Review for Final Exam
CMSC 828J Spring 2013

The final exam will cover topics mentioned during lectures. Topics mentioned during paper discussion that were not mentioned during lecture will not be tested. (Of course, in many cases paper discussion topics overlapped topics from lecture). These will include the following:

- PCA, LDA, MDS
- Fourier transforms and wavelets
- Linear combinations of point features
- Spherical harmonics for illumination subspaces
- Locality sensitive hashing
- Compressed Sensing
- PLS, CCA
- Riemannian manifolds
  - Tangent spaces, geodesics, local metrics
  - Matrix manifolds, (Steifel, Grassman, covariance)
- Kernel methods, including kernel PCA
- Manifold learning – including isomap, lle
- Shape spaces – including Kendall shape space

1) Suppose we are given a distance matrix, D, so that D(i,j) gives the distance between two points, i and j. How would you determine whether this matrix could come from the Euclidean distance between a bunch of 2D points?
2) Suppose we are given a set of points in 2D. We compute the distance between all pairs of points, and build a distance matrix. We then apply MDS (the version discussed in class) to this distance matrix, using the distances to build a set of 2D points.
   a. True or false: we will recover exactly the original set of points that we began with, up to a translation and rotation?
   b. Suppose for (2) we start with 3D points, build a distance matrix, and recover a set of 2D points. What will be the relationship between the 2D points we recover and the original set of 3D points?

3) Suppose we create a set of images of a scene. Each image is generated by a single directional light source, that produces no cast or attached shadows. Prove that the set of images we get has rank <= 3. Under what conditions will the set of images have rank 2?

4) Let C be a half cylinder in 3D, whose central axis is the y axis. We only consider the upper part of the cylinder. That is, we can describe the cylinder with the equations $x^2 + z^2 = 1$, $z>0$. Suppose we parameterize the cylinder by projecting it orthographically into the x-y plane.
   a. Give the local Riemannian metric for the manifold for each point in the plane.
   b. True or false: the line x = y is a geodesic given this Riemannian metric (or alternately, the curve defined by $x = y$, $x^2 + z^2 = 1$, $z>0$, is a geodesic curve on the cylinder). Prove that your answer is correct.

5) Mathematically, we say that a signal x is k-sparse when it has at most k nonzeros, i.e., $\|x\|_0 \leq k$. We let

   $$\Sigma_k = \{x : \|x\|_0 \leq k\}$$

   denote the set of all k-sparse signals. The spark of a given matrix A is the smallest number of columns of A that are linearly dependent. Prove the following theorem: For any vector y in R^m, there exists at most one signal x in Σ_k such that y=Ax if and only if spark(A)>2k.

6) Suppose we have a bunch of points on a 2D grid, with integer values (ie., we have the points (x,y) for x,y = 1,2,3, ….)

   a. If we apply ISOMAP to these points, what will be the result?

   b. Do you think that LLE will produce different results? Why or why not?