Midterm CMSC733 2013

Assigned, 10/31/13

Due, 11/7/13, 11am

This is a take home midterm. You must do these problems on your own. It is acceptable to review material that we have discussed in class with other students. However, it is not acceptable to discuss these specific problems with any other students. You may make use of other resources (papers, books, web pages) in solving these problems, but you should cite any material that you make use of. All problems will count equally.

1. **Diffusion**
   a. Consider Perona-Malik diffusion, as described in "A review of nonlinear diffusion filtering," by JoachimWeickert. equation (21). Suppose we smooth a 1D function using Perona-Malik diffusion. We can describe the smoothed function as f(x,t), a function of space and time. Is it true that
   $$\max_x f(x,t_1) \geq \max_x f(x,t_2) \quad \forall t_1, t_2 \quad t_2 > t_1?$$
   Prove this is true, or give an example to show it is not true.
   b. Answer the same problem with Gaussian smoothing instead of Perona-Malik.
2. **Markov Processes** Suppose Alice and Bob are playing a game. There is a token that starts with Alice. Alice and Bob each have a coin. Whoever has the token, flips their coin. If the coin lands heads, they pass the token to the other player. If the coin lands tails, the player keeps the token. This is repeated a million times. Whoever has the token at the end wins. You get to watch the first move of a few thousand games, and conclude that Alice is using a biased coin, that lands tails 80% of the time.

   . (a) If Bob's coin is fair, what is the probability that Alice will win the game?

   . (b) Suppose you also get to learn the outcome of a few thousand games, and discover that Bob is winning 60% of the time. Use this information to determine the probability that Bob's coin lands tails when it is flipped.

3. **Normalized Cut** Suppose we have a bunch of points, $x_1 \ldots x_n$, and we apply normalized cut to them using edge weights of $E(x_i,x_j) = \exp(-||x_i\text{-}xj_j|\text{^}2)$. Suppose we apply normalized cut to the points $x_1,x_1, x_2, x_2, \ldots x_n, x_n$, repeating each point twice.
   a. Is it possible we would get a different solution? Prove it is not possible, or show an example in which we would.
   b. What if, instead of applying Normalized Cut, we have an oracle that gives us the globally optimal solution to the normalized cut cost function? Would the answer change? Again, either prove that the result is always the same, or give an example in which it isn't.

4. **Markov Random Fields.** Suppose that you have run an edge detector on an image, producing a binary image, I, in which a pixel has a value of 1 if it is an edge, and 0 if it is not. You would like to segment the image to find a single object of interest. The object is known to contain the pixel (50,50), and to not touch the edge of the image. You wish to choose a segmentation so that the boundary between the object and the background contains as few non-edge pixels as possible.
    a. **FULLY** describe an MRF such that the MAP estimate of the labels in the MRF provides this segmentation.
    b. Fully describe a Gibbs distribution that is equivalent to the MRF in (a).
    c. Fully describe a graph that can be constructed for this MRF so that the min-cut of the graph provides the desired segmentation.

5. **Diffusion process** Suppose we have a 1D interval between 0 and 1. We have some quantity of material concentrated at ½. It diffuses over time. However, there is a wall bounding the interval at 0 and at 1, so that the material cannot diffuse beyond those limits. We model this by saying that if a particle is located at 0 (or 1) it can diffuse in the positive (negative) direction, but any attempt to diffuse in the negative (positive) direction results in the particle remaining where it is. Diffusion is otherwise homogenous and isotropic. The steady-state distribution is the distribution after some long period of time, at which point the distribution does not change.
    a. Assuming diffusivity is greater than 0, does the steady-state distribution depend on the diffusivity? Explain your answer. What is the steady state distribution? If it depends on diffusivity, give the distribution for three different choices of diffusivity.
    b. Suppose there is a region in the center of the interval, between ¼ and ¾, in which the diffusivity is twice as large as it is in the rest of the interval. Does the steady-state distribution depend on the exact values of the diffusivity? Explain your answer. What is the steady state distribution? If it depends on diffusivity, give the distribution for three different choices of diffusivity.

6. **Geometric Transformations** Let P be a 3x3 matrix that represents a 2-D projective transformation.
    a. Prove that P represents a rigid rotation only if P has one real eigenvalue and two complex eigenvalues that have non-zero imaginary components. Do not answer this question by saying: "We know that rotation matrices have a specific form of …." Any assertion of the form a rotation matrix must be proven.
    b. Note that above we say "only if". Is the above statement true if we say "if and only if"?