Image classification by a Two Dimensional Hidden Markov Model

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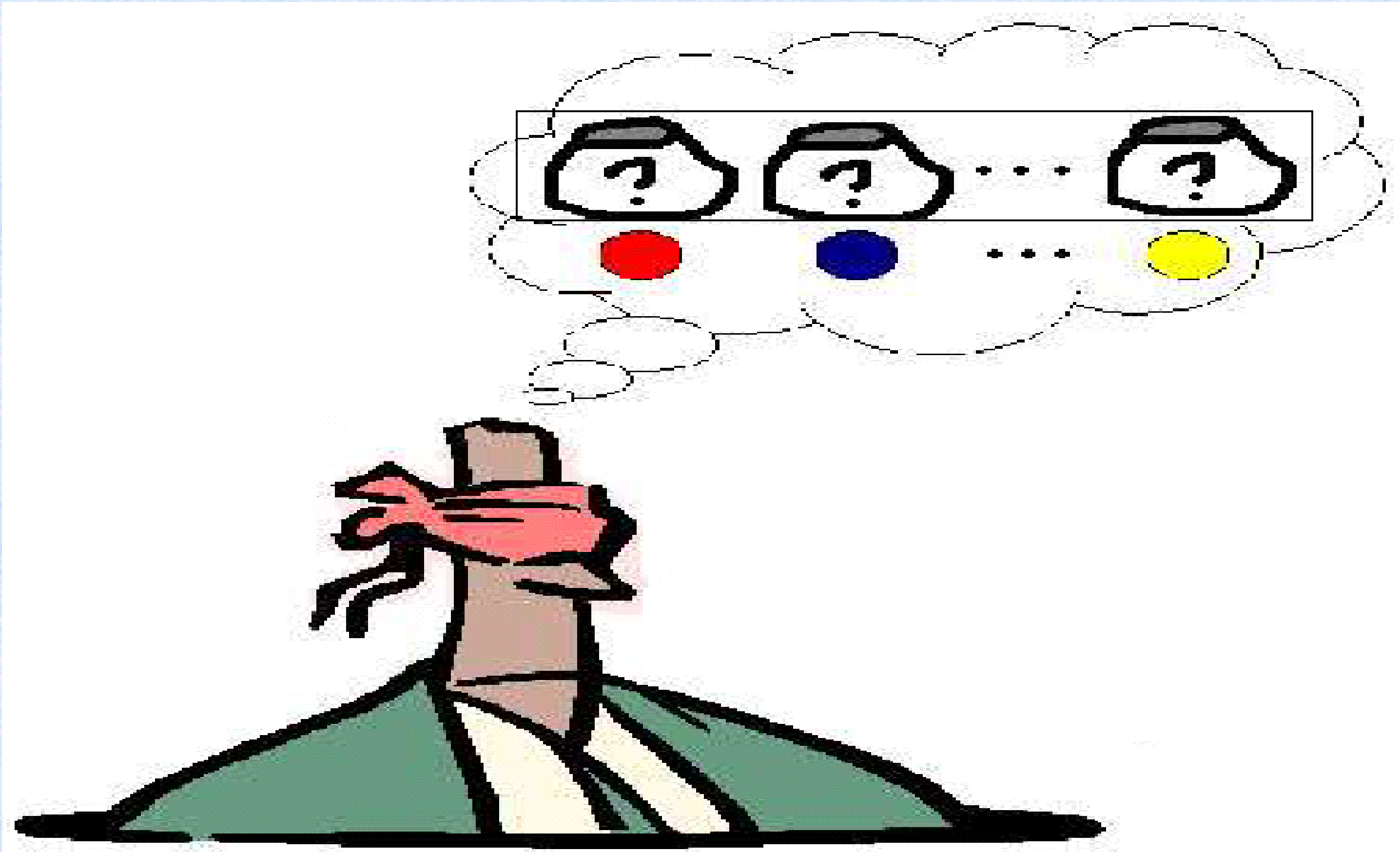
Hidden Markov Chain



Goal:

- To implement a novel classifier for image segmentation

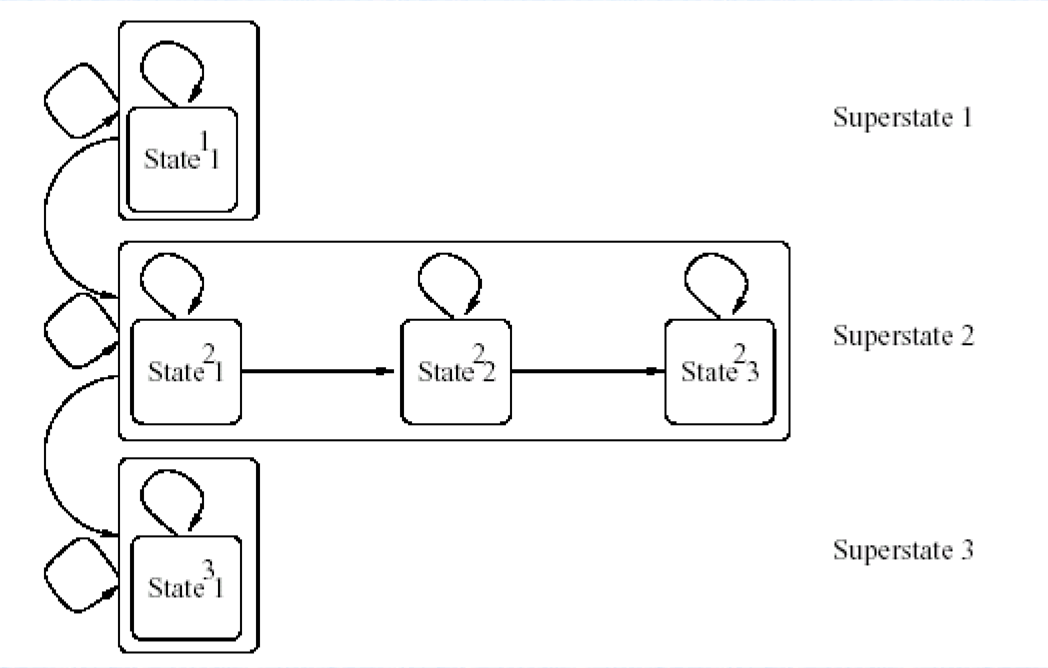
What's over there?

- LVQ (Learning vector quantization)
- BVQ (Bayes vector quantization)
- CART (Classification and regression trees)
- Pseudo-2D HMM
- Others… (PCA, LDA, neural networks, …)

Pseudo 2D HMM

- Extension of 1D case
- Not real 2D model since it does not connect all the possible states
- There is a "superstate" existing in the first element of each row. All the superstates consist of a markov chain.
- Each row consists of an independent markov chain.

Pseudo 2D HMM



Markov Random Field (MRF)

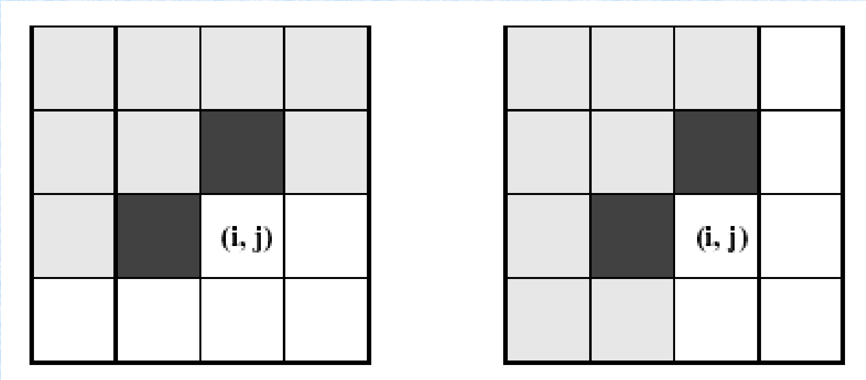
- A tool to encode contextual constraints into the prior probability.

  Define (1) neighbors (2) cliques (3) prior clique potential

  Derive (4) likelihood energy (5) posterior energy

Assumptions in the paper

- Causal Markov Random Field



$$P(s_{i,j} \mid s_{i',j'}, u_{i',j'} : (i',j') \in \tilde{\Psi} \cup \Psi) = P(s_{i,j} \mid s_{i-1,j}, s_{i,j-1})$$

Mainframe of the paper

- 2D HMM Model

  1. Expectation Maximum (EM)

  2. Viterbi Algorithm for 2D case

- Feature Selections:

  1. DCT coefficients & spatial derivatives of the average intensity value of blocks

  2. Wavelets & Laplacian Measurement

2D HMM

- 1. Training:

(a) Divide training images into non-overlapping blocks

(b) Extract the features of each block

(c) Select the number of states for the 2D-HMM

(d) Estimate model parameters based on the feature vectors (v) and their hand-labeled class (c)

Training Process

- Expectation Maximization
- Known: Mapping function from S and C

  E-Step: Compute Q(Φ| Φ^p)

$$E(\log f(\mathbf{x} \mid \phi') \mid \mathbf{y}, \phi^{(p)}) = \frac{1}{\alpha} \sum_{\mathbf{s}} P(\mathbf{s} \mid \mathbf{y}, \phi^{(p)}) \cdot \sum_{(i,j) \in \mathbb{N}} \log a'_{s_{i-1,j}, s_{i,j-1}, s_{i,j}} + \frac{1}{\alpha} \sum_{\mathbf{s}} P(\mathbf{s} \mid \mathbf{y}, \phi^{(p)}) \sum_{(i,j) \in \mathbb{N}} \log P(u_{i,j} \mid \mu'_{s_{i,j}}, \Sigma'_{s_{i,j}}).$$

where $P(\mathbf{s} \mid \mathbf{y}, \phi^{(p)}) = \frac{1}{\alpha} I(C(\mathbf{s}) = \mathbf{c}) \cdot \prod_{(i,j) \in \mathbb{N}} a^{(p)}_{s_{i-1,j}, s_{i,j-1}, s_{i,j}} \cdot \prod_{(i,j) \in \mathbb{N}} P(u_{i,j} \mid \mu^{(p)}_{s_{i,j}}, \Sigma^{(p)}_{s_{i,j}})$

Training Process (cont.)

- Expectation Maximization

  M-Step: Choose Φ to maximize $Q(\Phi | \Phi^p)$

$$a'_{m,n,l} = \frac{\sum_{(i,j)\in\mathbb{N}} H^{(p)}_{m,n,l}(i,j)}{\sum_{l=1}^{M}\sum_{(i,j)\in\mathbb{N}} H^{(p)}_{m,n,l}(i,j)} .$$

$$H^{(p)}_{m,n,l}(i,j) = \sum_{\mathbf{s}} I(m = s_{i-1,j}, n = s_{i,j-1}, l = s_{i,j}) P(\mathbf{s} \mid \mathbf{y}, \phi^{(p)})$$

$$\mu'_m = \frac{\sum_{i,j} L^{(p)}_m(i,j) u_{i,j}}{\sum_{i,j} L^{(p)}_m(i,j)} ,$$

$$L^{(p)}_m(i,j) = \sum_{\mathbf{s}} I(m = s_{i,j}) P(\mathbf{s} \mid \mathbf{y}, \phi^{(p)})$$

$$\Sigma'_m = \frac{\sum_{i,j} L^{(p)}_m(i,j)(u_{i,j} - \mu'_m)(u_{i,j} - \mu'_m)^t}{\sum_{i,j} L^{(p)}_m(i,j)} .$$

Compute it? Thank for diagonal

- Forward: $\theta_T(d) = P\{\mathbf{s}(d) = T, \mathbf{u}(\tau) : \tau \le d \mid \mathrm{M}\}$

$$\theta_{T_d}(d) = \sum_{T_{d-1}} \theta_{T_{d-1}}(d-1) \cdot P(T_d \mid T_{d-1}, \mathrm{M}) \cdot P(\mathbf{u}(d) \mid T_d, \mathrm{M})$$

- Bakward: $\beta_T(d) = P\{\mathbf{u}(\tau) : \tau > d \mid \mathbf{s}(d) = T, \mathrm{M}\}$

$$\beta_{T_d}(d) = \sum_{T_{d+1}} P(T_{d+1} \mid T_d, \mathrm{M}) \cdot P(\mathbf{u}(d+1) \mid T_{d+1}, \mathrm{M}) \cdot \beta_{T_{d+1}}(d+1)$$

Where Td: state sequences at dth diagonal

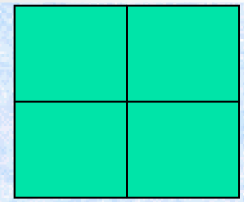
$$L_m(i,j) = \sum_{T_d : T_d(i,j)=m} \frac{\theta_{T_d}(\Delta(i,j)) \cdot \beta_{T_d}(\Delta(i,j))}{P(\mathbf{u}, \mathbf{c} \mid \mathrm{M})} \quad C(m) = c_{i,j}$$

$$H_{m,n,l}(i,j) = \sum_{T_d} \sum_{T_{d-1}} \frac{\theta_{T_{d-1}}(\Delta(i,j)-1)}{P(\mathbf{u}, \mathbf{c} \mid \mathrm{M})} \cdot \quad \begin{matrix} T_{d-1}(i-1,j)=m, T_{d-1}(i,j-1)=n \\ T_d(i,j)=l \end{matrix}$$

$$[P(T_d \mid T_{d-1}, \mathrm{M}) P(\mathbf{u}(d) \mid T_d, \mathrm{M}) \cdot \beta_{T_d}(\Delta(i,j))]$$

$$C(m) = c_{i-1,j}, C(n) = c_{i,j-1}, C(l) = c_{i,j}$$

Simple example



1. Define # of states
2. Divided the image into sub-blocks and extract the features (U)of each block.
3. Given C(i,j) and random assign $a_{m,n,l}$, μ_m and Σ_m
4. Use the parameters to find forward and backward probability based on each diagonal
5. Calculate $L_m(i, j)$ and $H_{m,n,l}(i, j)$
6. Use L and H to update $a_{m,n,l}$, μ_m and Σ_m
7. Go back to step 4

2D HMM

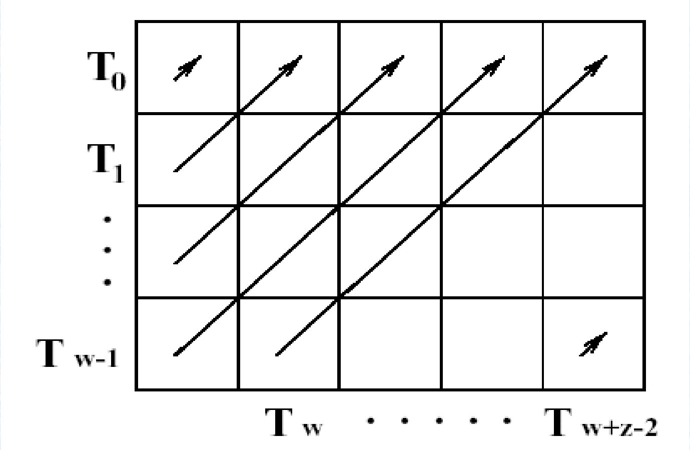
- Recognition

(a) Generate feature vectors for the testing image

(b) Search for the set of classes with maximum a posteriori probability given the feature vectors according to the trained 2D HMM

2D Viterbi algorithm

- The computational complexity is $(NxM-1)^{mo}$
- Any way to simplify?
- Using forward probabilities and applying the blocks of the diagonal to create markovian isolation sequence



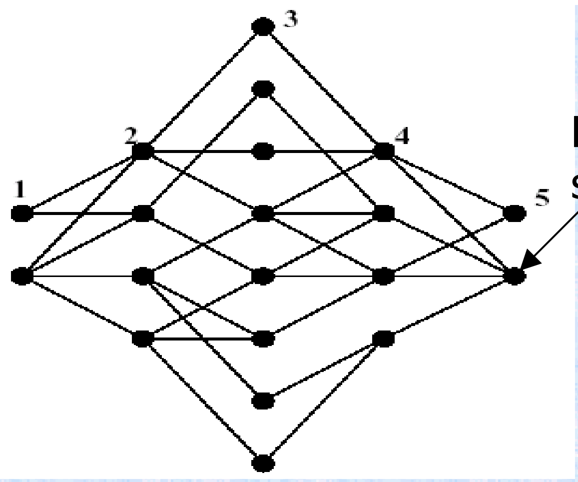
2D Viterbi algorithm

- Reminder

  Viterbi is used to maximize P(I,O| Φ)

- For 2D, maximize $P\{s_{i,j}, u_{i,j} : (i,j) \in \mathbb{N}\}$

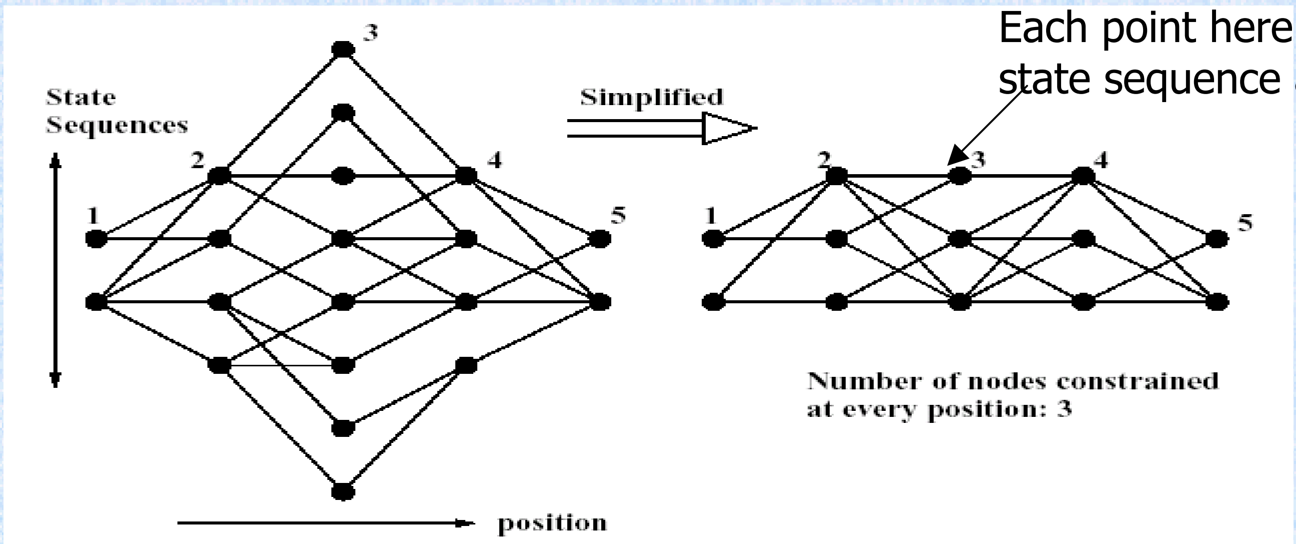
$$P(T_0) \cdot P(T_1 \mid T_0) \cdot P(T_2 \mid T_1) \cdots P(T_{w+z-2} \mid T_{w+z-3}) \prod_{(i,j)\in\mathbb{N}} P(u_{i,j} \mid s_{i,j})$$



Each point here infers a state sequence at Td

2D Viterbi Algorithm (cont.)

- It is infeasible. NP completeness
- Proposition: path-constrained Viterbi algorithm



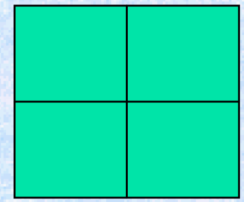
2D Viterbi Algorithm (cont.)

- In each diagonal, find the best N routes based on $\sum_{i=1}^{v} \gamma_{i,s_i}$ where v is the number of diagonal terms.
- We can simplify the constraints above by finding $\max_{s_i}^{-1} \gamma_{i,s_i}$ for each i
- Best route searching
- Not a precise model. Two improvements

  1. Choose large N

  2. Divide original image into sub-images

Simple Example



- 1. Image input
- 2. Extract the features
- 3. In each diagonals, pick the best N sequences.
- 4. Use the normal viterbi to calculate

$$P(T_0)\cdot P(T_1 \mid T_0)\cdot P(T_2 \mid T_1)\cdots P(T_{w+z-2} \mid T_{w+z-3}) \prod_{(i,j)\in \mathbb{N}} P(u_{i,j} \mid s_{i,j})$$

Now Ti is limited by the local maximizor

5. Find the highest score of S and C(s)=C

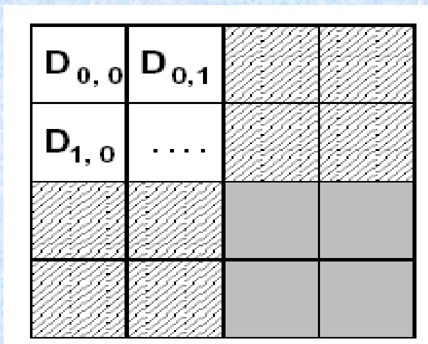
Aerial Image Segmentation

- Goal: Try to differentiate the man-made or nature scenes from the aerial image
- # of states: 5 (3~6 are all available) for nature scene. 9 (7~10 are all available) for man-made scene
- Size of N: 32
- Block size: 4x4

Aerial Image Segmentation

- Feature selection:

1. Intra-block feature: DCT coefficients



1. $f_1 = D_{0,0}$; $f_2 = |D_{1,0}|$; $f_3 = |D_{0,1}|$;

2. $f_4 = \sum_{i=2}^{3} \sum_{j=0}^{1} |D_{i,j}|/4$;

3. $f_5 = \sum_{i=0}^{1} \sum_{j=2}^{3} |D_{i,j}|/4$;

4. $f_6 = \sum_{i=2}^{3} \sum_{j=2}^{3} |D_{i,j}|/4$.

2.Inter-block feature: Average intensity difference between neighbors (U,L)

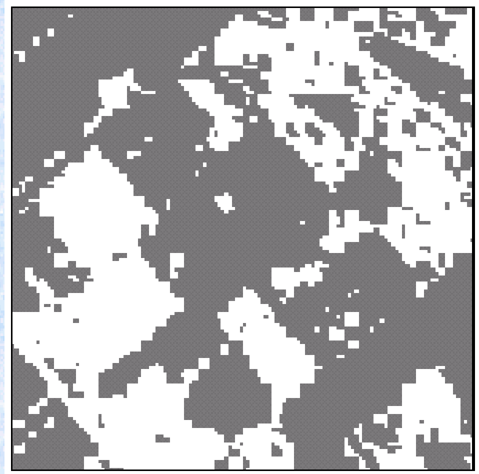
Results

- Man-made scene as targets
- Compared with three other methods (CART, LVQ (learning vector quantization) and BVQ)

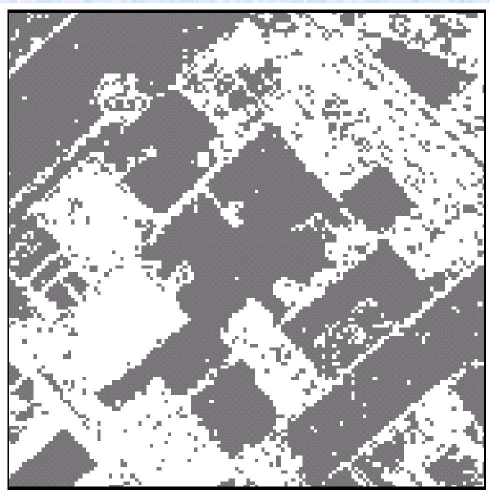
Algorithm	sensitivity	specificity	PVP	P_e
2-D HMM	0.7795	0.8203	0.8381	0.1880
CART 1	0.8528	0.7126	0.7530	0.2158
CART 2	0.8097	0.7340	0.7505	0.2408
LVQ1	0.8187	0.7419	0.7691	0.2183

Results

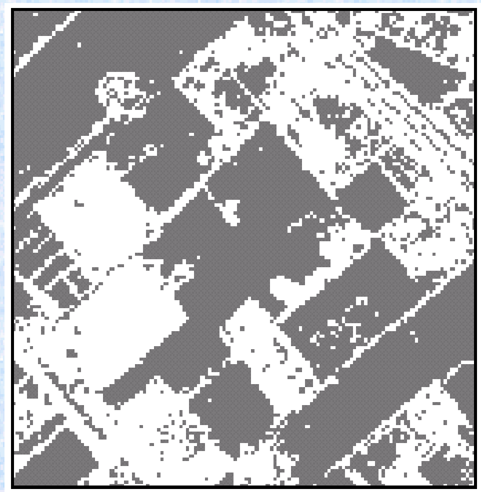
Source



HMM



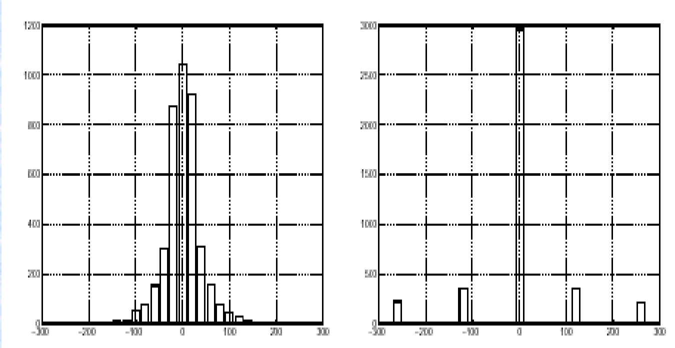
CART1



LVQ1

Documentation segmentation

- Goal: Segmentation of document images into text and photograph
- # of states: 4 (2~5 are available)
- Block size: 8x8
- Features:



Wavelet coefficients (Haar wavelet)

(1) Measure the goodness of match between the observed distribution and the Laplacian distribution. (χ^2)

(2) Measure the likelihood of the wavelet coefficients being composed by highly concentrated values. (L)

Documentation segmentation

- How the hell does it work? (LH HL HH)

χ^2 : Divide the histogram of the wavelet coefficients into bins. fi: relative frequency of bin i and Fi: probability of the Laplacian distribution.

$$\chi^2 = \sum_{i=1}^{k} (fi - Fi)^2 / Fi$$

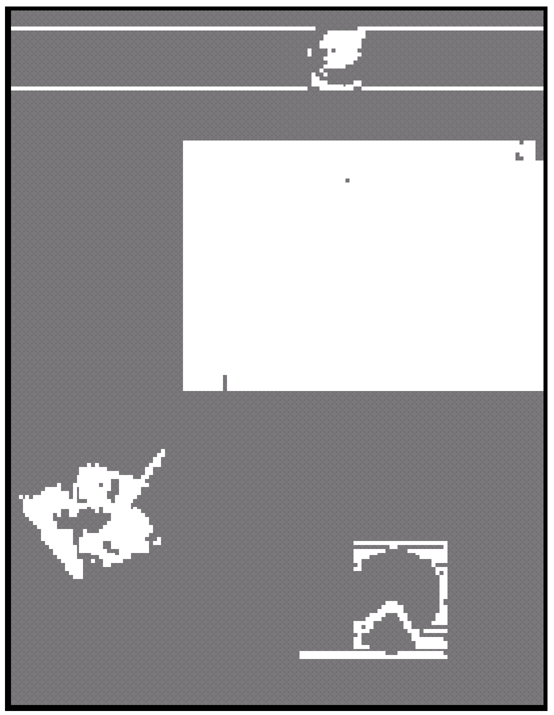
L : the weight sum of the concentration level β i.

β i is defined as the percentage of the data in a narrow neighborhood based on the total number of data in the zone.

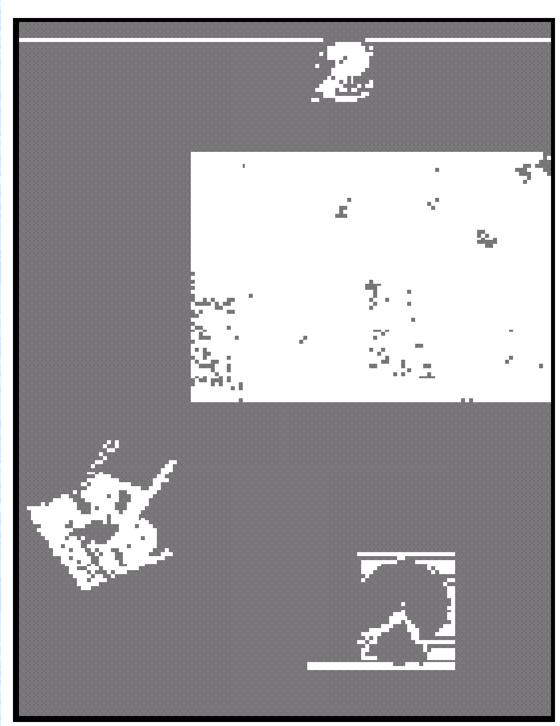
Results

Error rate: fewer than 2% for HMM and CART

Source



HMM



CART

Conclusion

- 2D HMM takes the inter-blocks relationship into account. That's why it stands out in most of the current classifier.
- 2D HMM is still limited by the computational power of the machine. This paper provides a roughly correct version.

Questions

- The computational cost of this algorithm is still high. Maybe not a good real-time processing algorithm.
- Parallel computing can be applied.
- Multi-scale (pyramid) processing may be able to assist the inter-block relations we lost in path-constrained viterbi algorithm.

Reference

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Your opinion