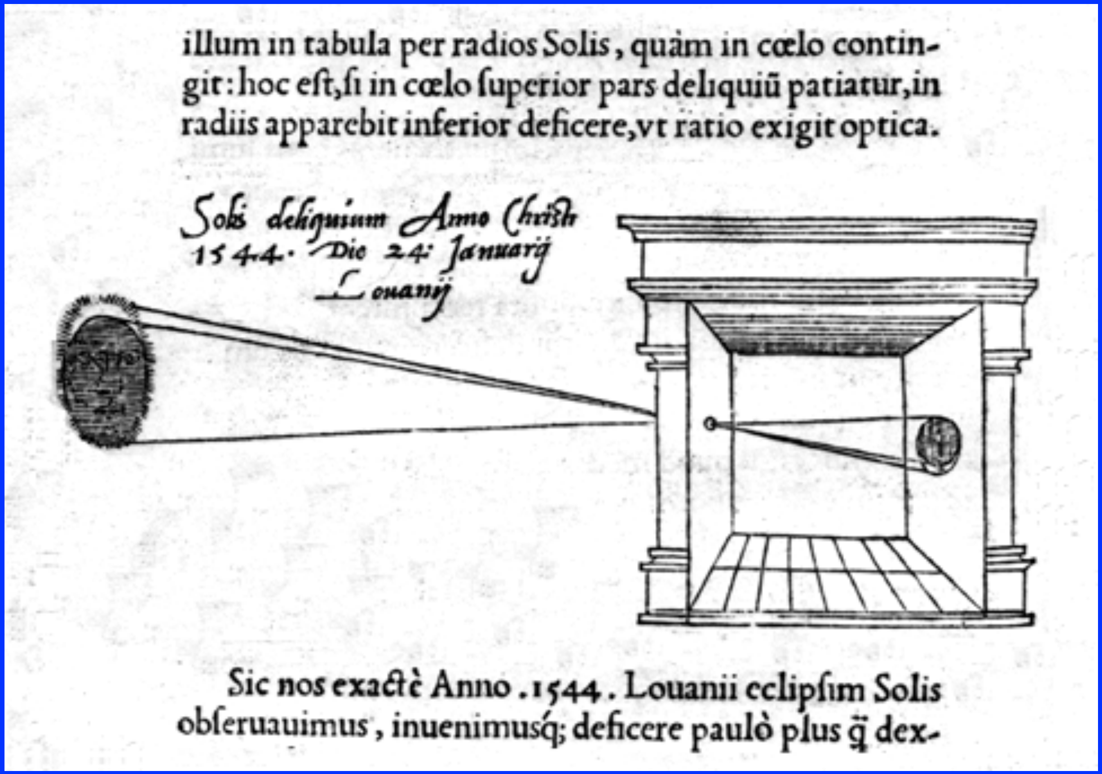
Camera Obscura



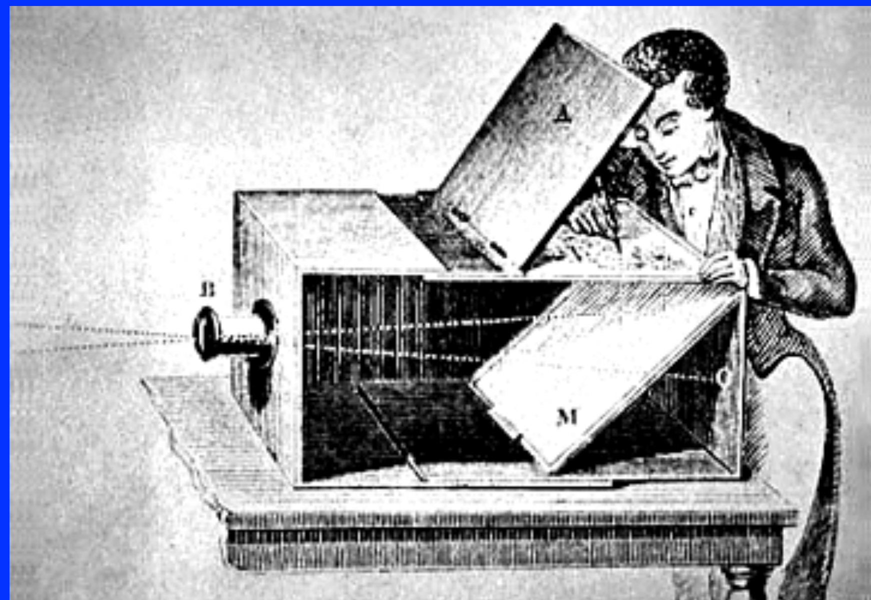
illum in tabula per radios Solis, quàm in cœlo contin-
git: hoc eſt, ſi in cœlo ſuperior pars deliquiũ patiatur, in
radiis apparebit inferior deficere, vt ratio exigit optica.

Sic nos exactè Anno .1544. Louanii eclipſim Solis
obſeruauimus, inuenimusq; deficere paulò plus q̃ dex-

"When images of illuminated objects ... penetrate through a small hole into a very dark room ... you will see [on the opposite wall] these objects in their proper form and color, reduced in size ... in a reversed position, owing to the intersection of the rays".

Da Vinci

http://www.acmi.net.au/AIC/CAMERA_OBSCURA.html (Russell Naughton)



- Used to observe eclipses (eg., Bacon, 1214-1294)
- By artists (eg., Vermeer).

Jetty at Margate England, 1898.

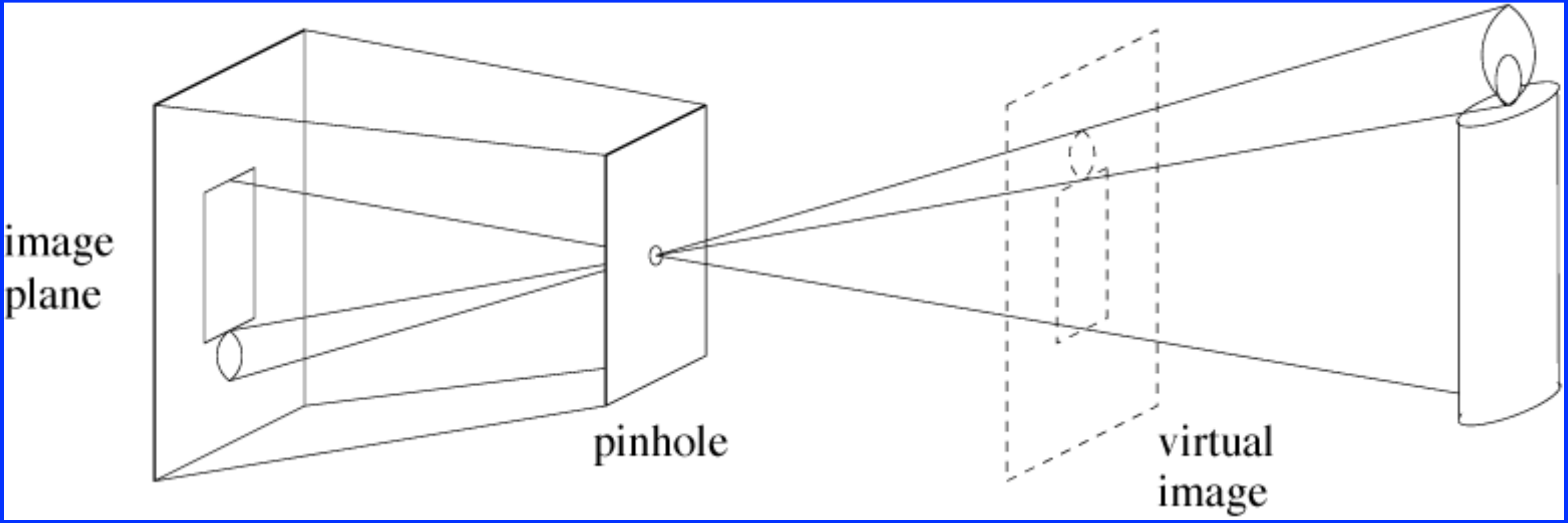
http://brightbytes.com/cosite/collection2.html (Jack and Beverly Wilgus)

Cameras

- First photograph due to Niepce
- First on record 1822

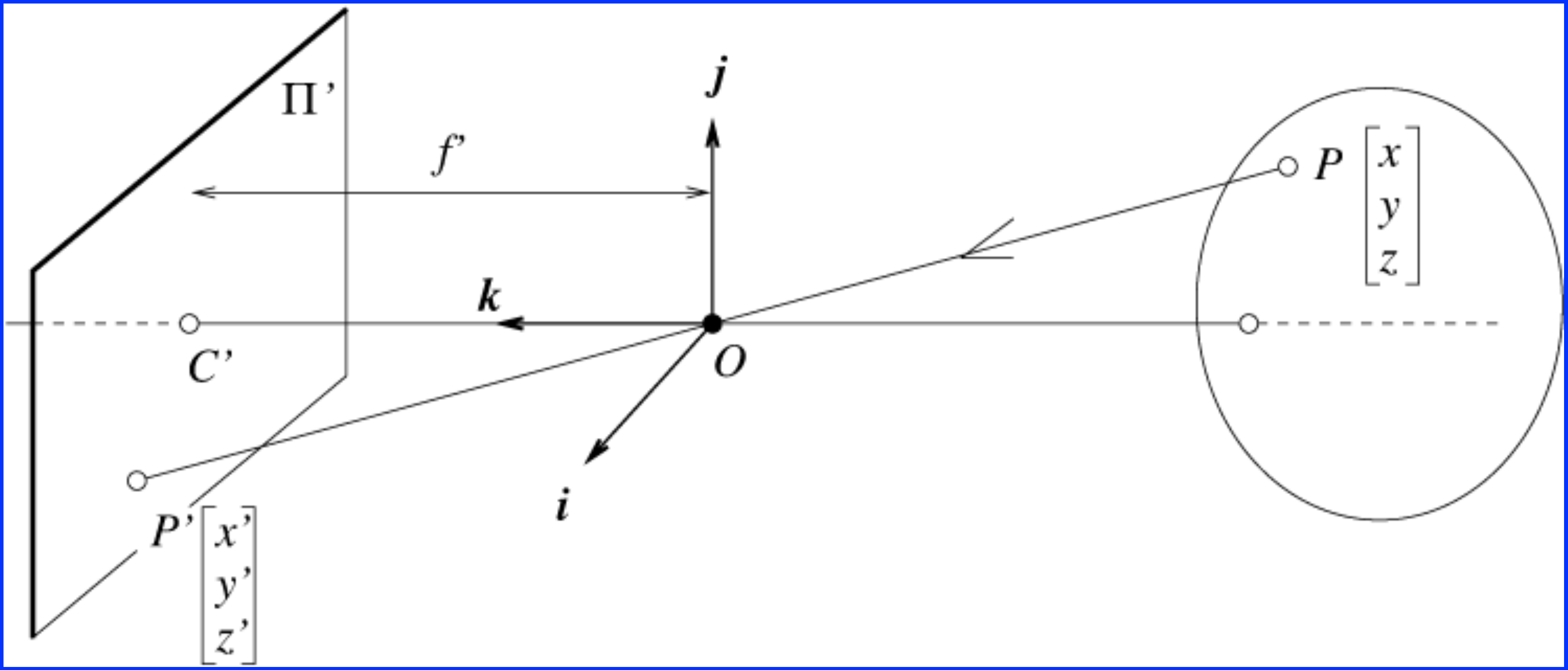
Pinhole cameras

- Abstract camera model - box with a small hole in it
- Pinhole cameras work in practice



(Forsyth & Ponce)

The equation of projection



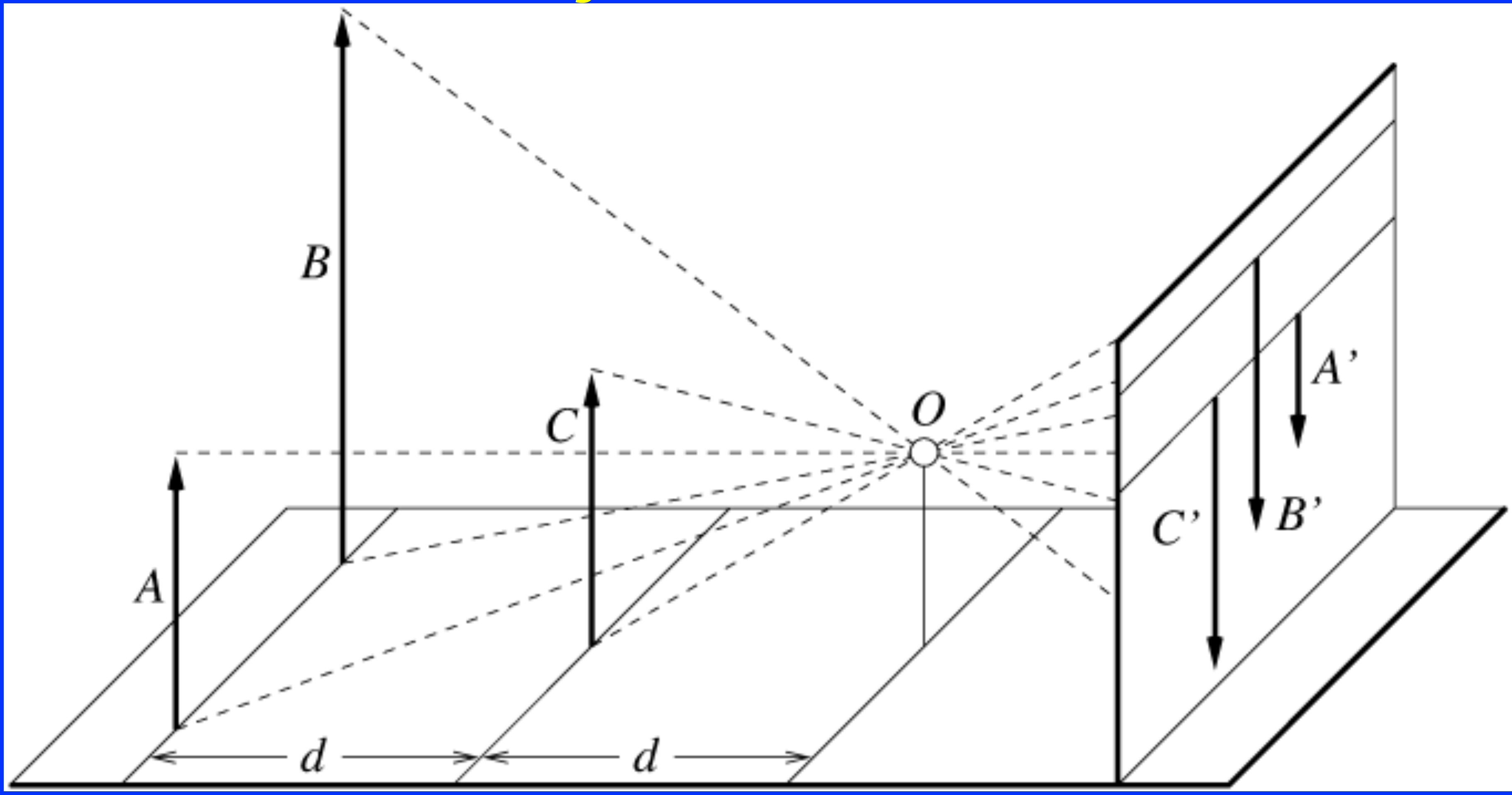
(Forsyth & Ponce)

The equation of projection

- Cartesian coordinates:
  - We have, by similar triangles, that (x, y, z) -> (f x/z, f y/z, f)
  - Ignore the third coordinate, and get

$$(x, y, z) \rightarrow (f\frac{x}{z}, f\frac{y}{z})$$

Distant objects are smaller



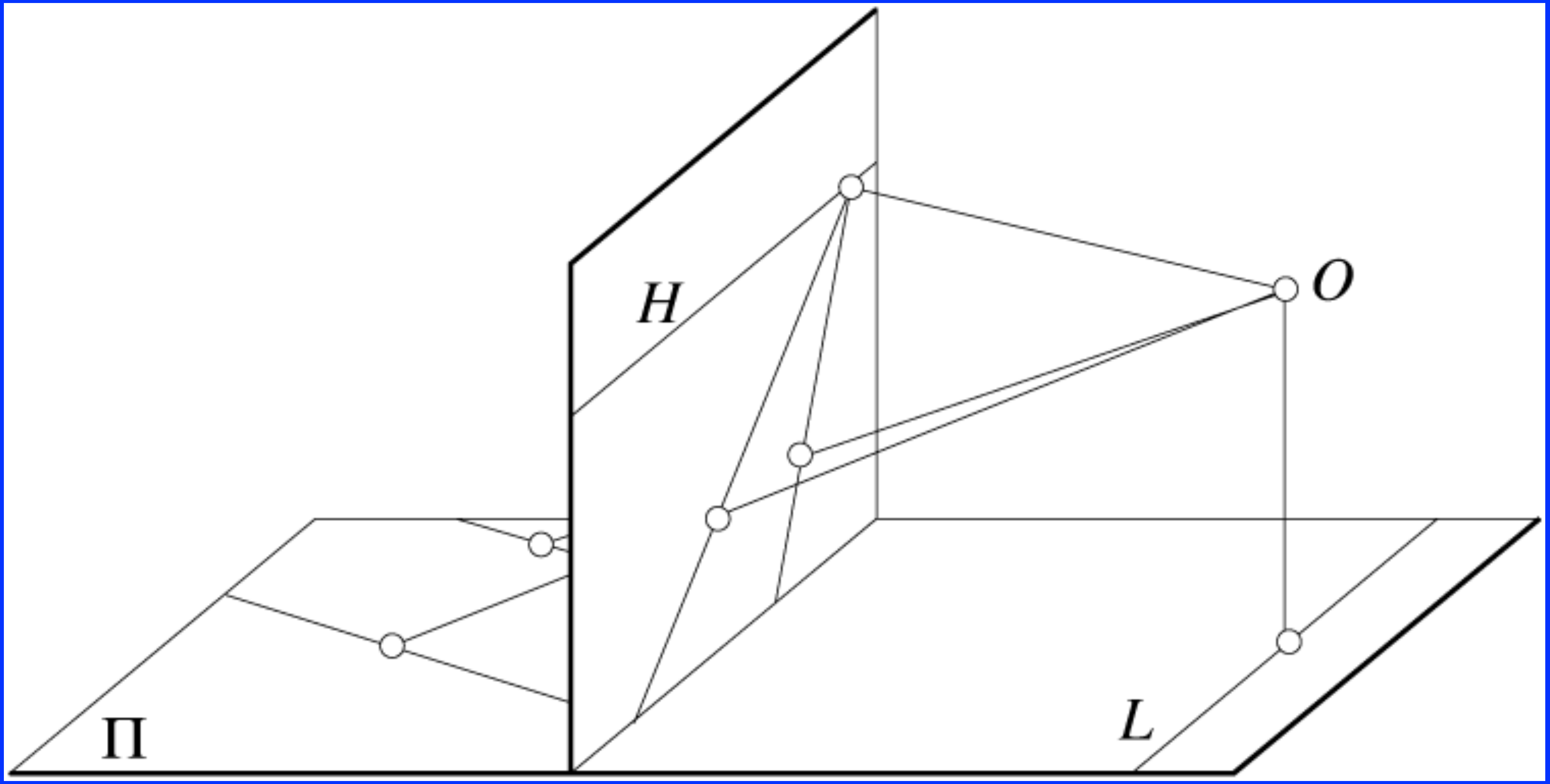
(Forsyth & Ponce)

For example, consider one line segment from (x, 0,z) to (x,y,z), and another from (x,0,2z) to (x,y, 2z). These are the same length.

These project in the image to a line from (fx/z,0) to (fx/z, fy/z) and from (fx/z,0) to (fx/2z, fy/2z), where we can rewrite the last point as: (1/2)(fx/z,fy/z). The second line is half as long as the first.

Parallel lines meet

Common to draw image plane *in front* of the focal point.
Moving the image plane merely scales the image.



(Forsyth & Ponce)

Vanishing points

- Each set of parallel lines meets at a different point
  - The *vanishing point* for this direction
- Sets of parallel lines on the same plane lead to *collinear* vanishing points.
  - The line is called the *horizon* for that plane

For example, let’s consider a line on the floor. We describe the floor with an equation like: y = -1. A line on the floor is the intersection of that equation with x = az + b. Or, we can describe a line on the floor as: (a, -1, b) + t(c, 0, d) (Why is this correct, and why does it have more parameters than the first way?)

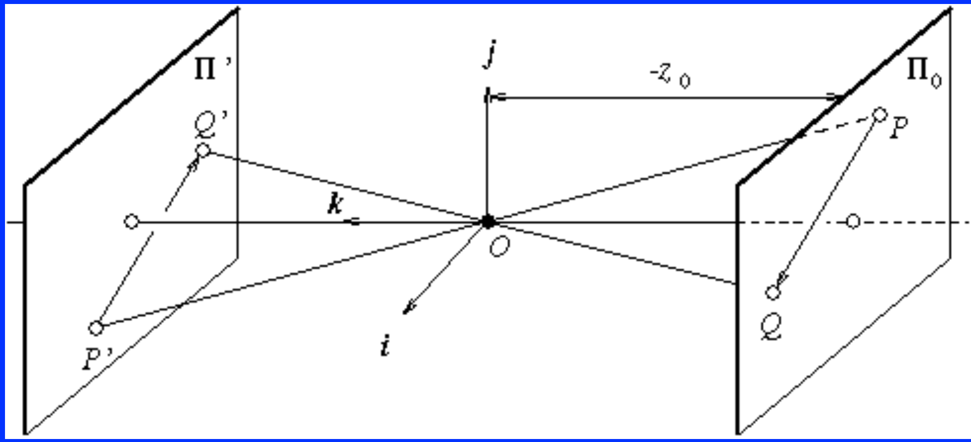
As a line gets far away, z -> infinity. If (x,-1,z) is a point on this line, its image is f(x/z,-1/z). As z -> infinity, -1/z -> 0. What about x/z? x/z = (az+b)/z = a + b/z -> a. So a point on the line appears at: (a,0). Notice this only depends on the slope of the line x = az + b, not on b. So two lines with the same slope have images that meet at the same point, (a,0), which is on the horizon.

Properties of Projection

- Points project to points
- Lines project to lines
- Planes project to the whole image
- Angles are not preserved
- Degenerate cases
  - Line through focal point projects to a point.
  - Plane through focal point projects to line
  - Plane perpendicular to image plane projects to part of the image (with horizon).

Weak perspective (scaled orthographic projection)

- Issue
  - perspective effects, but not over the scale of individual objects
  - collect points into a group at about the same depth, then divide each point by the depth of its group



(Forsyth & Ponce)

The Equation of Weak Perspective

$$(x, y, z) \rightarrow s(x, y)$$

- *s* is constant for all points.
- Parallel lines no longer converge, they remain parallel.

Pros and Cons of These Models

- Weak perspective much simpler math.
  - Accurate when object is small and distant.
  - Most useful for recognition.
- Pinhole perspective much more accurate for scenes.
  - Used in structure from motion.
- When accuracy really matters, must model real cameras.

Projective Transformations

- Mapping from plane to plane.
- Form a group.
  - They can be composed
  - They have inverses.
  - Projective transformations equivalent to set of images of images.

Linear Representation of Projective transformation

- Problem: x/z, y/z is non-linear.
- Answer: Homogenous coordinates.
  - By definition, (x,y,w) = (kx,ky,kw)
  - So (x,y,w) = (x/w, y/w, 1)

3D rigid motion + projection (because of homogenous coordinates, there is no projection)

$$\begin{pmatrix} 1 & 0 & 0 & t_x \\ 0 & 1 & 0 & t_y \\ 0 & 0 & 1 & t_z \end{pmatrix} \begin{pmatrix} r_{1,1} & r_{1,2} & r_{1,3} & 0 \\ r_{2,1} & r_{2,2} & r_{2,3} & 0 \\ r_{3,1} & r_{3,2} & r_{3,3} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix} \equiv \begin{pmatrix} r_{1,1} & r_{1,2} & r_{1,3} & t_x \\ r_{2,1} & r_{2,2} & r_{2,3} & t_y \\ r_{3,1} & r_{3,2} & r_{3,3} & t_z \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

For Planar Objects

$$\begin{pmatrix} r_{1,1} & r_{1,2} & r_{1,3} & t_x \\ r_{2,1} & r_{2,2} & r_{2,3} & t_y \\ r_{3,1} & r_{3,2} & r_{3,3} & t_z \end{pmatrix} \begin{pmatrix} x \\ y \\ 0 \\ 1 \end{pmatrix} \equiv \begin{pmatrix} r_{1,1} & r_{1,2} & t_x \\ r_{2,1} & r_{2,2} & t_y \\ r_{3,1} & r_{3,2} & t_z \end{pmatrix} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$$

The first two columns on right are orthonormal. Scale is irrelevant. So there are 6 degrees of freedom.

We ignore constraints to get 8. This is called a **projective transformation.** By convention we scale the matrix so the lower right value is 1.

Points at Infinity

- Note that we can have points like (1,1,0)
- If we thought of the third coordinate as z, and the (x,y) coordinates as (1/0,1/0), this point would be at infinity.
- This will make sense. These are points that are infinitely far away, in the x or y direction. We need to represent them because these points can become visible, eg., the horizon.

Example: Suppose we take a fronto-parallel planar surface, at z = 0, and transform it so that it is the ground plane, using a transformation like:

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 1 \end{pmatrix}$$

This takes a point like (x,y,1) and transforms it to (x, -1, y+1). In 3D we can think of this as transforming it to lie on the y = -1 plane.

Now consider a line, of the form (x,y,1) + (tu,tv,0) = (x+tu,y+tv,1), for all values of t. Applying this transformation gives: (x+tu, -1, y+tv+1). As t -> infinity, this converges to (u/v, 0, 1). So a point at infinity is being mapped to a point on the horizon, which has y = 0.

Another way to think of this is to consider the point (u,v,0). This is a point at infinity. Applying the transformation to this point produces: (u, 0, v), which is the same as (u/v, 0, 1). The point (u,v,0) is the same point as (x+tu,-1,y+tv+1) in the limit as t goes to infinity. And this transformation is mapping this point to a point that is not at infinity, but that is visible.

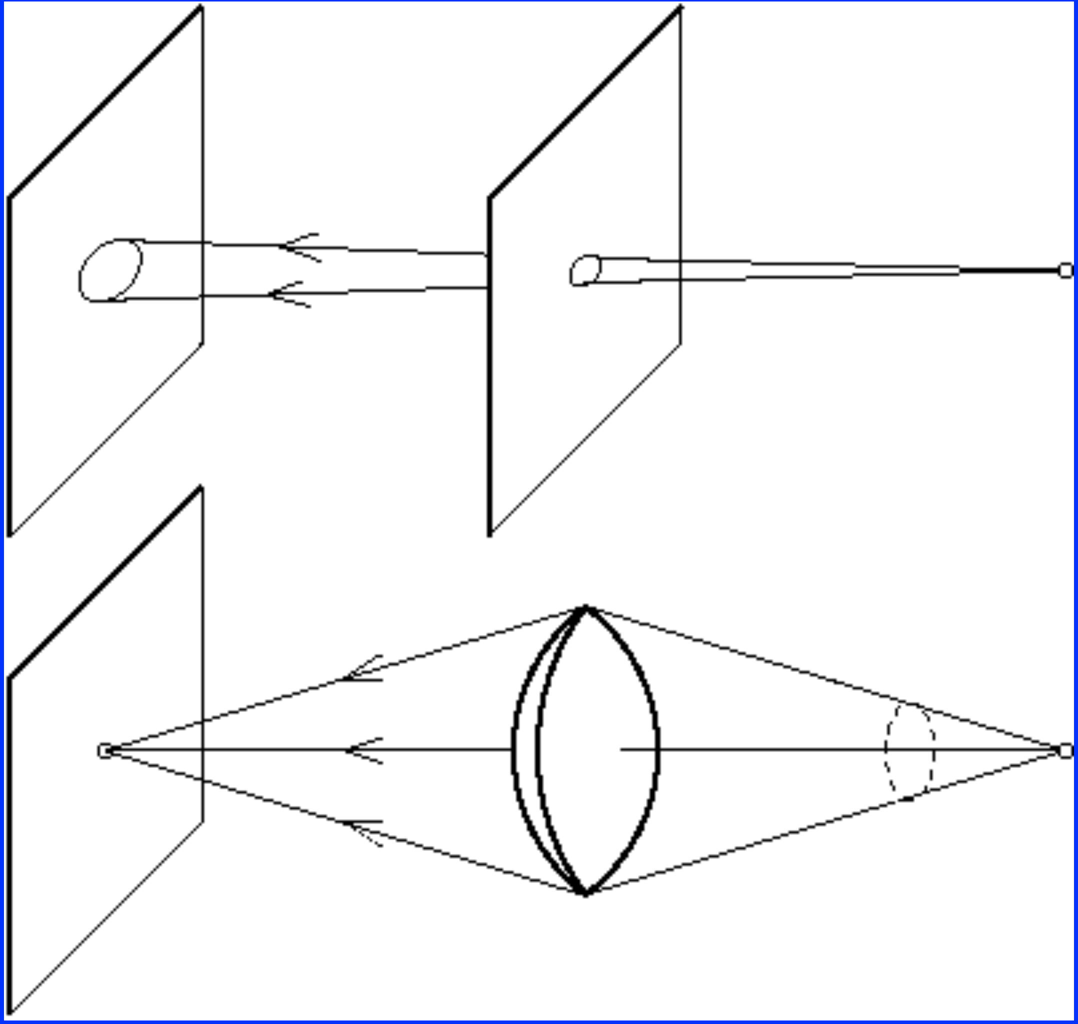
Solving for Projective Transformation w/ Points

- One point: Px = ku, where x is point in one image, u in second image.
- 3 Linear equations with 8 unknowns for P and one for k.
- Each new points provides 3 equations, one unknown.
- 4 points means 12 equations, 12 unknowns.

Lines

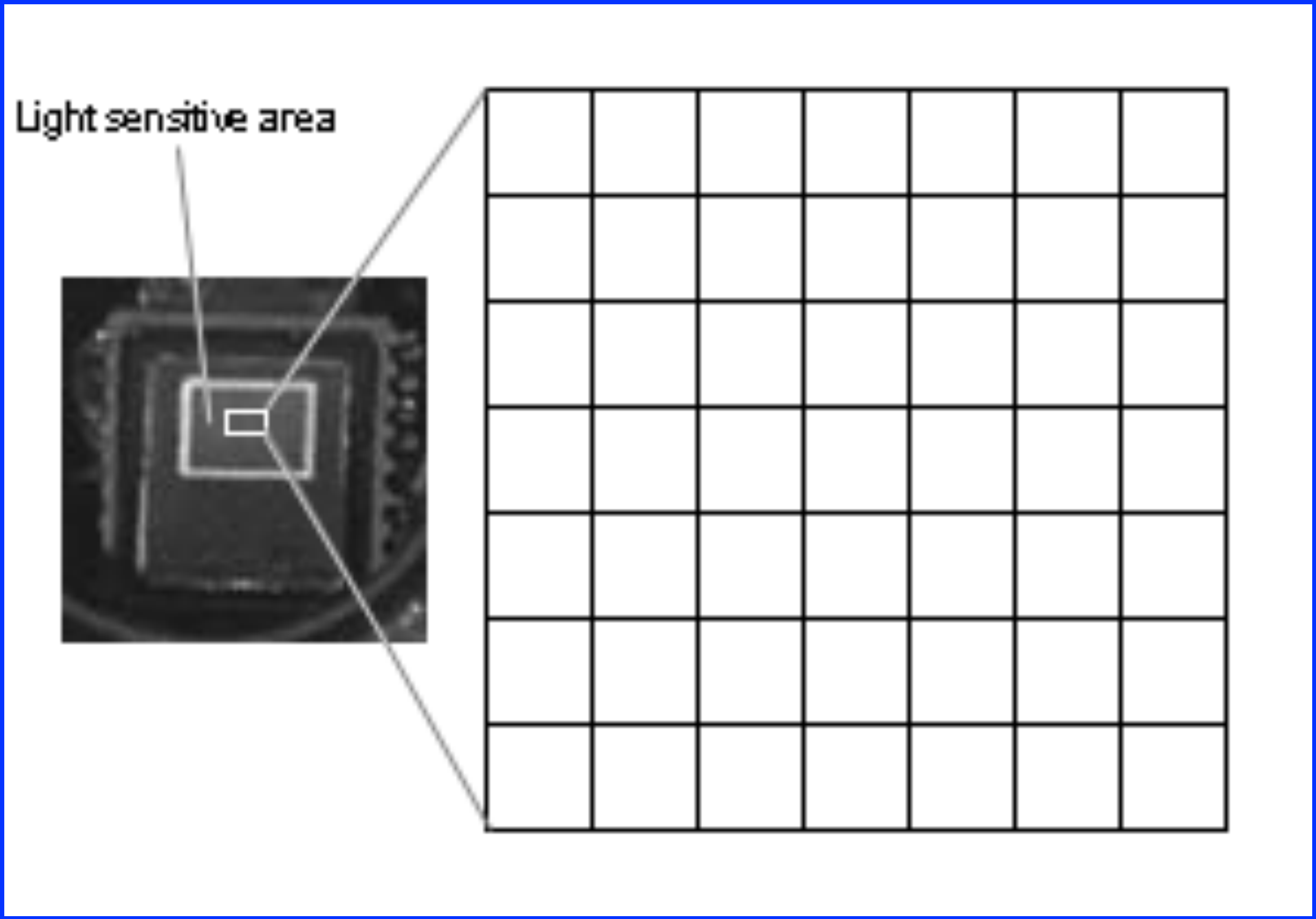
- We can represent a line in projective space as $L = (a,b,c)^T$.
- Then the equation for a point to lie on a line is $L^T p = 0$.
- Given two lines, L and L’, we can find their intersection by solving for $L^T p = 0$, $L'^T p = 0$.
- These are two linear equations, so all pairs of lines intersect. (unless L = L’, which are the same line).
- So parallel lines intersect. Lines are parallel but different when k(a,b) = (a’,b’), with kc <> c’. So ax +by+cw = 0 = kax+kby+c’w=0. So w = 0. Intersection point is at infinity.

Cameras with Lenses



(Forsyth & Ponce)

CCD Cameras



http://huizen.ddsw.nl/bewoners/maan/imaging/camera/ccd1.gif