# A Parametric Texture Model Based on Joint Statistics of Complex Wavelet Coefficients

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# Texture Characterization - Traditional models

- Julesz hypothesis
  - Nth-order empirical densities of image pixels
- Markov Random Fields
  - Statistical interactions within local neighborhood
- Linear kernels
  - Multiple orientations and scales.

## Contributions of this paper : What's new ?

- Universal(?) parametric model for visual texture
  - Overcomplete multi-scale complex wavelet representation
  - Markov statistical descriptors : pairs of wavelet coefficients at adjacent spatial locations, orientations and scales
- Novel method for sampling from this model
  - Iterative projection onto sets
- Revisit Julesz conjecture

# How do we represent a texture ?

- A random field
    - $X(n, m)$ on a finite lattice $(n, m)$
    - A set of constraint functions $\{\phi_k(X),\ k = 1 \ldots N_c\}$, such that

$$\mathcal{E}(\phi_k(X)) = \mathcal{E}(\phi_k(Y)), \forall k$$

$\Rightarrow$ samples of X and Y are perceptually equivalent

▷ Universal ?

▷ What about the reverse ?

# Testing a Representation/Model

- Julesz Conjecture
  - Perceptual equivalence ?
  - Individual images versus Statistics of RFs ?
    - ▷ Requires ergodicity.
- Proposed synthesis-by-analysis approach
  - Practical ergodicity is enough

$$P_X(|\overline{\phi(x(n,m))} - \mathcal{E}(\phi(X))| < \epsilon) \geq p$$

# The Complete Framework requires

- a real two-dimensional homogeneous random field
- candidate constraint functions
- a method for estimating statistical paramaters
- an algorithm for sampling a RF satistfying the statistical constraints
- a method for measuring percptual similarity of two texture images.

# Random fields from Statistical Constraints

- Density with maximum entropy
  - Optimal - no other constraints imposed
  - Difficult to compute
- Alternative
  - Sampling from "Julesz Ensemble"

$$T_{\vec{\phi},\vec{c}} = \{\vec{x} : \overline{\phi_k(\vec{x})} = c_k, \forall k\}$$

  - Equivalent to maximal entropy distribution as lattice size grows to infinity

# Sampling via Projection

- Assuming $X_0$ is a homogeneous RF, and

$$p_{\vec{\phi},\vec{c}} : \mathbb{R}^{|L|} \longrightarrow T_{\vec{\phi},\vec{c}}$$

we can get

$$X_t = p_{\vec{\phi},\vec{c}}(X_0)$$

  - Choice of $X_0$
    - ▷ That maximizes the entropy of $X_t$ - equally difficult
    - ▷ High-entropy distribution for $X_0$ - Gaussian white noise

# Projection onto Constraint Surfaces

- Difficult to construct a single $p_{\vec{\phi},\vec{c}}$
- Alternative : an iterative solution
  - Set of functions

$$p_k : \mathbb{R}^{|L|} \longrightarrow T_k$$

where

$$T_k = \{\vec{x} : \overline{\phi_k(\vec{x})} = c_k\}$$

# How do we project ?

- Gradient Projection

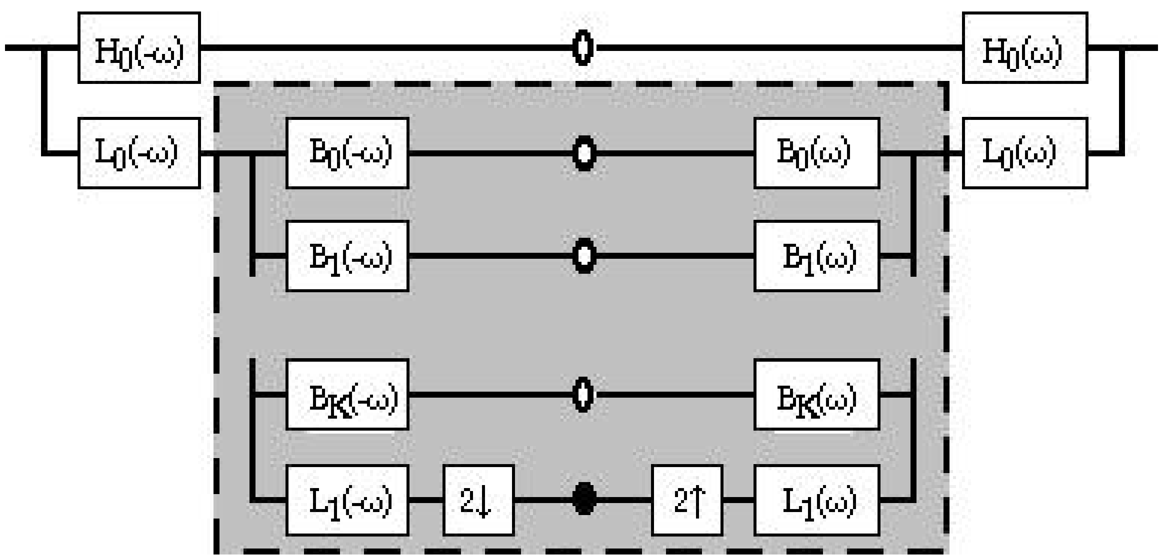
$$\vec{x}' = \vec{x} + \lambda_k \vec{\nabla} \phi_k(\vec{x})$$

where $\lambda_k$ is chosen such that

$$\phi_k(\vec{x}') = c_k$$

# Texture Model

- Constraint functions ??
  - On pixel values / some other basis
    - ▷ Biological motivation - localized oriented bandpass linear filters
    - ▷ Steerable pyramid



# Statistical Constraints - Perceptual Criteria

- Following approach used to incrementally augment the set of constraint functions
    - Initialization - some basic parameters
    - Gather synthesis failure
    - New statistical constraint
    - Verify that the new constraint works !
    - Verify that the old constraints are still necessary

# Constraints used : Marginal Statistics

- Normalized moments, range of the lowpass images computed at each level

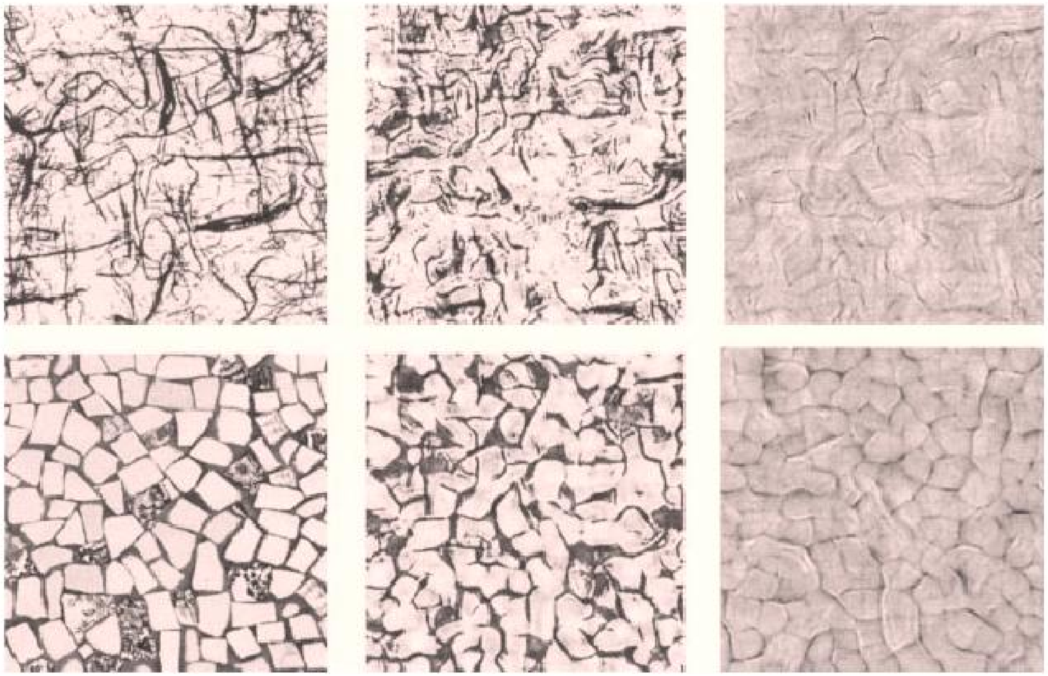


*Figure 3.* Necessity of marginal constraints. Left column: original texture images. Middle: Images synthesized using full constraint set. Right: Images synthesized using all but the marginal constraints.

# Constraints used : Coefficient Correlation

- Local auto-correlation of the lowpass images computed at each level

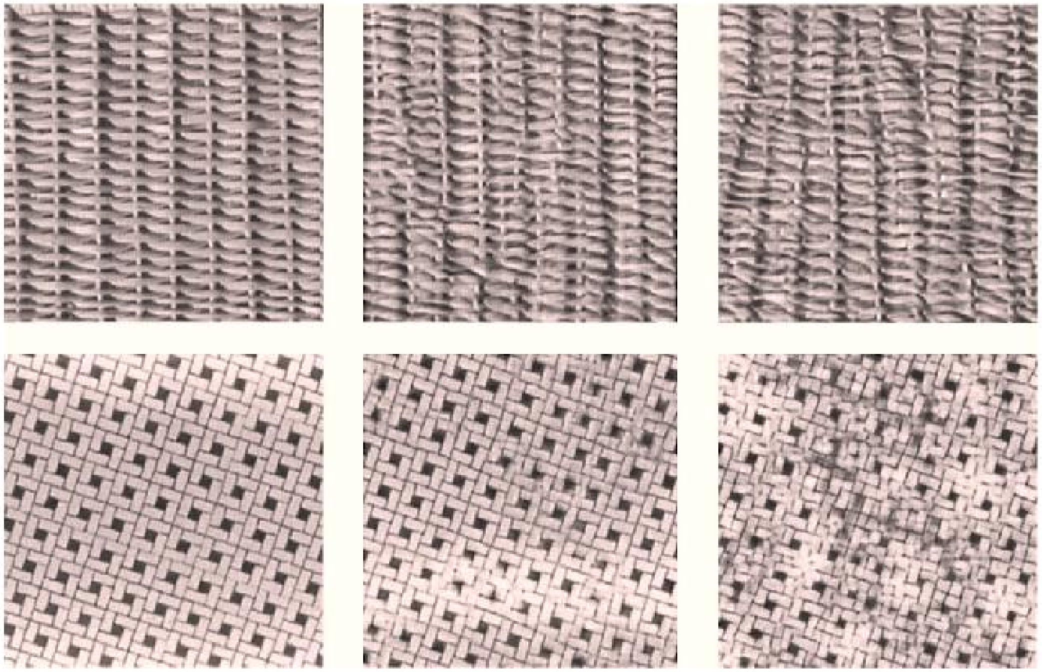


*Figure 4.* Necessity of raw autocorrelation constraints. Left column: original texture images. Middle: Images synthesized using full constraint set. Right: Images synthesized using all but the autocorrelation constraints.

# Constraints used : Magnitude Correlation

- Correlation of complex magnitude of pairs of coefficeints at adjacent positions, orientations and scales

- Normalized magnitude responses :

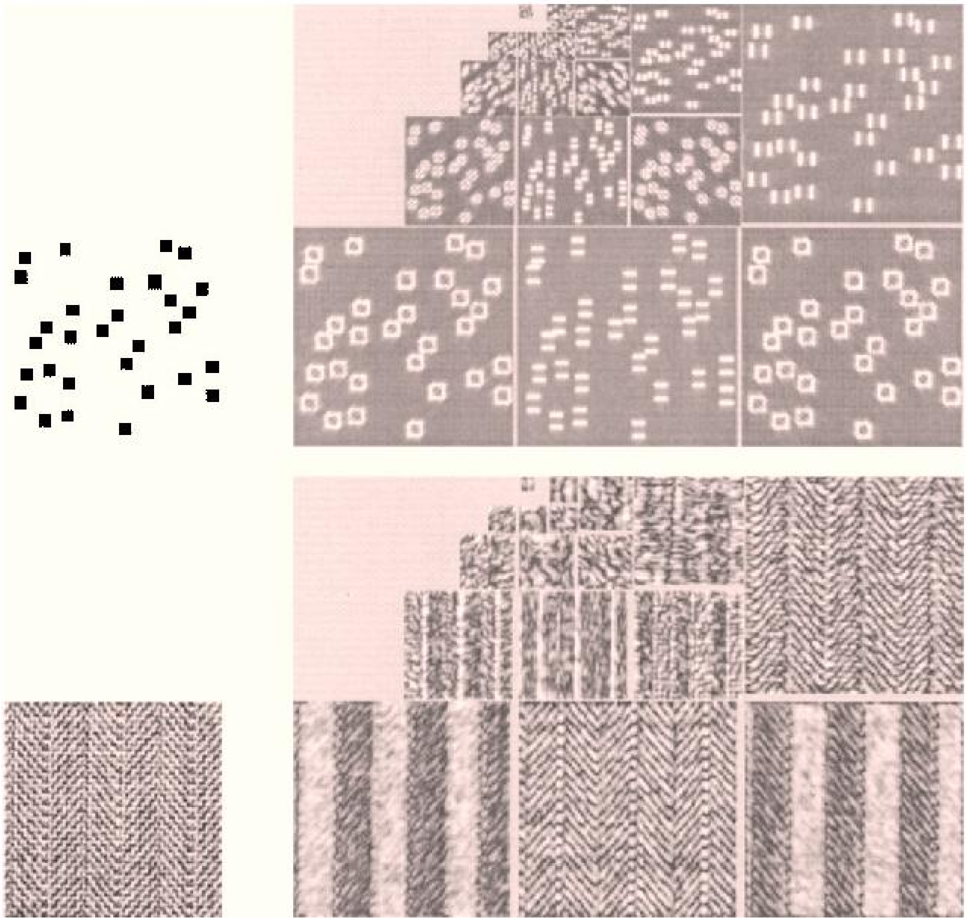


*Figure 5.* Normalized magnitude responses of the steerable pyramid subbands for two example textures images (shown at left).

- Necessary ?

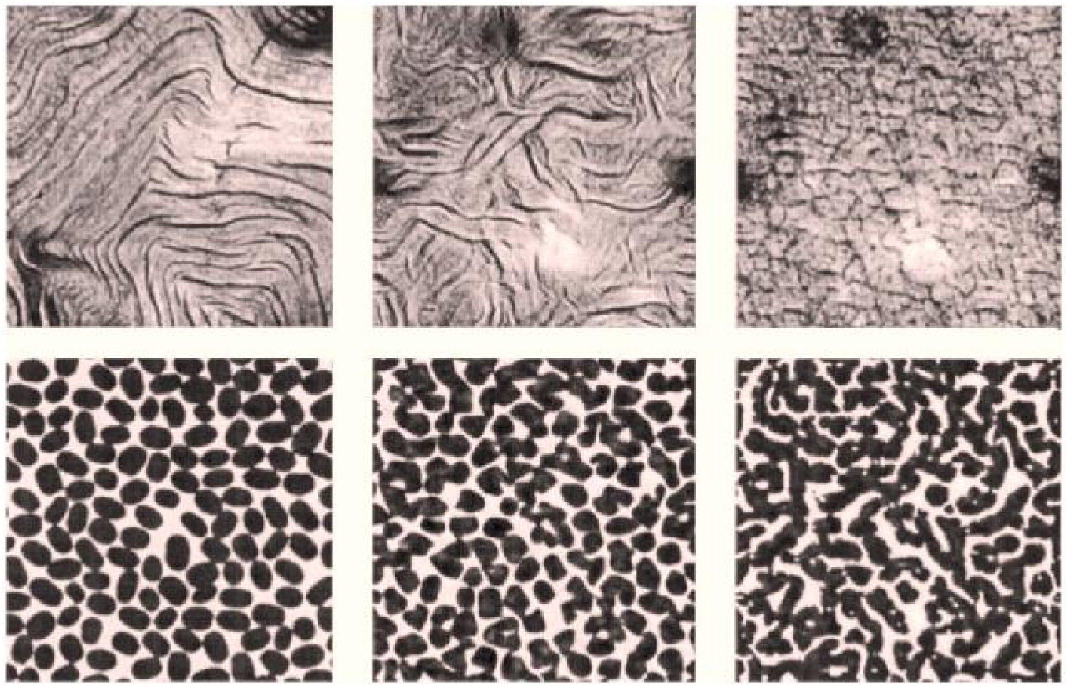


*Figure 6.* Necessity of magnitude correlation constraints. Left column: original texture images. Middle: Images synthesized using full constraint set. Right: Images synthesized using all but the magnitude auto- and cross-correlation constraints.

# Constraints used : Cross-Scale Phase Statistics

- Cross-correlation of the real part of the coefficients with both the real and imaginary part
- Edges/lines dilemma ?

- Necessary ?

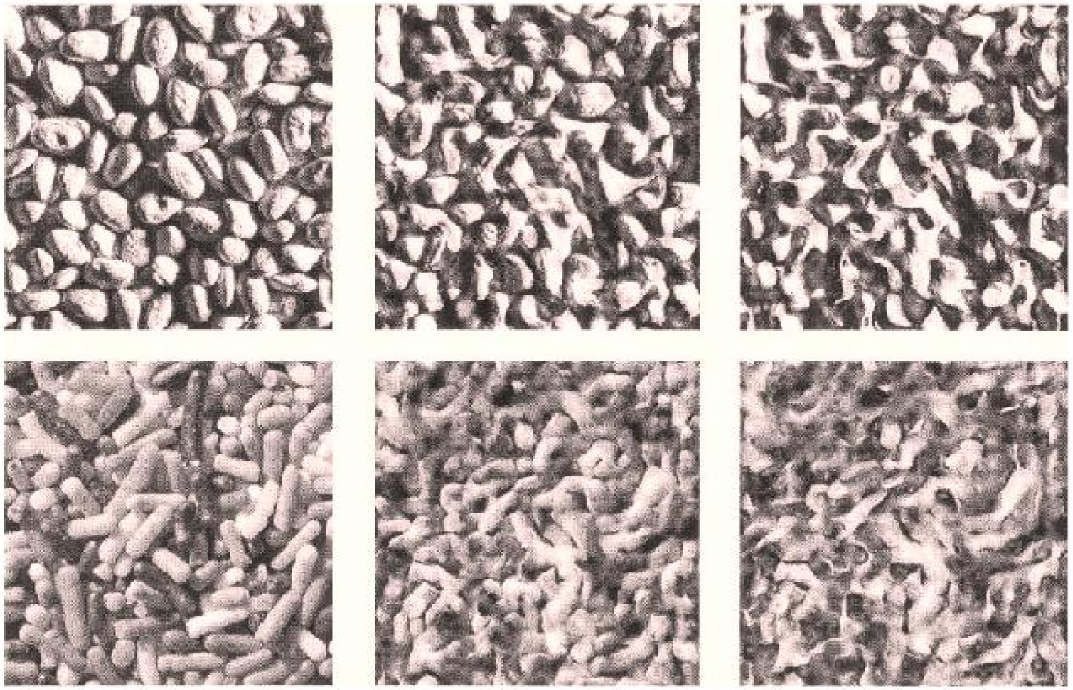


*Figure 8.* Necessity of cross-scale phase constraints. Left column: original texture images. Middle: Images synthesized using full constraint set. Right: Images synthesized using all but the cross-scale phase constraints.

# How good is the synthesis ?

- On classic counterexamples of Julesz conjecture :

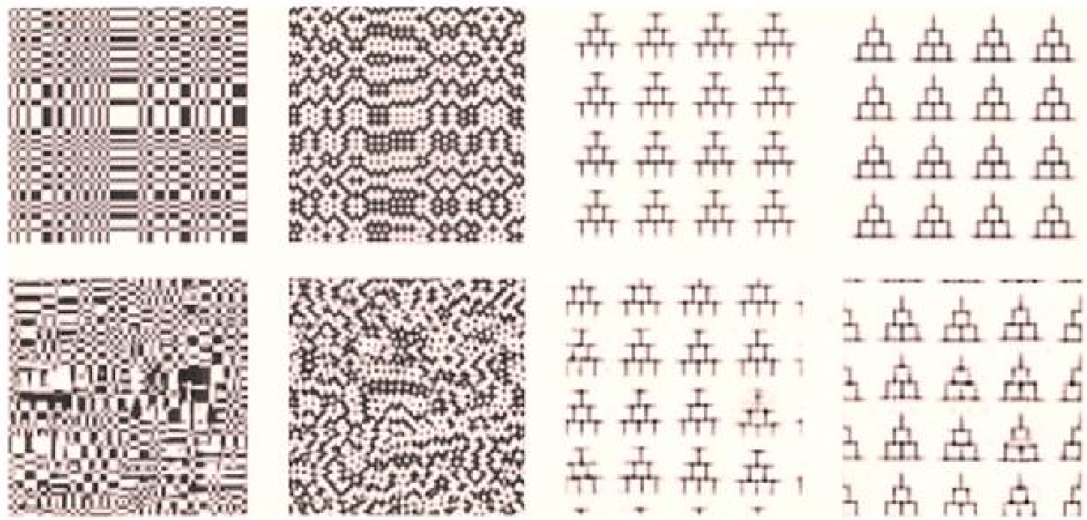


*Figure 13*. Synthesis of classic counterexamples to the Julesz conjecture (Julesz et al., 1978; Yellot, 1993) (see text). Top row: original artificial textures. Bottom row: Synthesized textures.

- Failures:

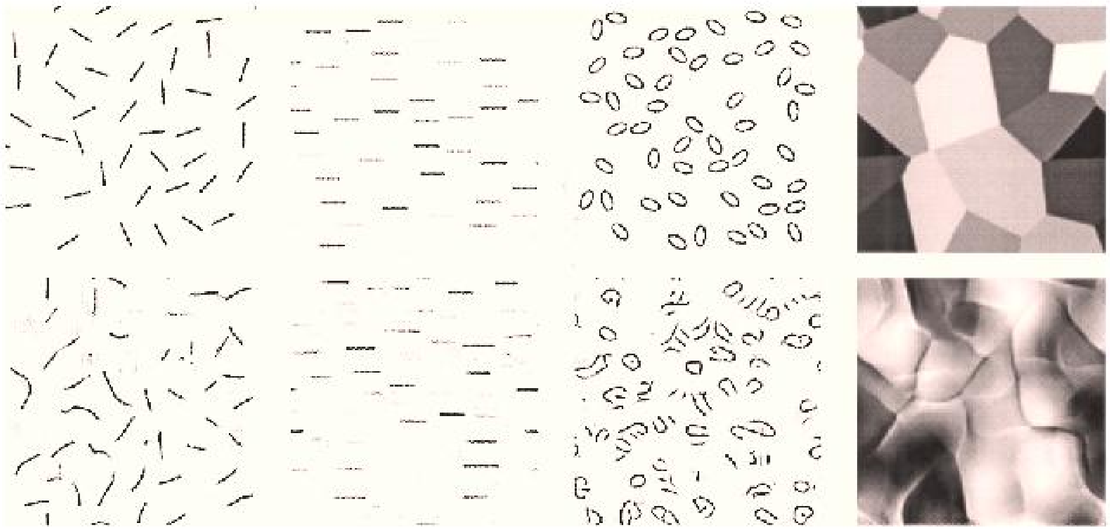


*Figure 18.* Artificial textures illustrating failure to synthesize certain texture attributes. See text.

- On interesting "non"-textures:

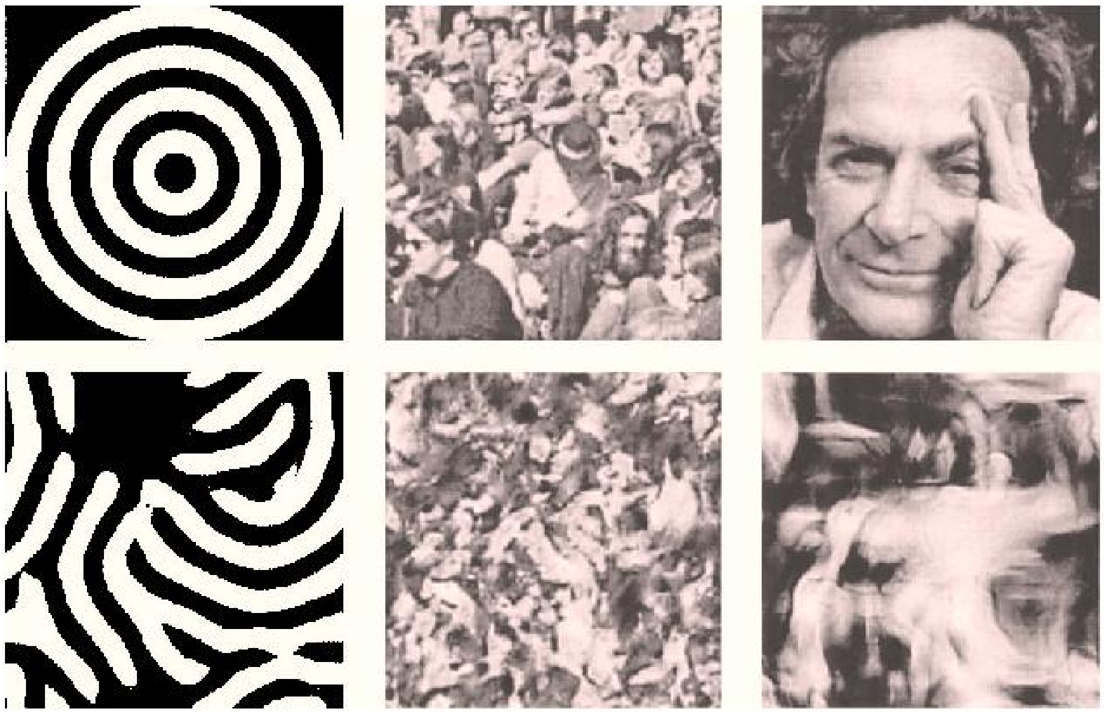


*Figure 17.* Synthesis results on inhomogeneous photographic images not usually considered to be “texture”.