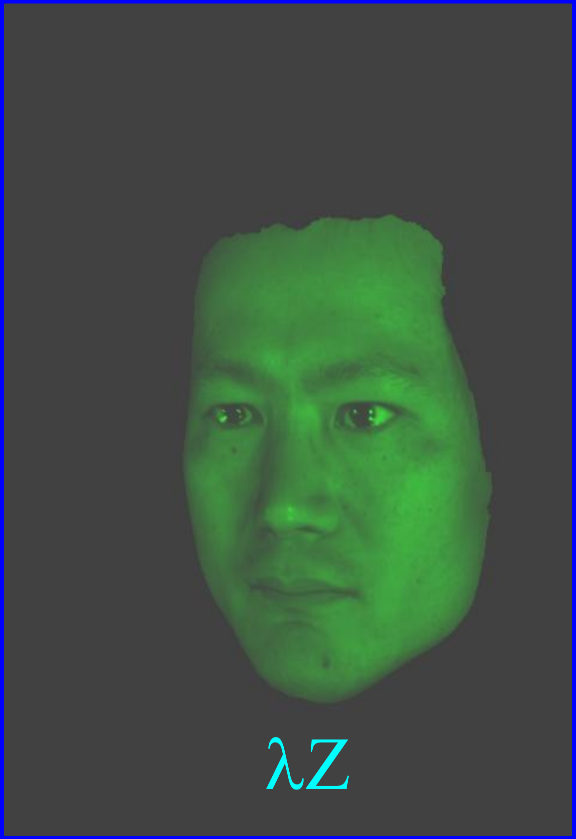
$\lambda(X^2-Y^2)$

λXY

λXZ

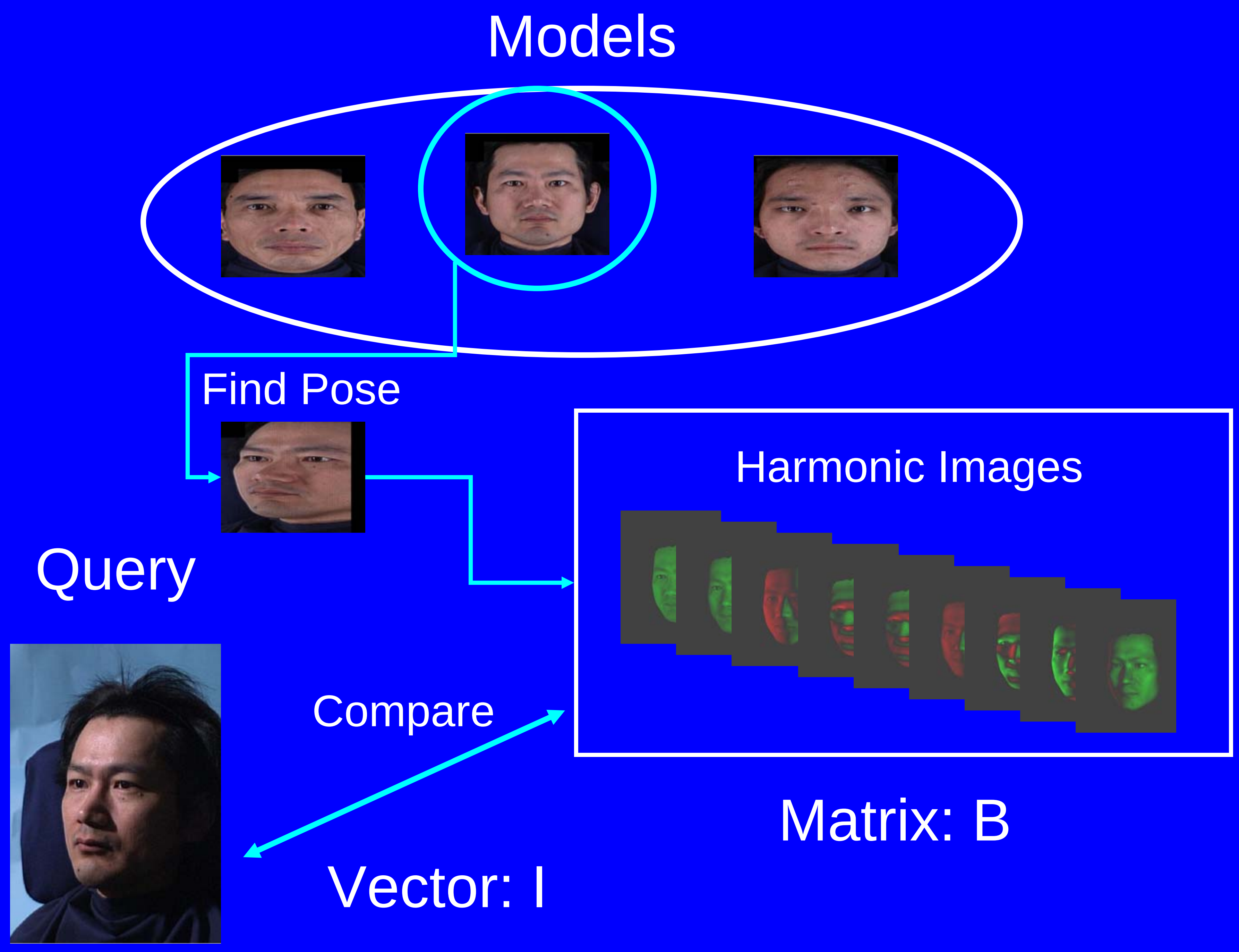
λYZ

Compare this to 3D Subspace



Accuracy of Approximation of Images

- Normals present to varying amounts.
- Albedo makes some pixels more important.
- *Worst case approximation arbitrarily bad.*
- *“Average” case approximation should be good.*


Models
Find Pose
Harmonic Images
Query
Compare
Matrix: B
Vector: I



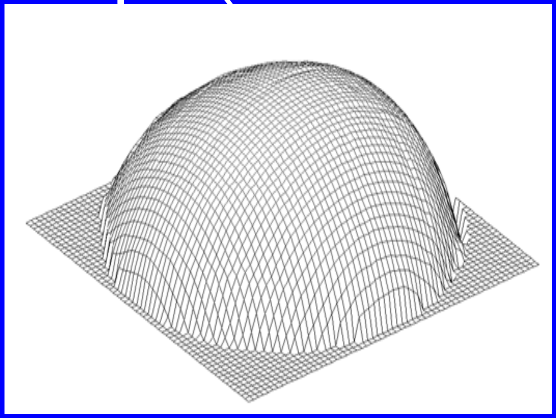




Photometric Stereo

Albedos

Shape (normals only)

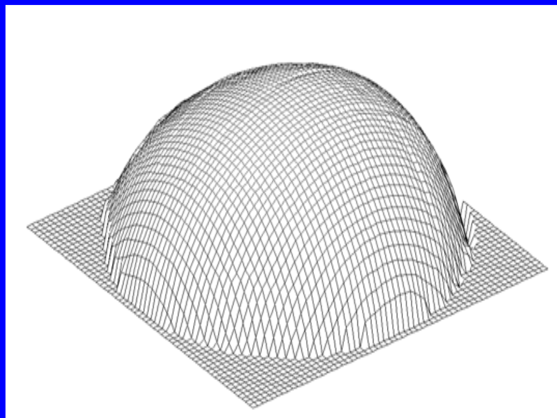


Two Stages of Photometric Stereo

- Local: given many images of pixel, determine surface normal.
- Global: Given set of surface normals, convert them into a surface.
  - “Surface” means z = f(x,y).
  - This has one DOF per pixel
  - Normal has two DOFs per pixel
  - Surface -> nonlinear constraint on normals.

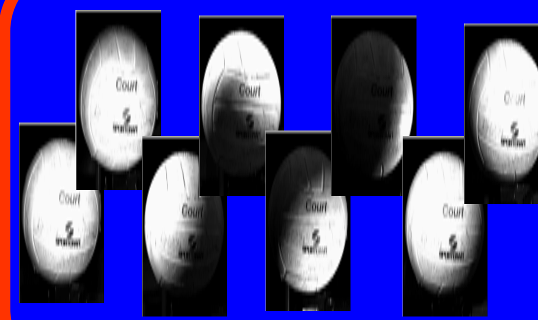
Local

- $I(x,y) = \langle L, \lambda\, n(x,y) \rangle$
  - With known lighting and albedo, this is linear with 3 unknowns, non-linear constraint.
    - Linear with unknown albedo/lighting intensity.
  - With unknown lighting, linear with global ambiguity in lighting
    - Global surface constraints disambiguate this up to a “bas-relief” transformation.
  - With known lighting, albedo, global constraints, one unknown per pixel.
    - This is shape from shading; solvable with boundary conditions.



Harmonics

I, IX, IY



PCA

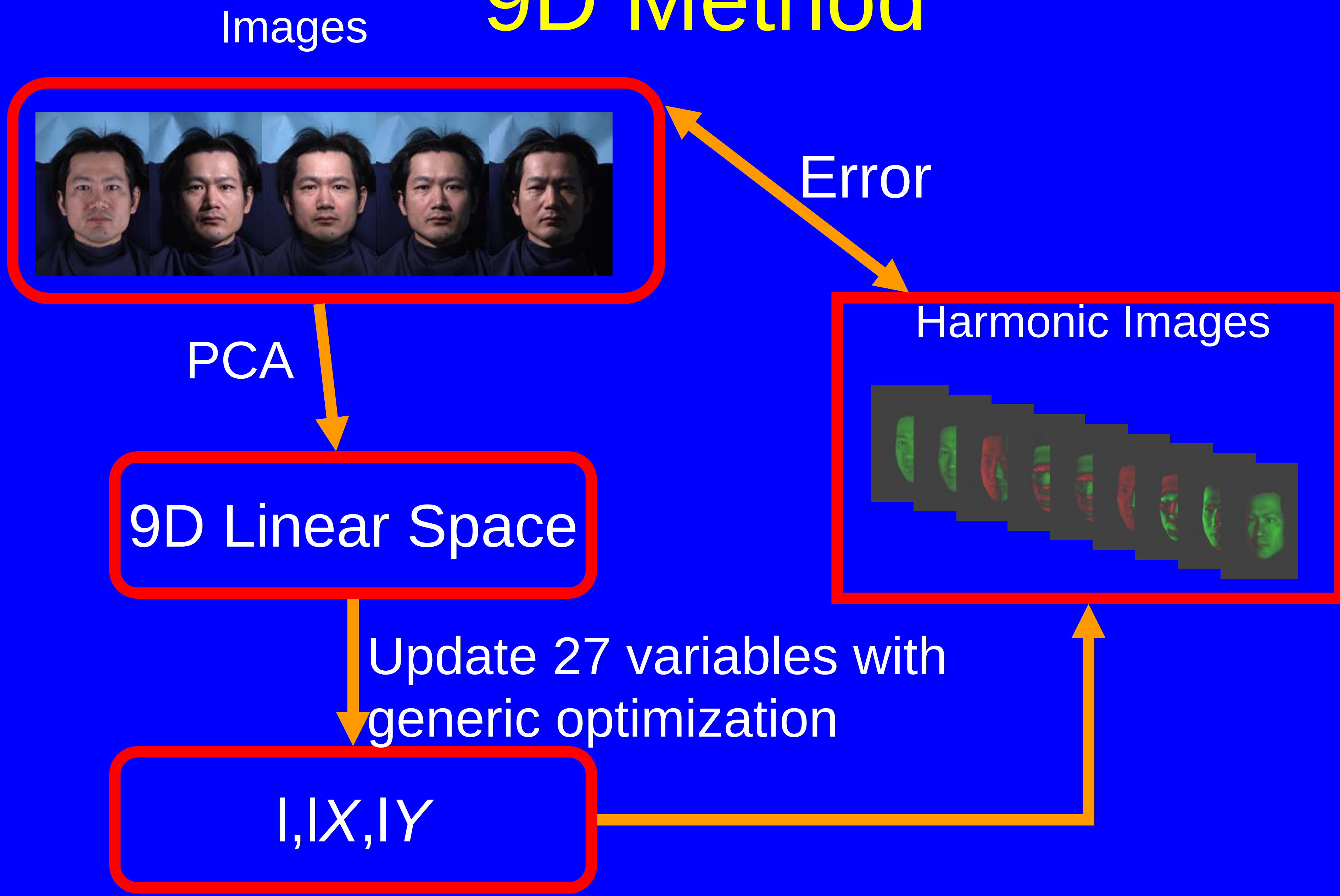
$\tilde{S}_1, \tilde{S}_2, \tilde{S}_3, \ldots$

Linear Transformation

Constraint: I, IX, IY must generate harmonic space near images ($\tilde{S}_1, \tilde{S}_2, \tilde{S}_3, \ldots$).

- 4D: solved with simple linear algebra.
- 9D: solved iteratively.

9D Method


Images
Error
Harmonic Images
PCA
9D Linear Space
Update 27 variables with
generic optimization
I,IX,IY

9D Method (cont'd)

- Optimization efficient:
  - 27 unknowns not 3n.
  - Simple, linear method of computing error.
- Good starting point.
- Linear ambiguity in solution:

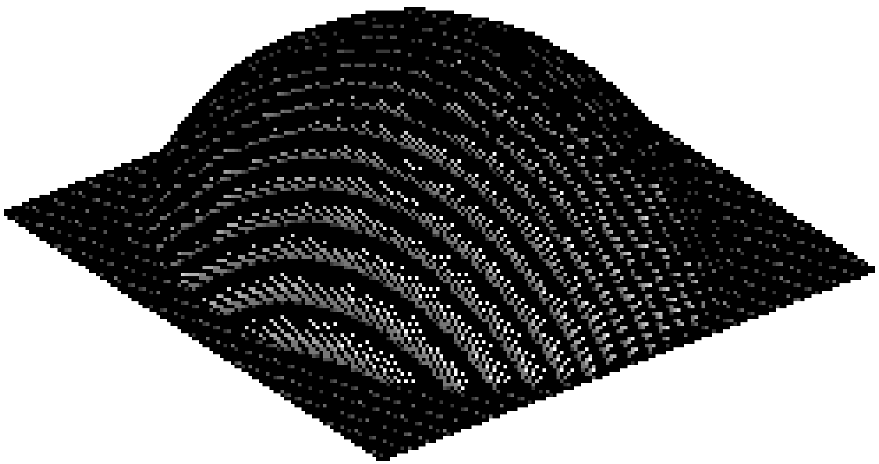
  - Linear transformation of $(\lambda X, \lambda Y, \lambda Z)$ leaves harmonic images unchanged, to 1^{st} order.

Experiments

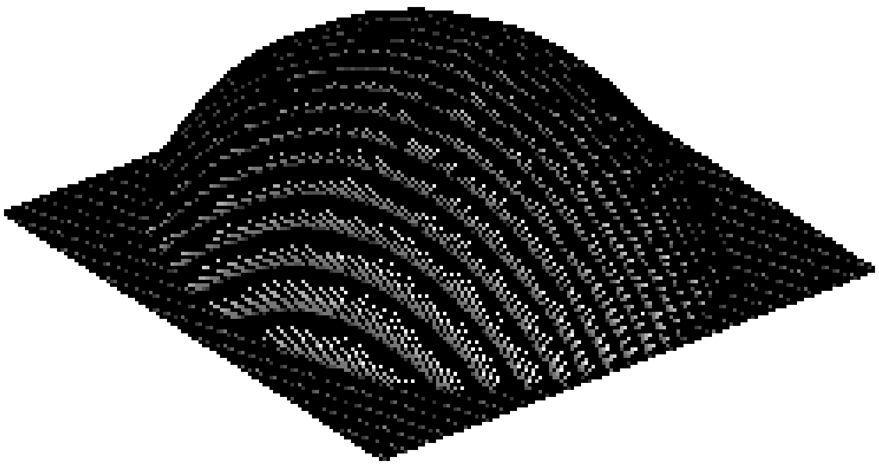
- 64 pictures of volleyball, single point source.
- 32 pictures of face, two sources each.
- 20 pictures each of other objects, multiple sources
- Saturated points filled in.
- Linear ambiguities resolved by hand.
- Normals -> surface using standard method.

Experiment 1

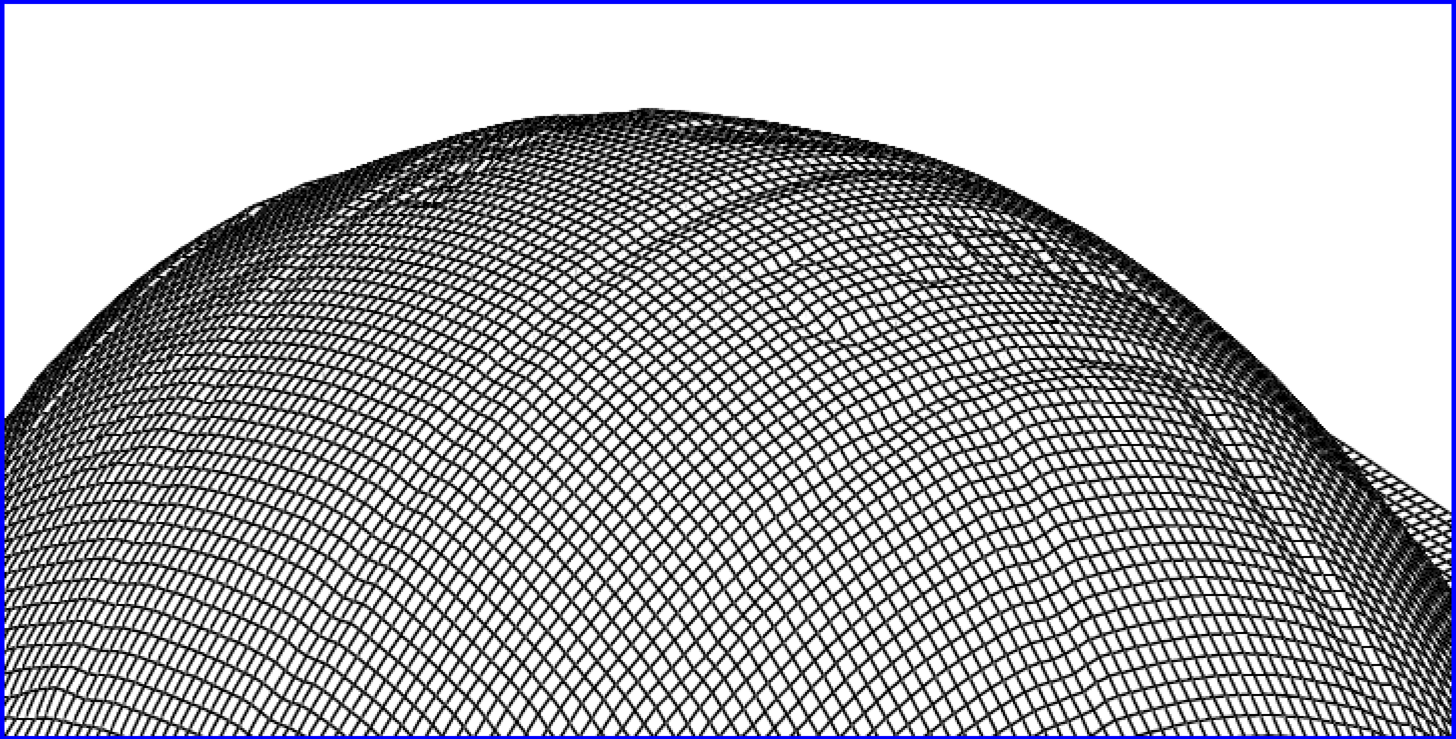
4D



9D

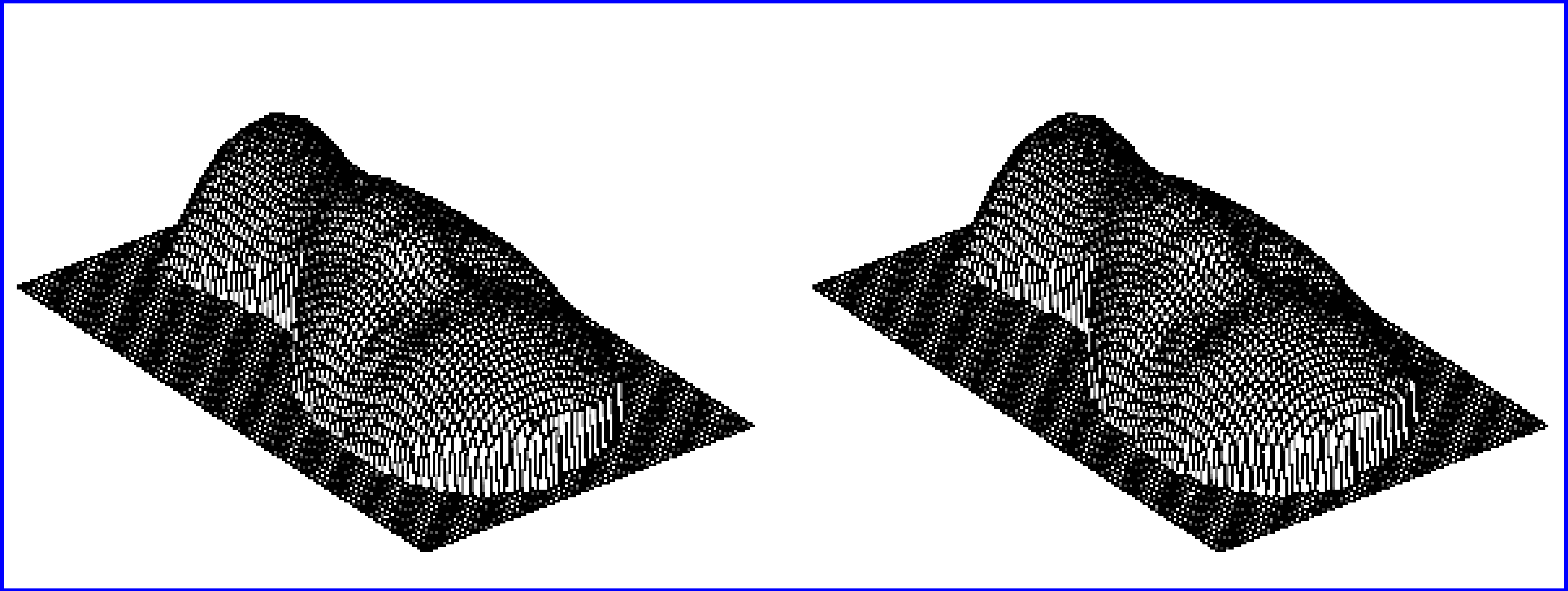



Court
SPORTCRAFT



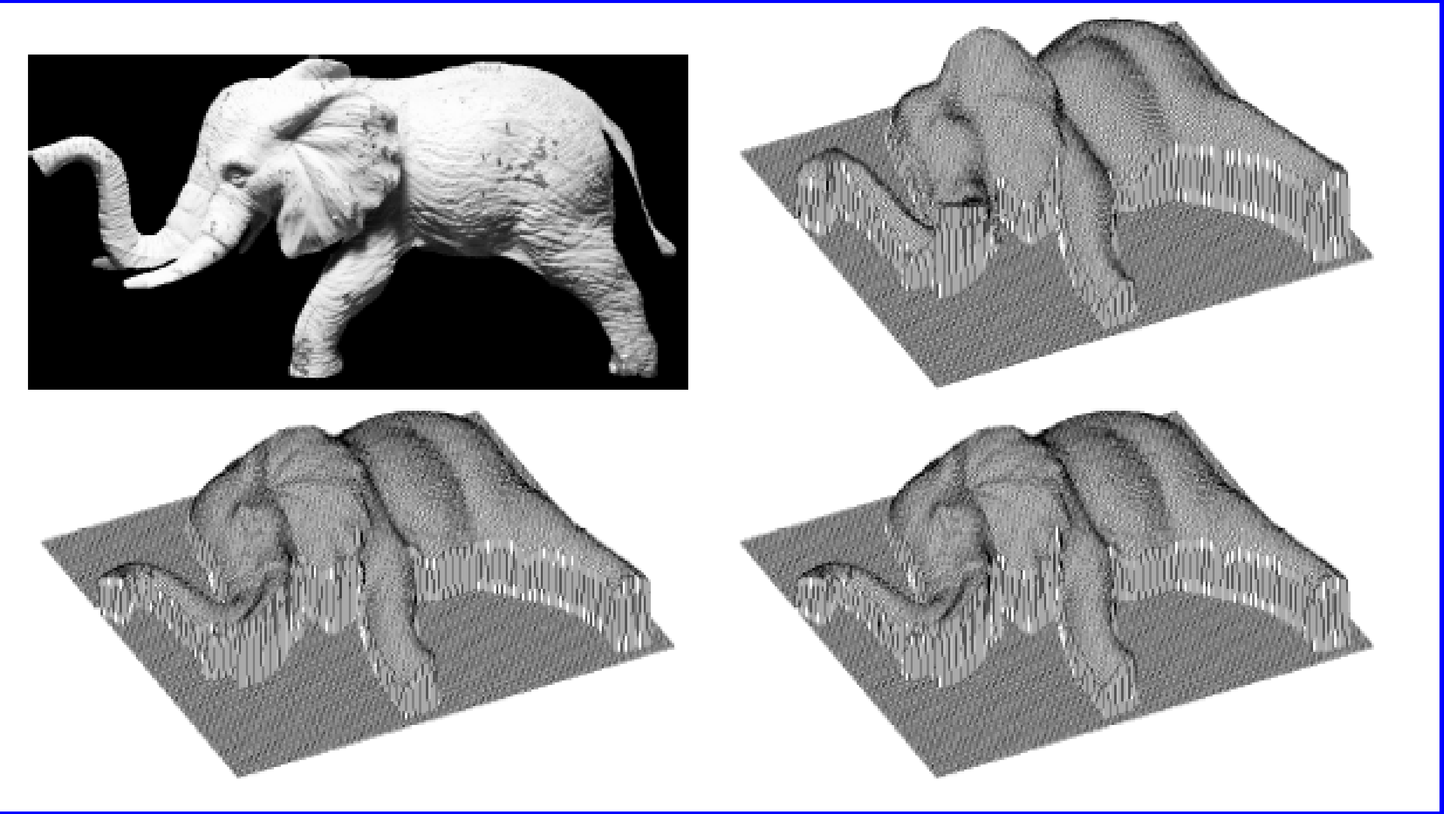
Experiment 2

4D 9D



More Experiments

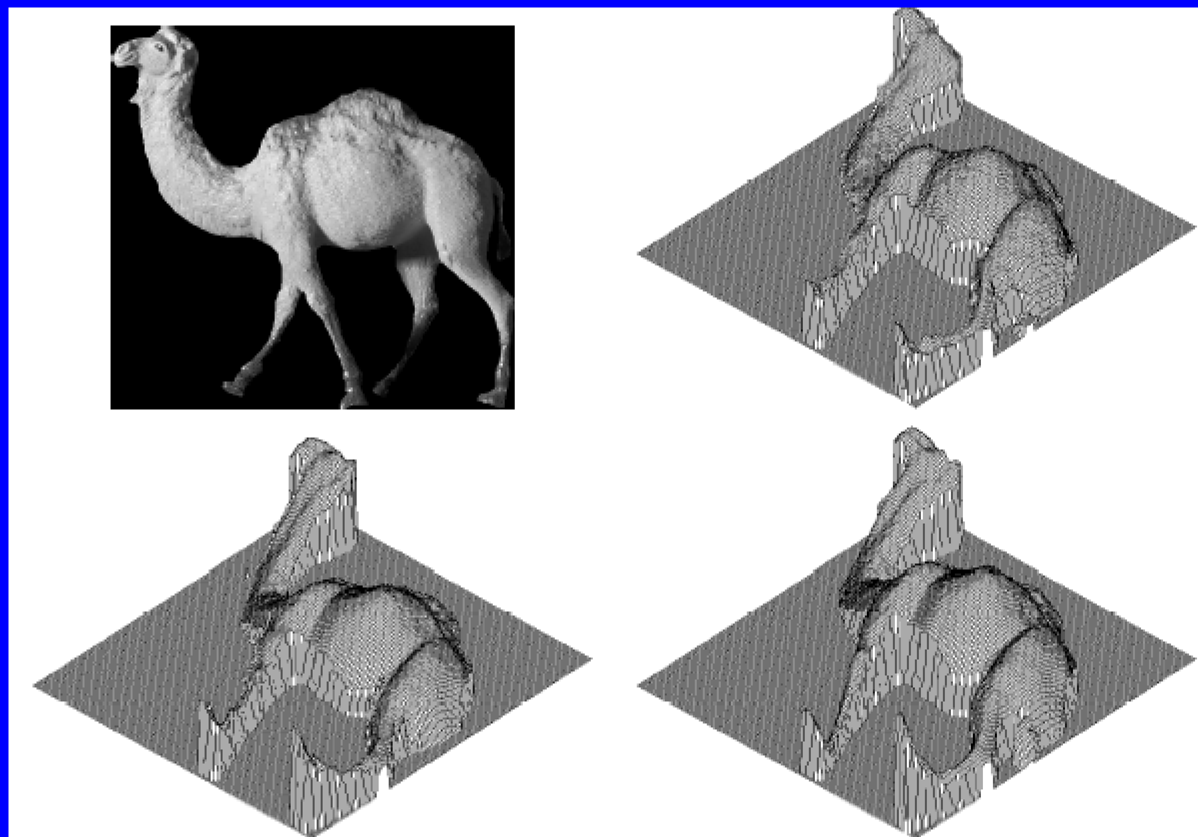
Ground Truth



4D 9D

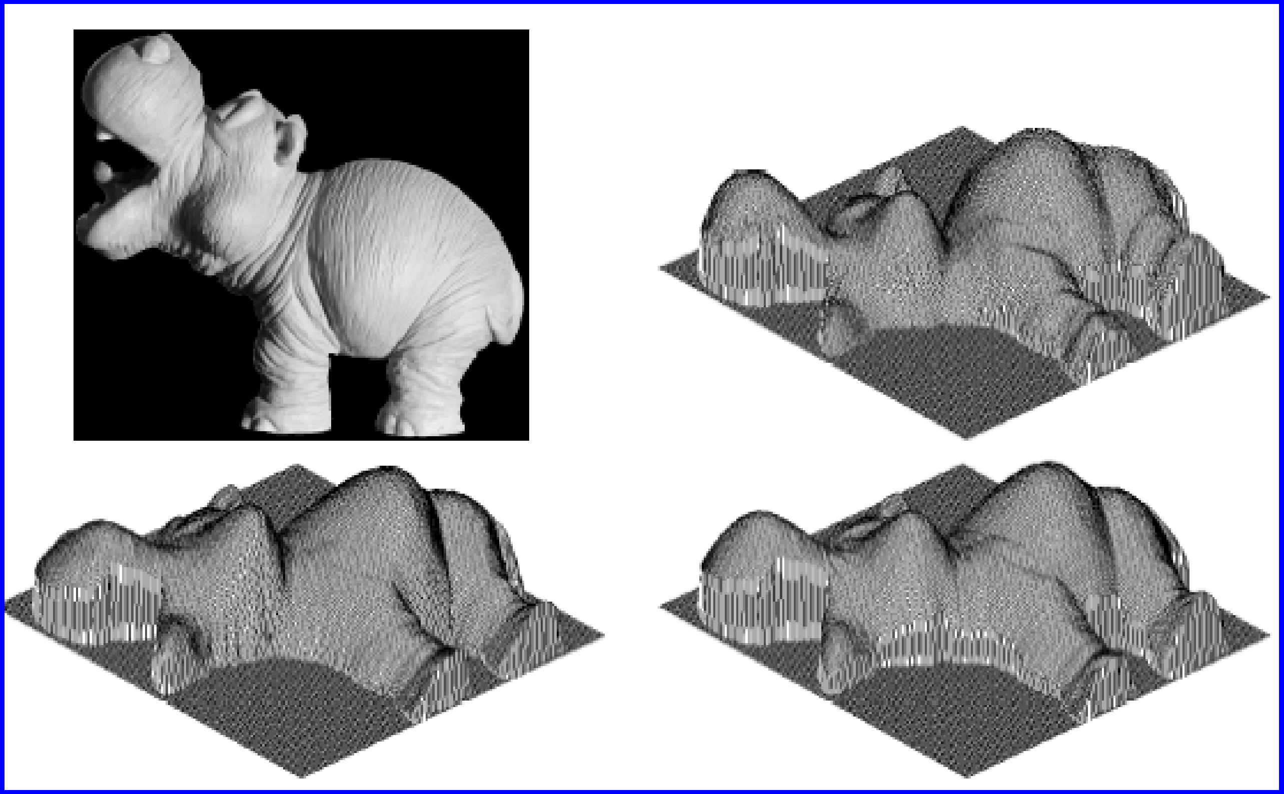
More Experiments

Ground Truth



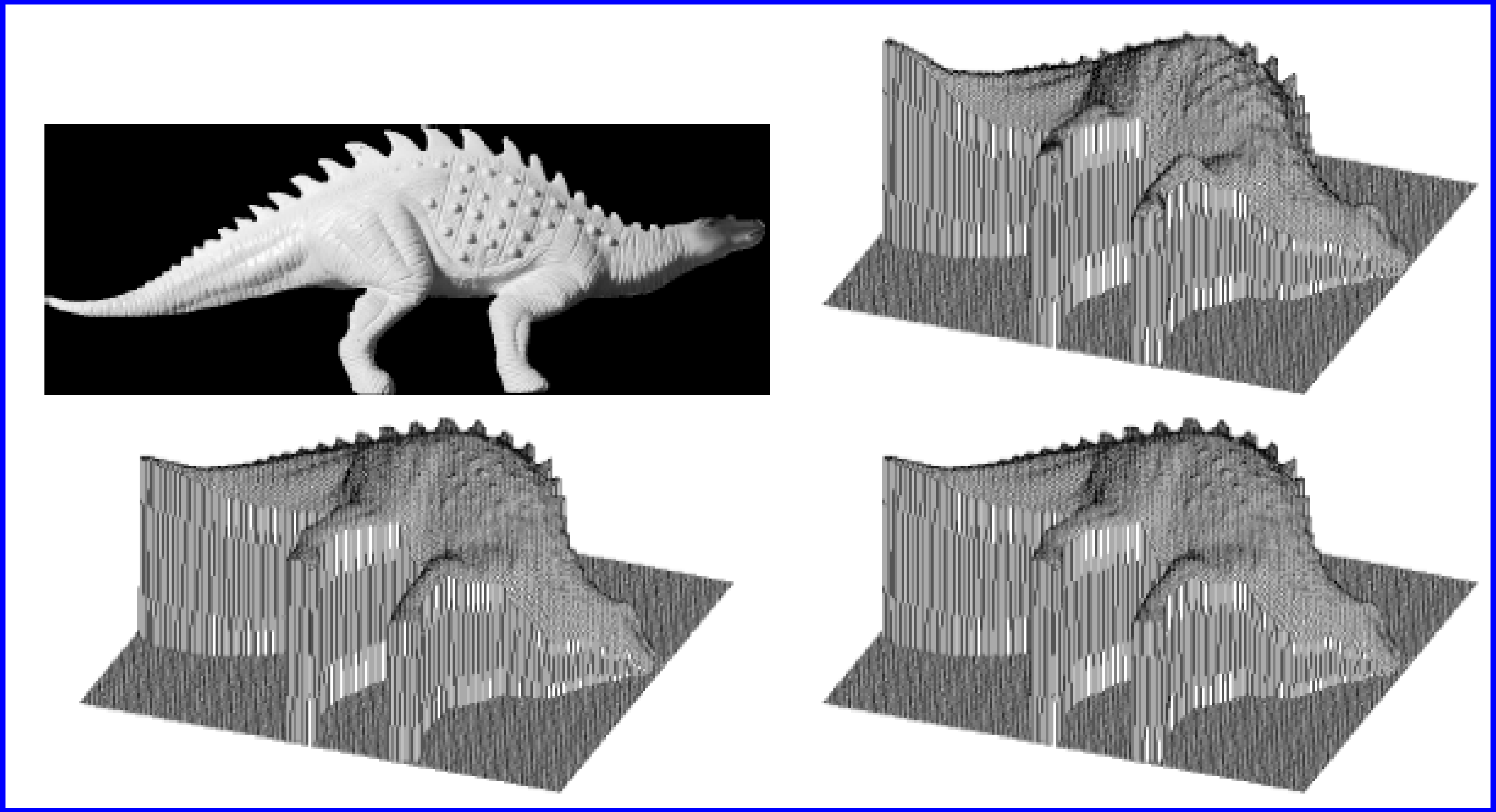
4D 9D

More Experiments



More Experiments

Ground Truth



4D

9D