Lighting affects appearance

How do we represent light? (1)

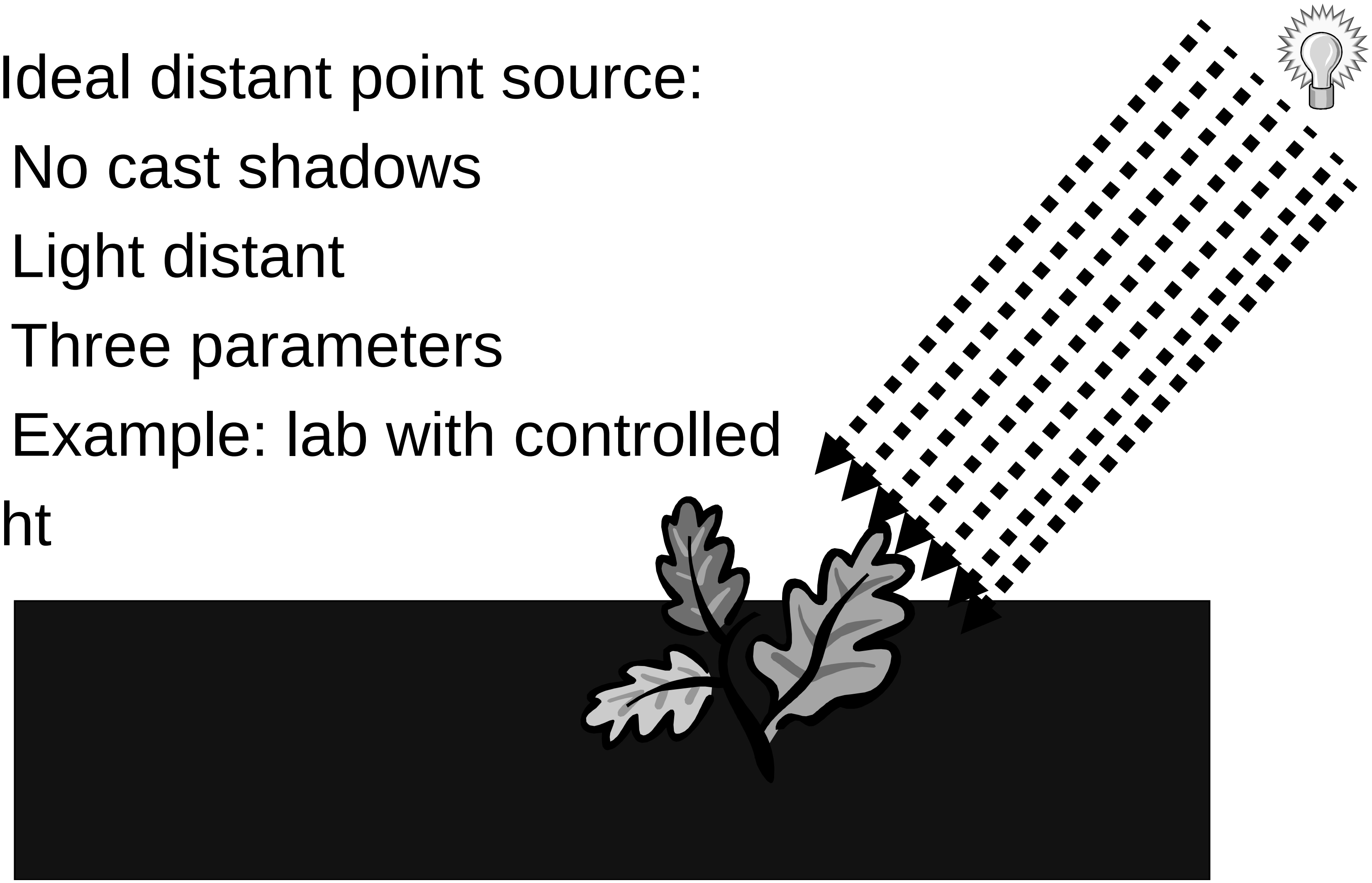
- Ideal distant point source:

  - No cast shadows

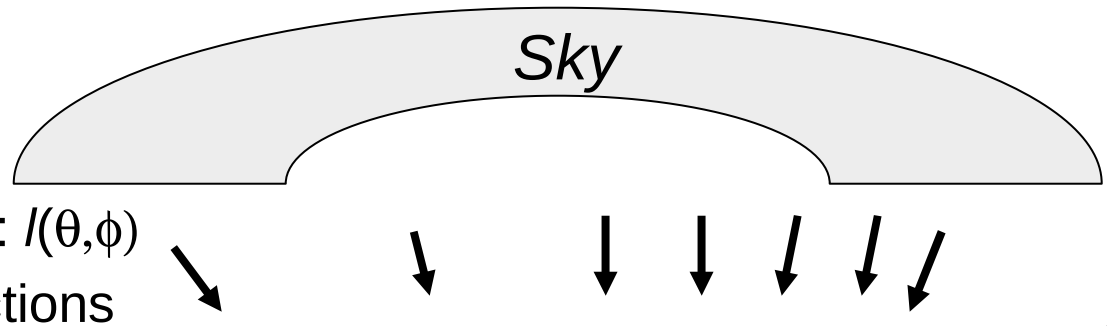
  - Light distant

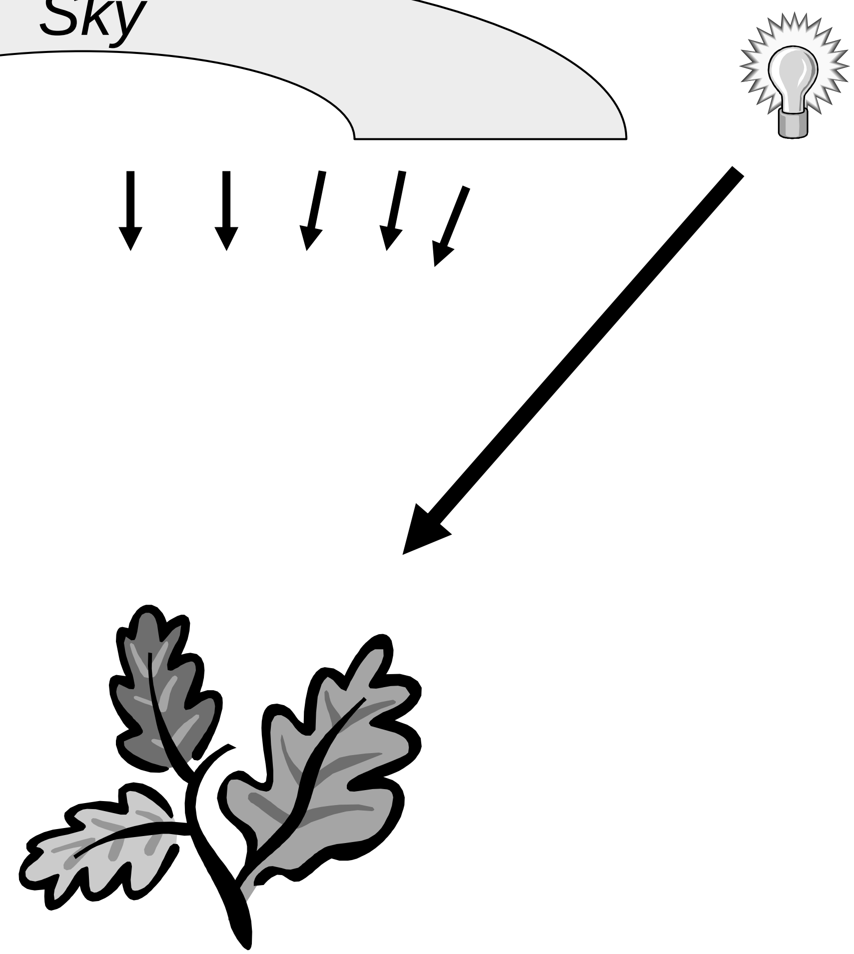
  - Three parameters

  - Example: lab with controlled light

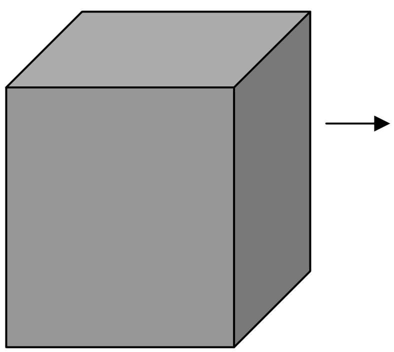


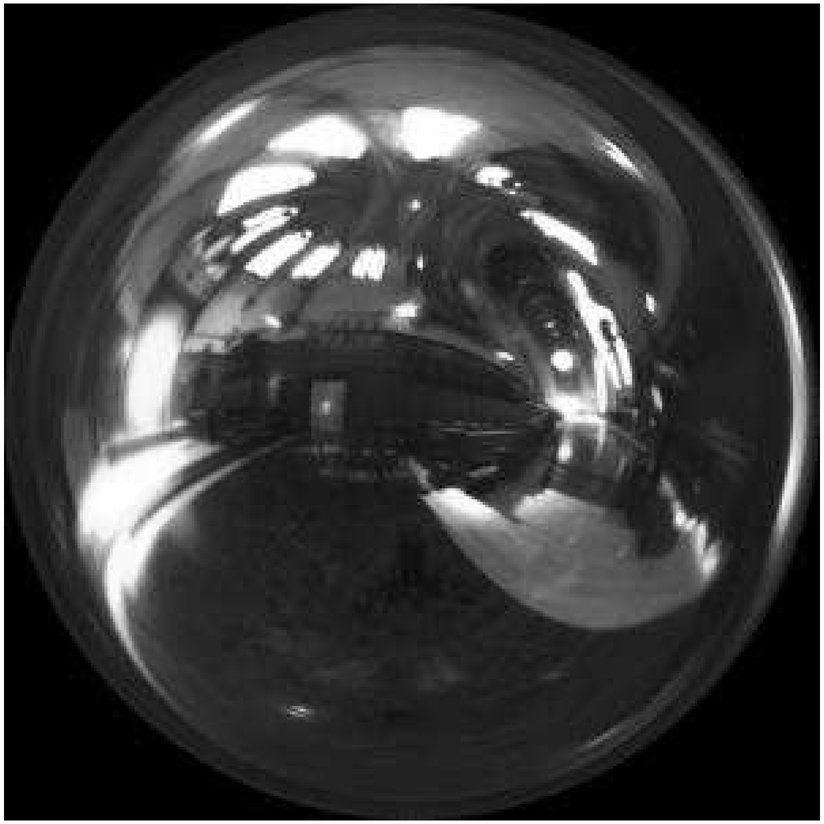
How do we represent light? (2)





- Environment map: $I(\theta,\phi)$
  - Light from all directions
  - Diffuse or point sources
  - Still distant
  - Still no cast shadows.
  - Example: outdoors (sky and sun)





Lambertian + Point Source

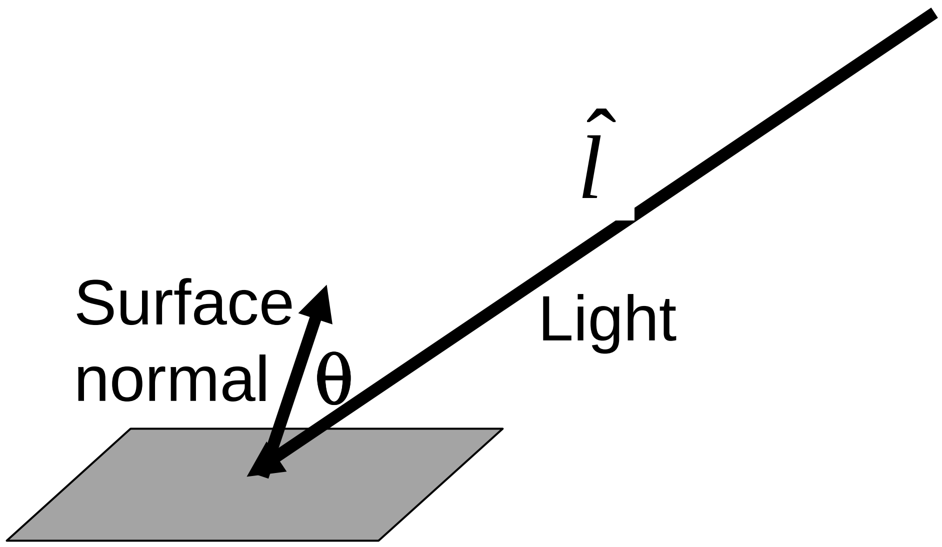
$$\vec{l} = l \bullet \vec{l} \quad \begin{cases} \vec{l} \text{ is direction of light} \\ l \text{ is intensity of light} \end{cases}$$

$$i = \max(0, \lambda(\vec{l} \bullet \hat{n}))$$

i is radiance

λ is *albedo*

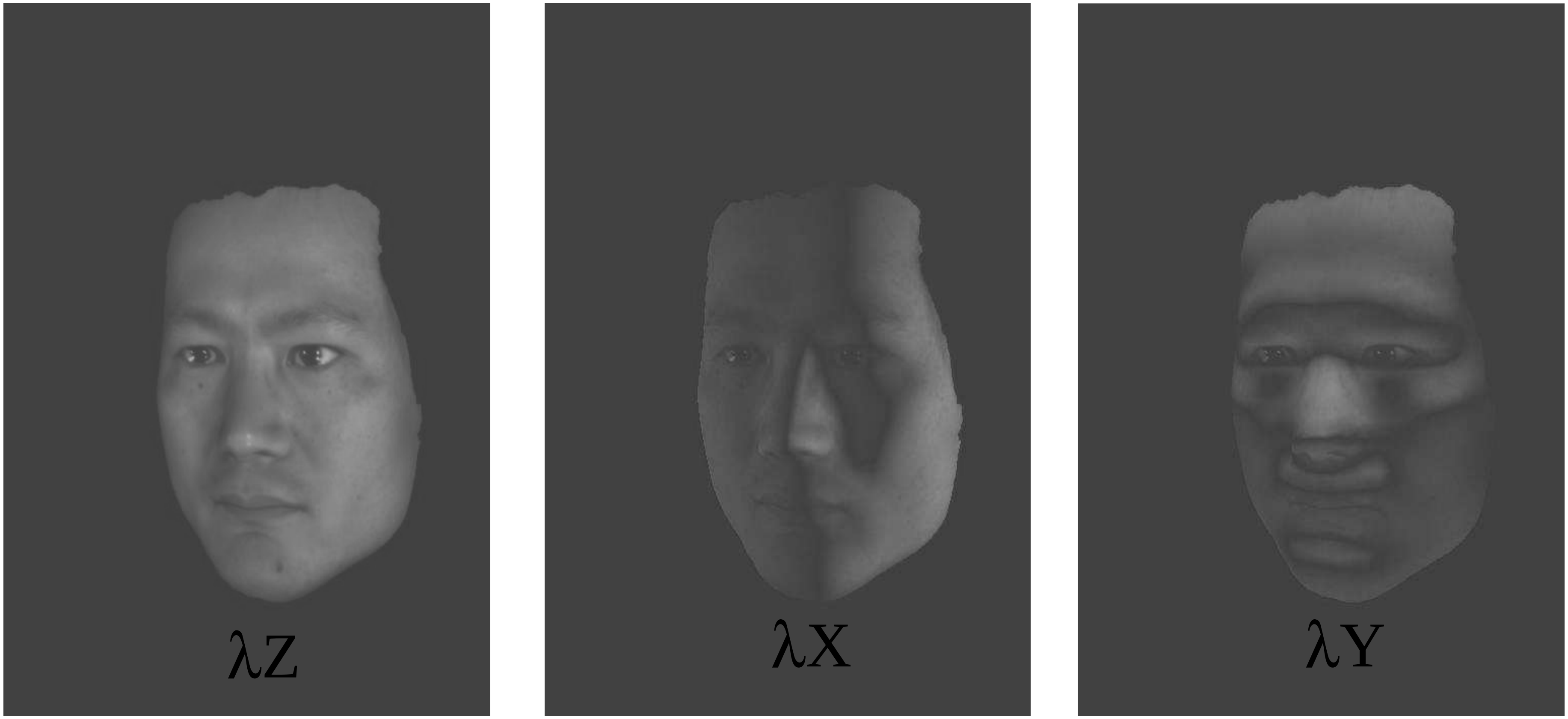
$\hat{n}$ is surface normal



Lambertian, point sources, no shadows. (Shashua, Moses)

- *Whiteboard*
- Solution linear
- Linear ambiguity in recovering scaled normals
- Lighting not known.
- Recognition by linear combinations.

Linear basis for lighting



A brief Detour: Fourier Transform, the other linear basis

- Analytic geometry gives a coordinate system for describing geometric objects.
- Fourier transform gives a coordinate system for functions.

Basis

- P=(x,y) means P = x(1,0)+y(0,1)
- Similarly:

$$f(\theta) = a_{11}\cos(\theta) + a_{12}\sin(\theta)$$
$$+ a_{21}\cos(2\theta) + a_{22}\sin(2\theta) + \dots$$

Note, I'm showing non-standard basis, these are from basis using complex functions.

Example

$$\forall c, \exists a_1, a_2 \text{ such that}:$$

$$\cos(\theta + c) = a_1 \cos\theta + a_2 \sin\theta$$

Orthonormal Basis

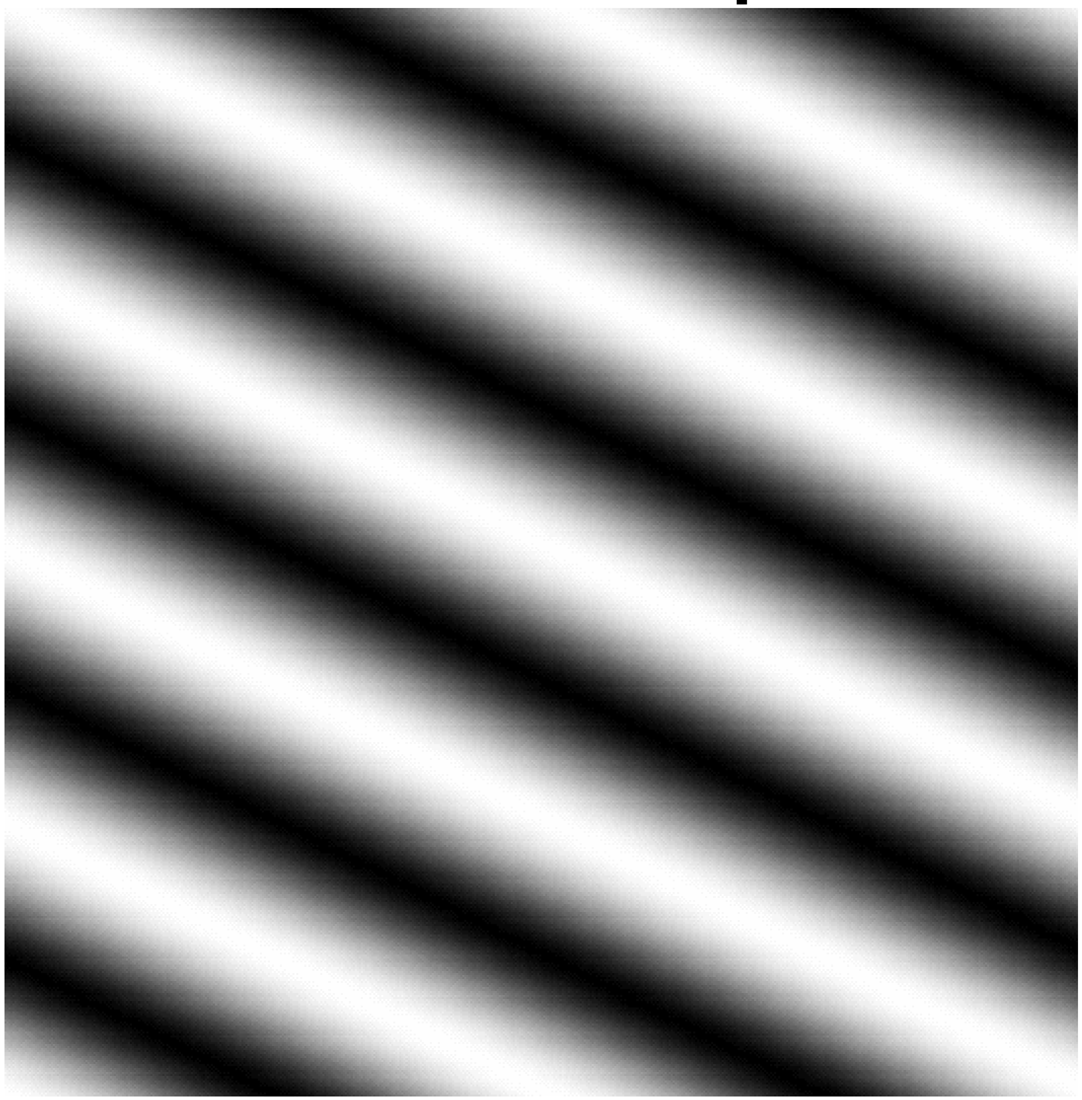
- $\|(1,0)\|=\|(0,1)\|=1$
- $(1,0).(0,1)=0$
- Similarly we use normal basis elements eg:

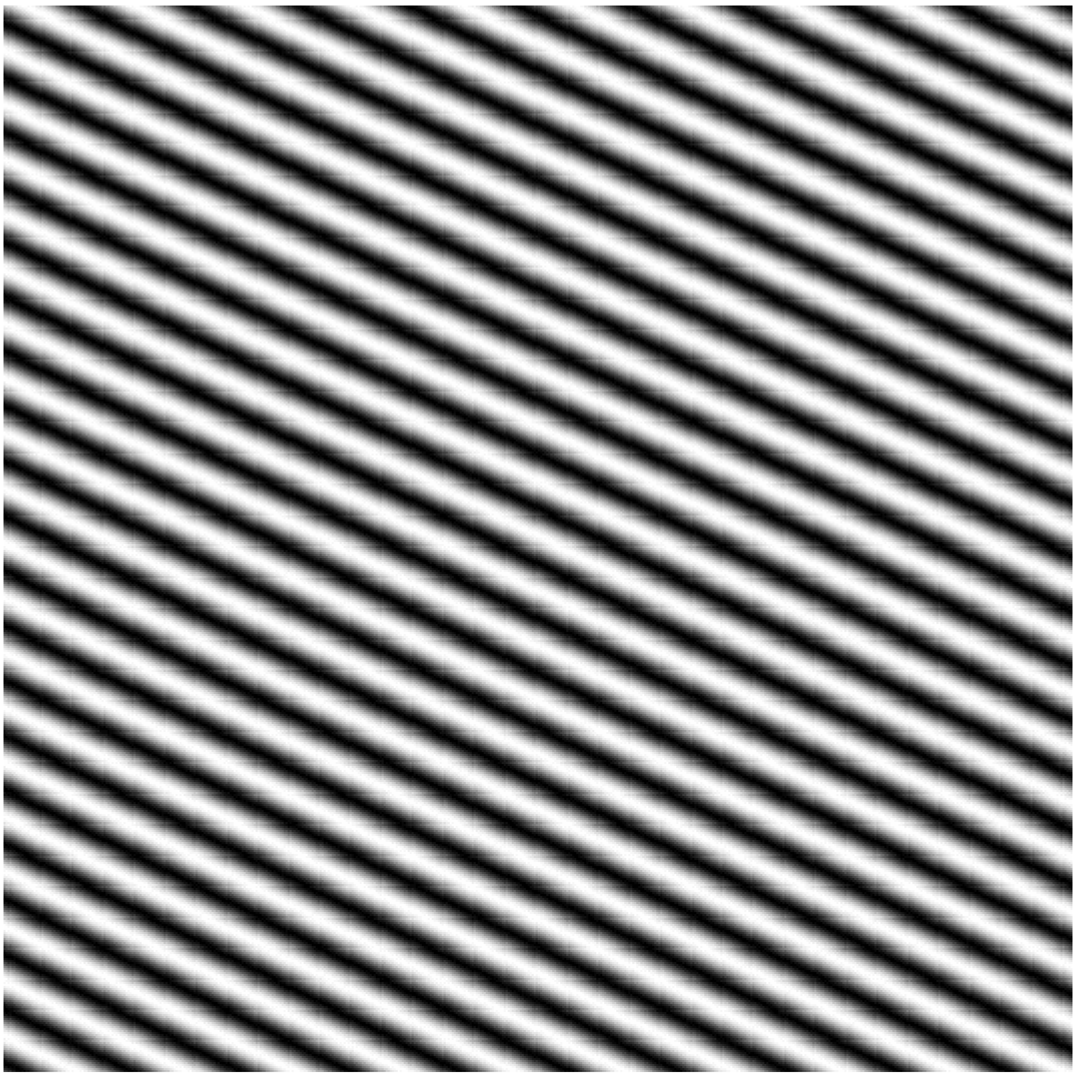
$$\frac{\cos(\theta)}{\|\cos(\theta)\|} \qquad \|\cos(\theta)\| = \sqrt{\int_0^{2\pi} \cos^2\theta \, d\theta}$$

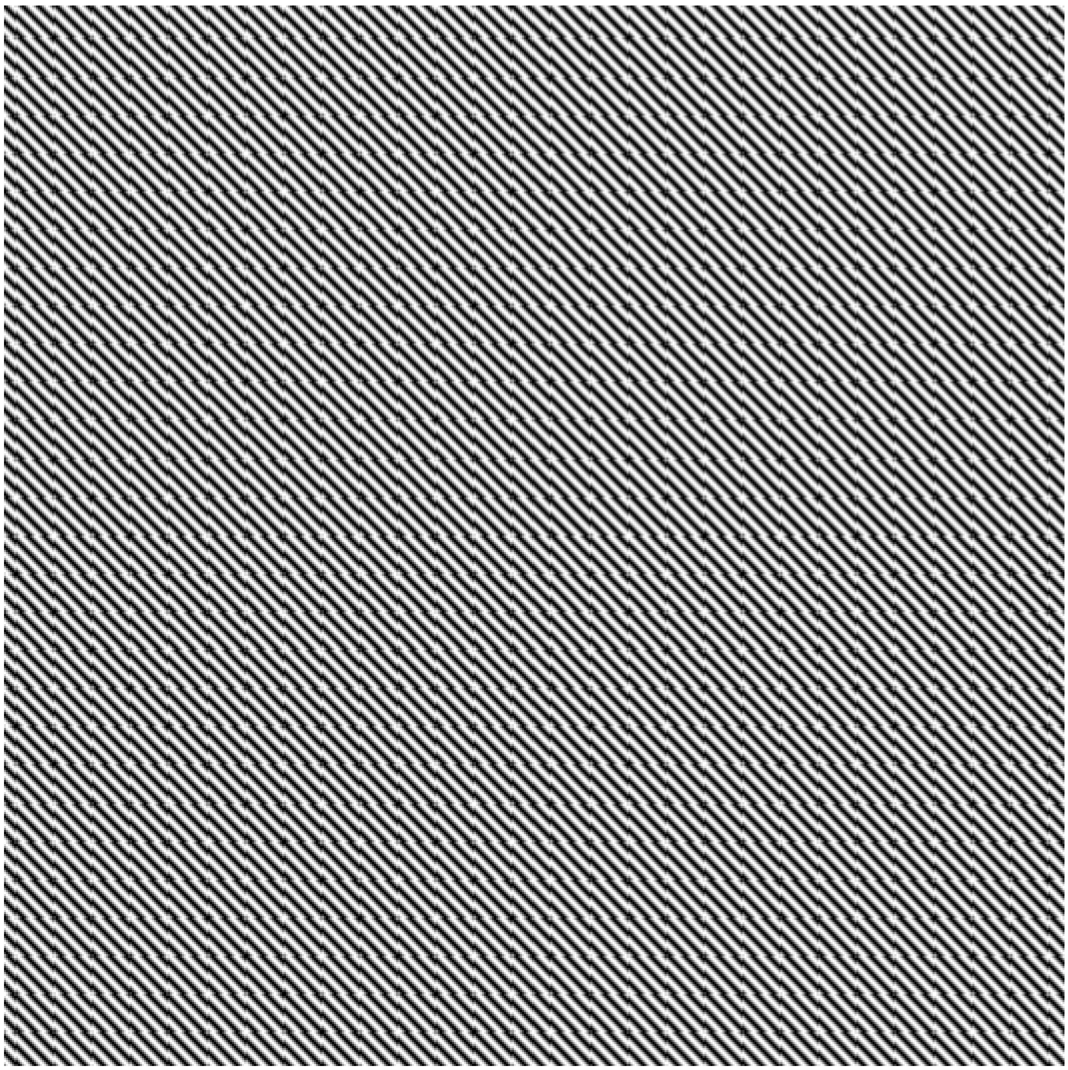
- While, eg:

$$\int_0^{2\pi} \cos\theta \sin\theta \, d\theta = 0$$

2D Example







Convolution

$$f(x) = g * h = \int g(x - x_0) h(x_0) dx_0$$

Imagine that we generate a point in *f* by centering *h* over the corresponding point in *g*, then multiplying *g* and *h* together, and integrating.

Convolution Theorem

$$f \otimes g = T^{-1} F * G$$

- *F,G* are transform of *f,g*

That is, *F* contains coefficients, when we write *f* as linear combinations of harmonic basis.

Examples

$$\cos\theta \otimes \cos\theta = ?$$

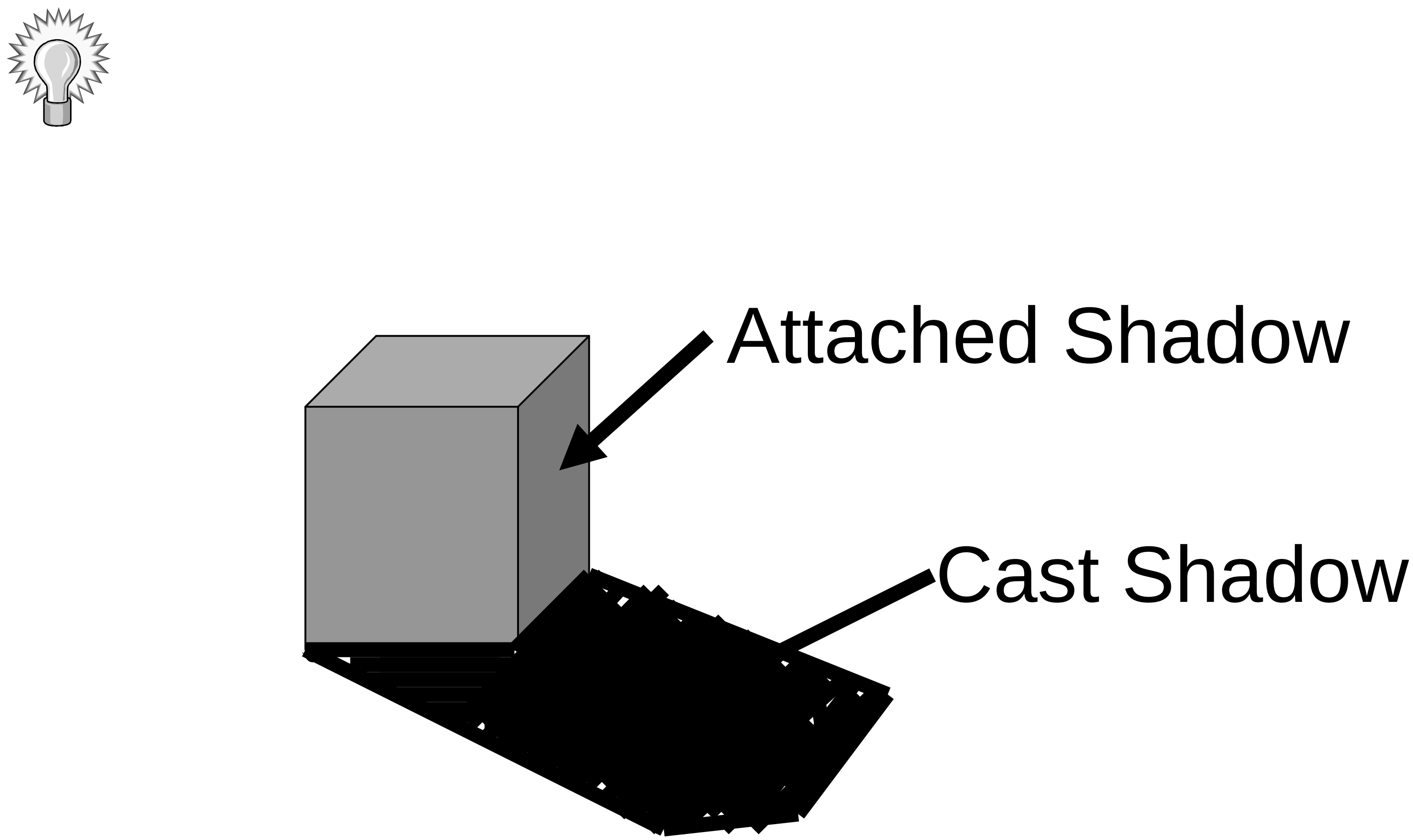
$$\cos\theta \otimes \cos 2\theta = ?$$

$$\cos\theta \otimes f = ?$$

$$(\cos\theta + .2\cos 2\theta + .1\cos 3\theta) \otimes f = ?$$

Low-pass filter removes low frequencies from signal. Hi-pass filter removes high frequencies. Examples?

Shadows



With Shadows: PCA

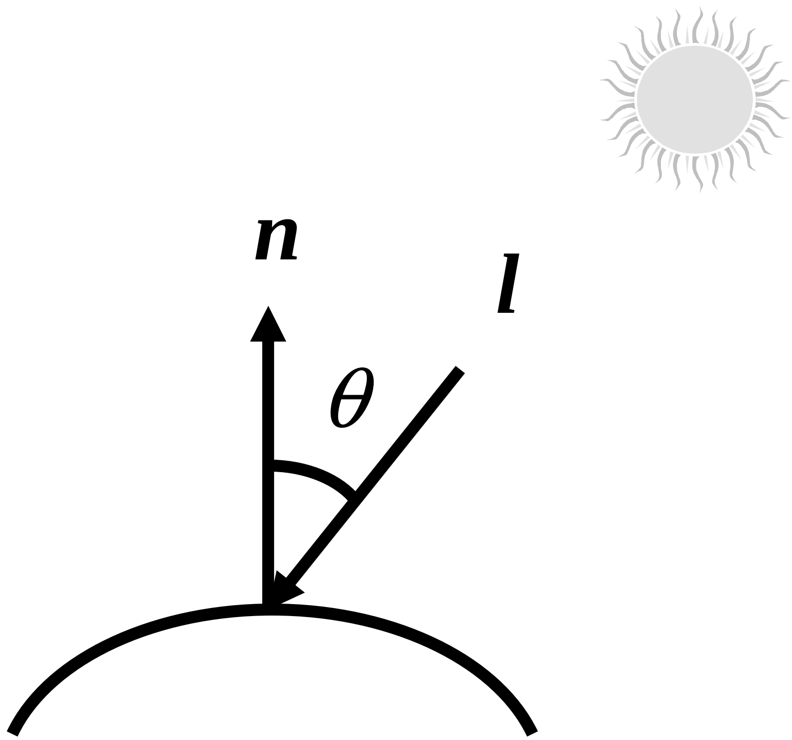
(Epstein, Hallinan and Yuille;
see also Hallinan; Belhumeur and Kriegman)

	Ball	Face	Phone	Parrot
#1	48.2	53.7	67.9	42.8
#3	94.4	90.2	88.2	76.3
#5	97.9	93.5	94.1	84.7
#7	99.1	95.3	96.3	88.5
#9	99.5	96.3	97.2	90.7

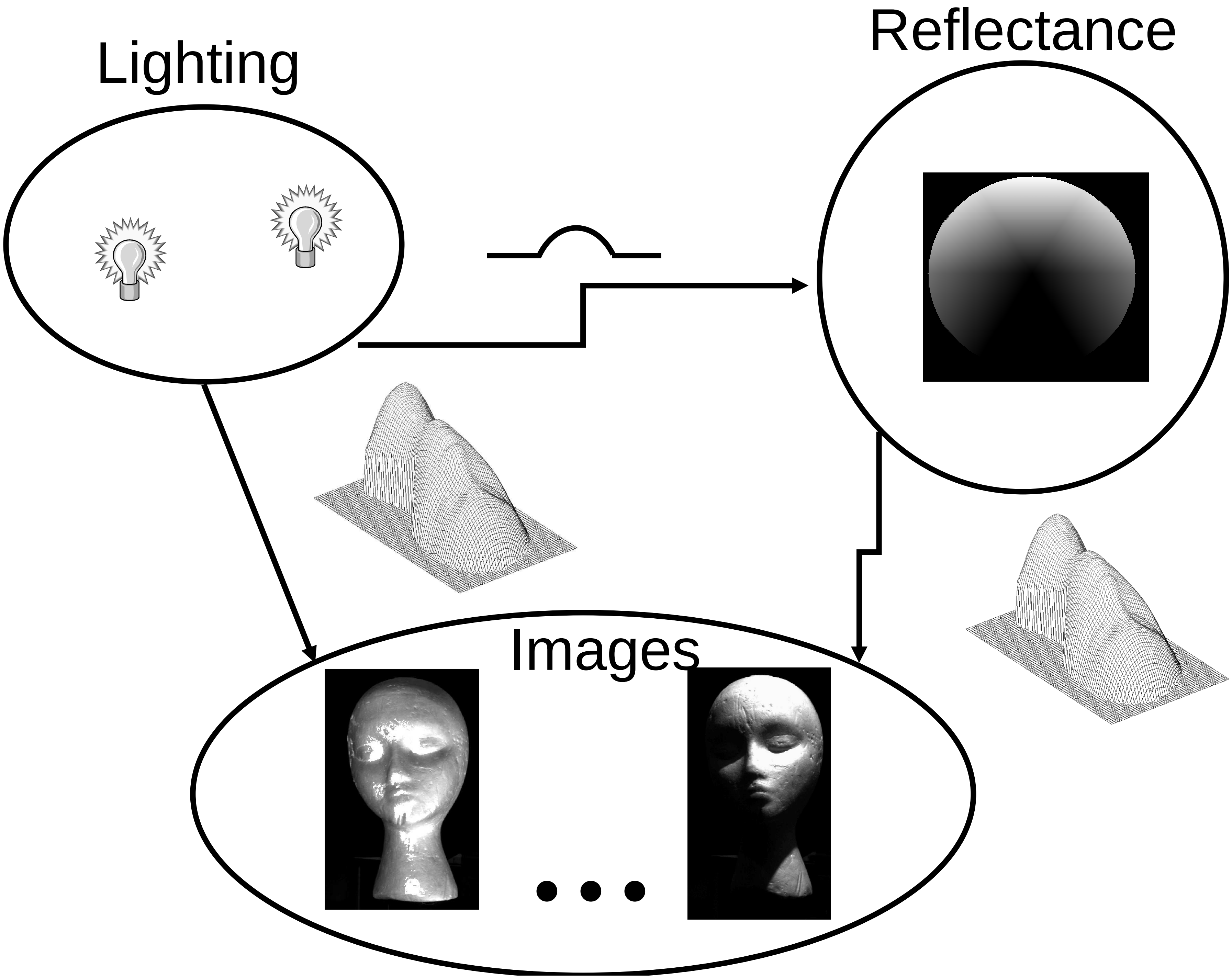
Dimension: 5 ± 2D

Domain

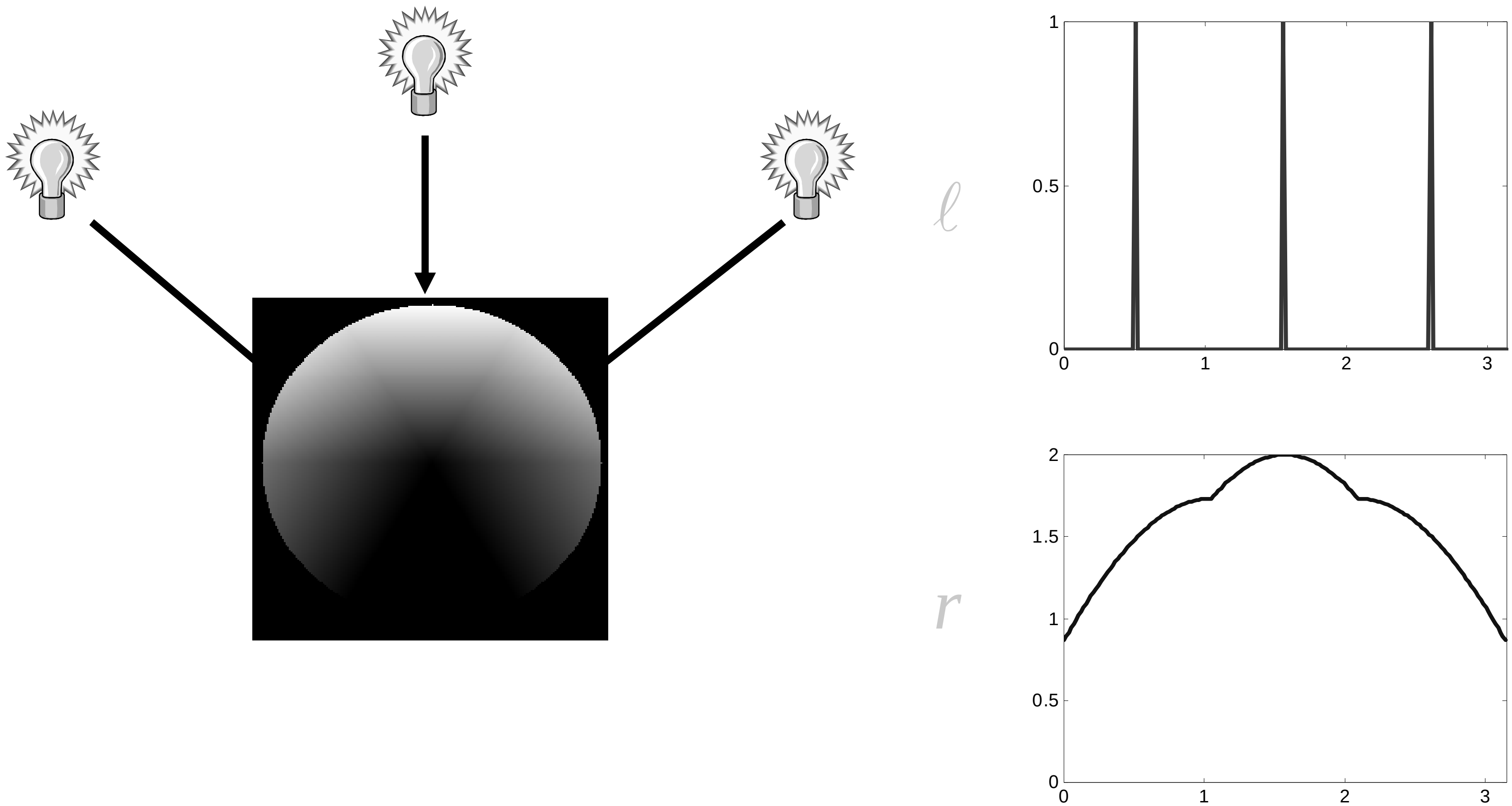
- Lambertian
- Environment map

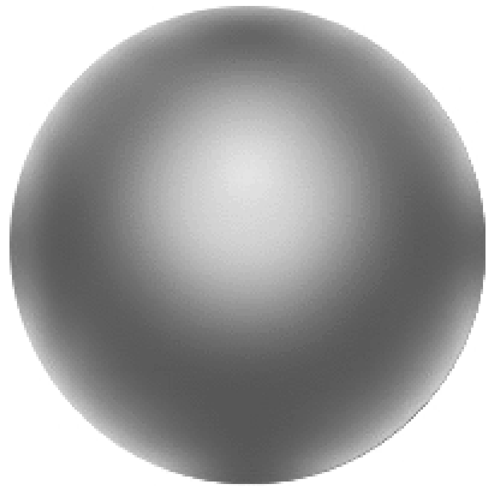


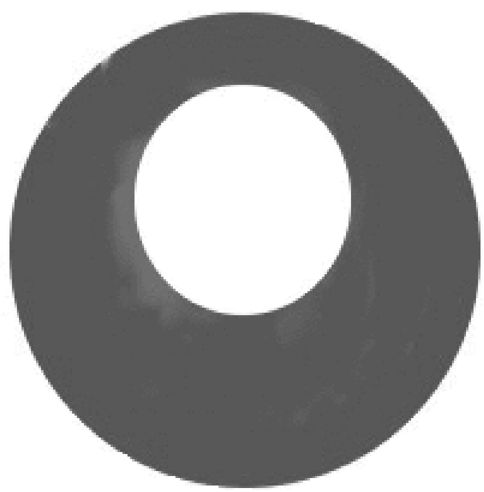
$$l\lambda \max(\cos\theta, 0)$$

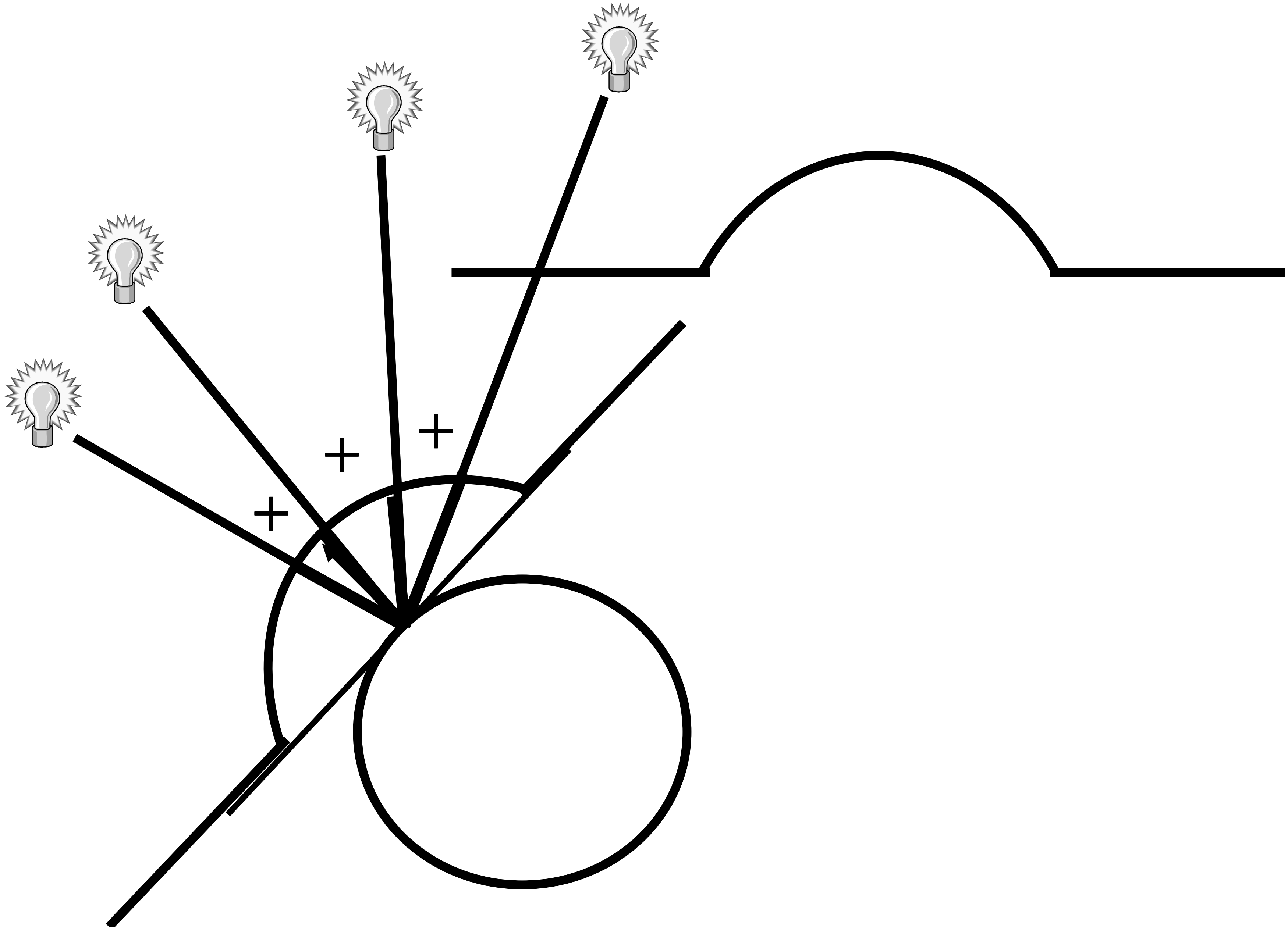

Lighting
Reflectance
Images

Lighting to Reflectance: Intuition









(See D’Zmura, ‘91; Ramamoorthi and Hanrahan ‘00)

Spherical Harmonics

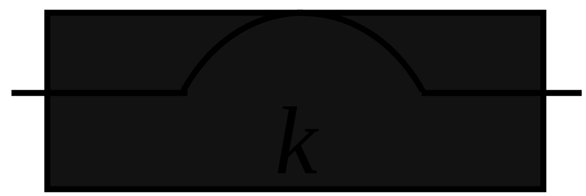
- Orthonormal basis, h_{nm}, for functions on the sphere.
- *n*'th order harmonics have 2*n*+1 components.
- Rotation = phase shift (same *n,* different *m*).
- In space coordinates: polynomials of degree *n.*
- S.H. used for BRDFs (Cabral et al.; Westin et al;). (See also Koenderink and van Doorn.)

$$h_{nm}(\theta,\phi) = \sqrt{\frac{(2n+1)}{4\pi}\frac{(n-m)!}{(n+m)!}}P_{nm}(\cos\theta)e^{im\phi}$$

$$P_{nm}(z) = \frac{(1-z^2)^{m/2}}{2^n n!}\frac{d^{n+m}}{dz^{n+m}}(z^2-1)^n$$

S.H. analog to convolution theorem

- Funk-Hecke theorem: “Convolution” in function domain is multiplication in spherical harmonic domain.
- *k* is low-pass filter.

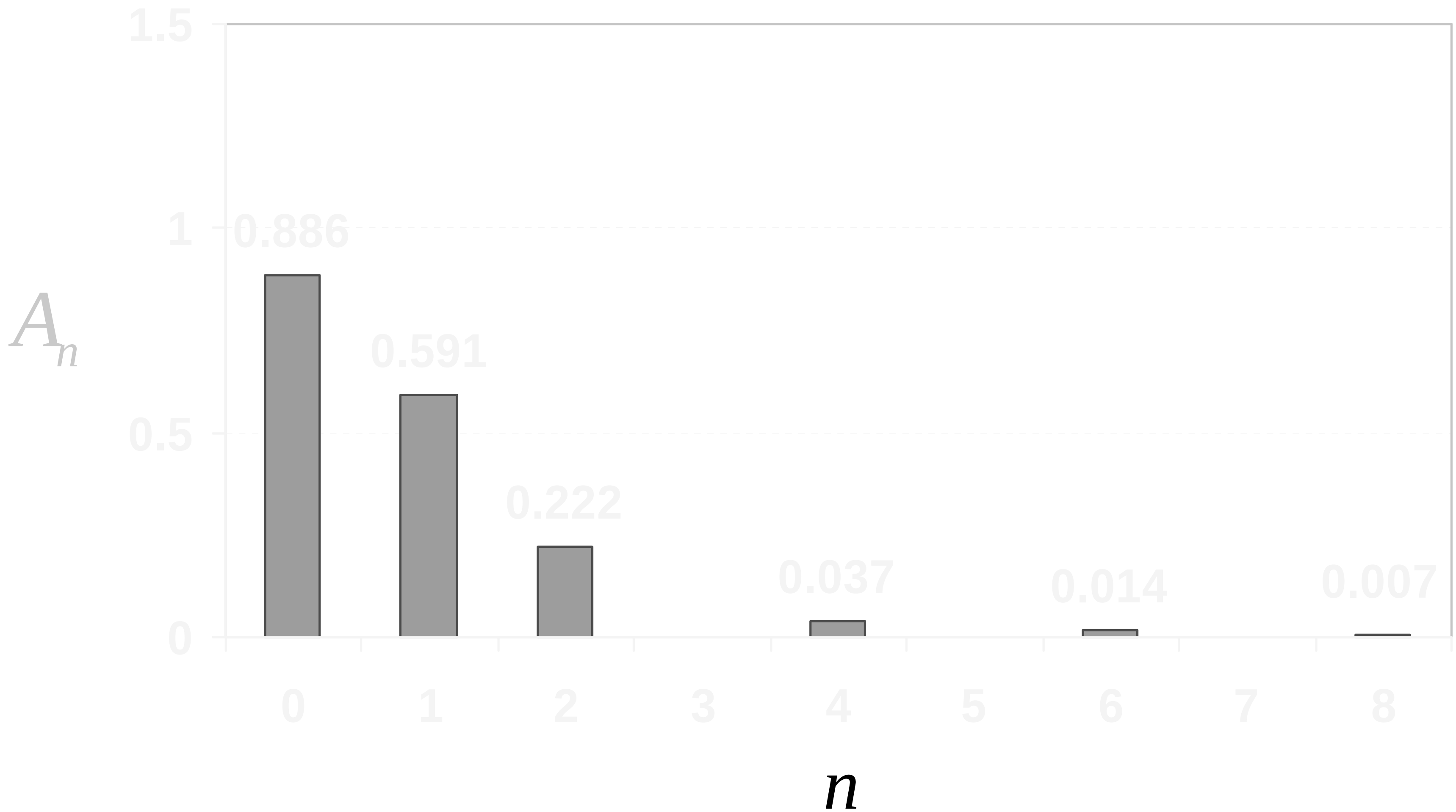


Harmonic Transform of Kernel

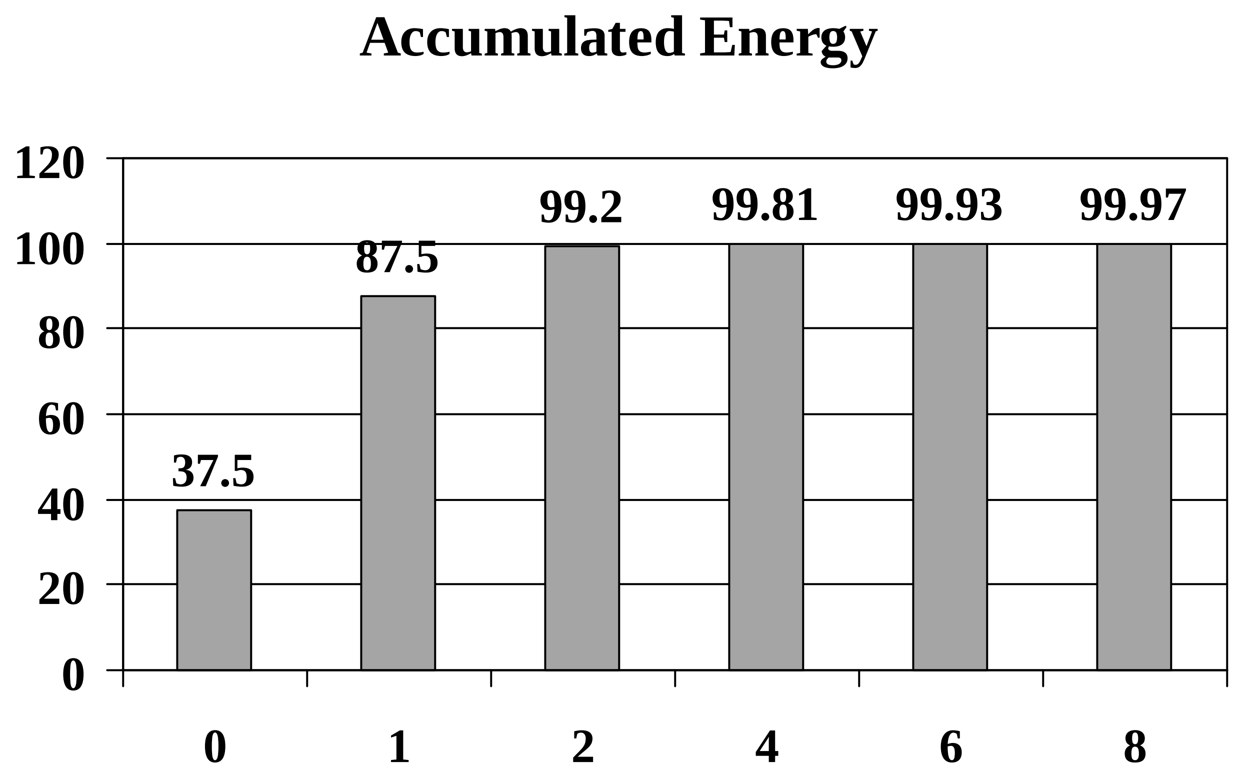
$$k(\theta) = \max(\cos\theta, 0) = \sum_{n=0}^{\infty} k_n h_{n0}$$

$$k_n = \begin{cases} \dfrac{\sqrt{\pi}}{2} & n = 0 \\ \sqrt{\dfrac{\pi}{3}} & n = 1 \\ (-1)^{\frac{n}{2}+1} \dfrac{(n-2)!\sqrt{(2n+1)\pi}}{2^n(\frac{n}{2}-1)!(\frac{n}{2}+1)!} & n \geq 2, \text{even} \\ 0 & n \geq 2, \text{odd} \end{cases}$$

Amplitudes of Kernel



Energy of Lambertian Kernel in low order harmonics



Reflectance Functions Near Low-dimensional Linear Subspace

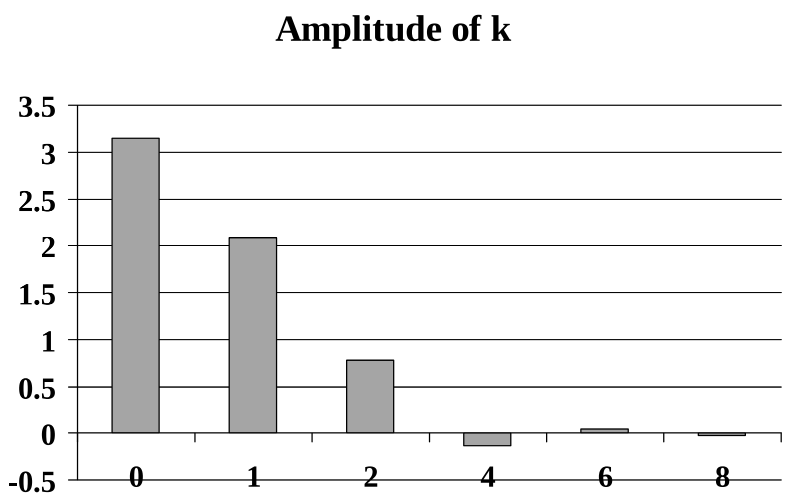
$$r = k * l = \sum_{n=0}^{\cancel{\infty}} \sum_{m=-n}^{n} (K_{nm} L_{nm}) h_{nm}$$

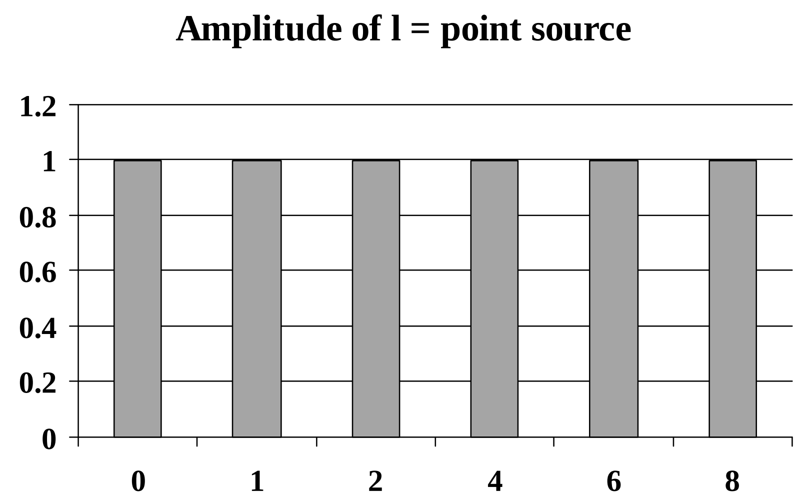
$$\approx \sum_{n=0}^{2} \sum_{m=-n}^{n} (K_{nm} L_{nm}) h_{nm}$$

Yields 9D linear subspace.

How accurate is approximation? Point light source

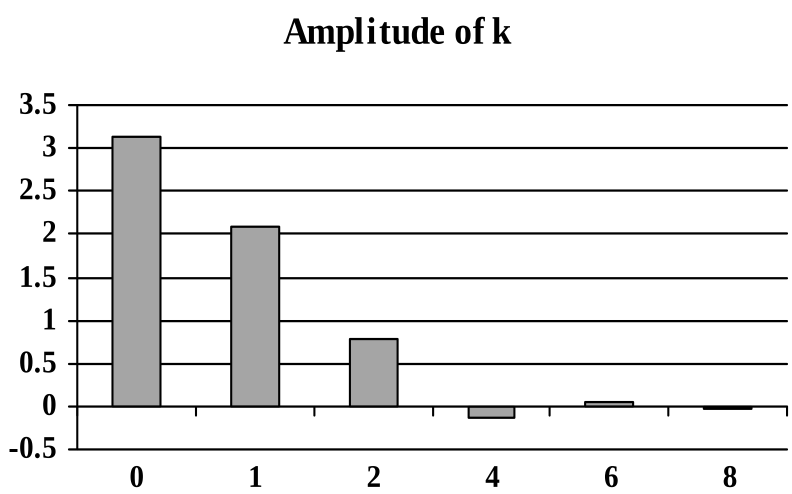
$$r = k * l = \sum_{n=0}^{\infty} \sum_{m=-n}^{n} (K_{nm} L_{nm}) h_{nm} \approx \sum_{n=0}^{2} \sum_{m=-n}^{n} (K_{nm} L_{nm}) h_{nm}$$

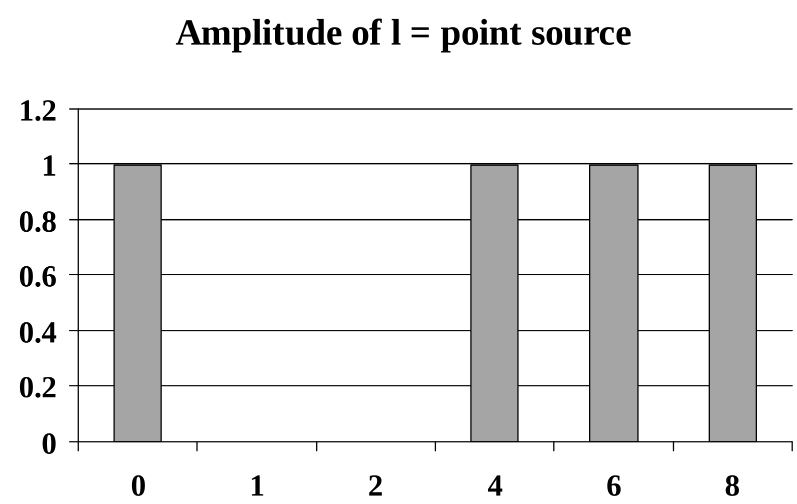




9D space captures 99.2% of energy

How accurate is approximation? (2)
Worst case.



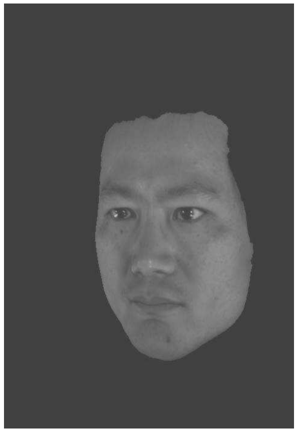


- DC component as big as any other.
- 1st and 2nd harmonics of light could have zero energy

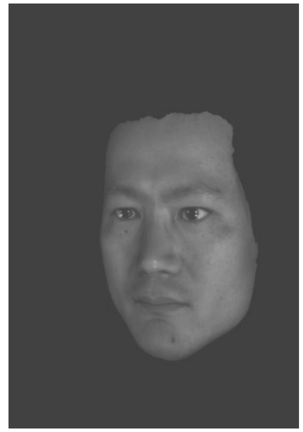
9D space captures 98% of energy

Forming Harmonic Images

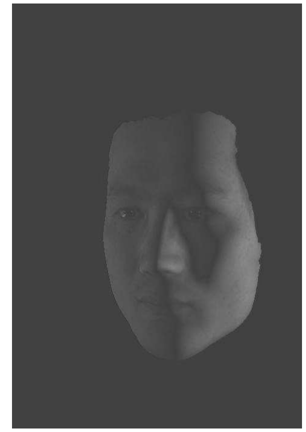
$$b_{nm}(p)=\lambda r_{nm}(X,Y,Z)$$



λ

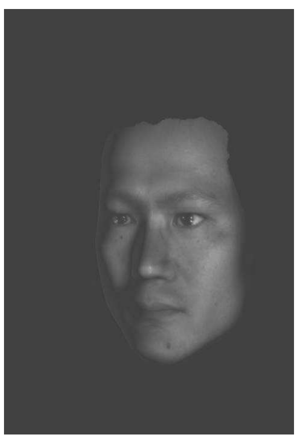


λZ



λX

λY



$2\lambda(Z^2-X^2-Y^2)$

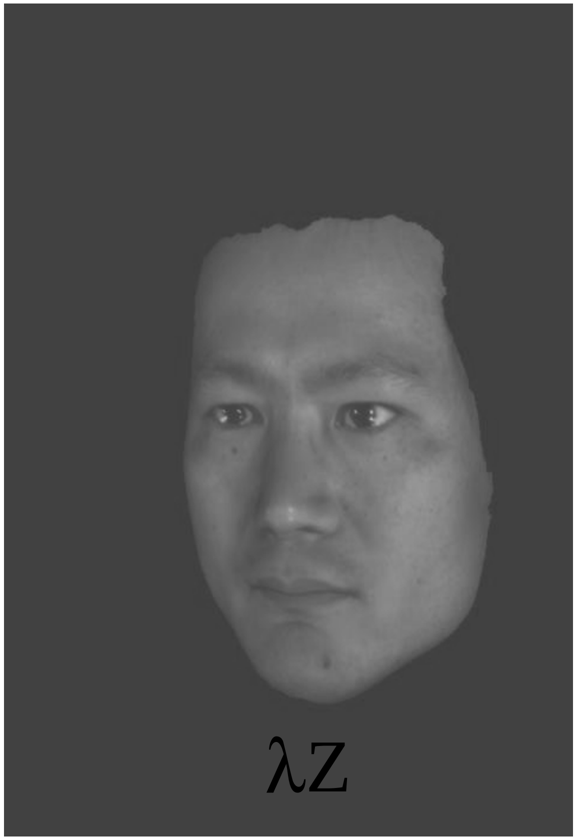
$\lambda(X^2-Y^2)$

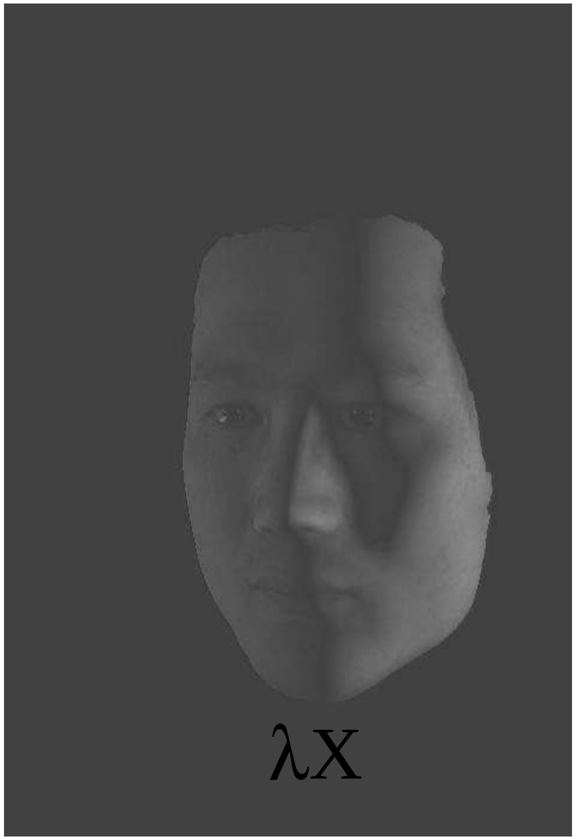
λXY

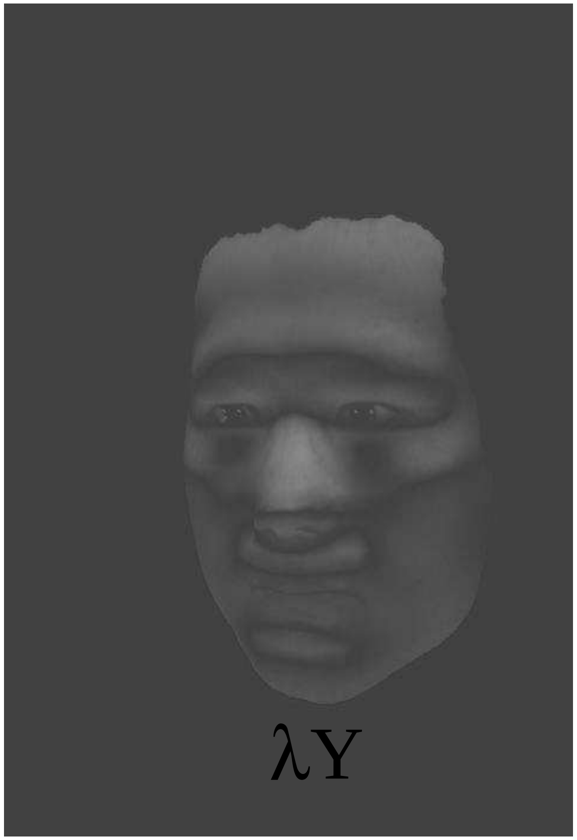
λXZ

λYZ

Compare this to 3D Subspace

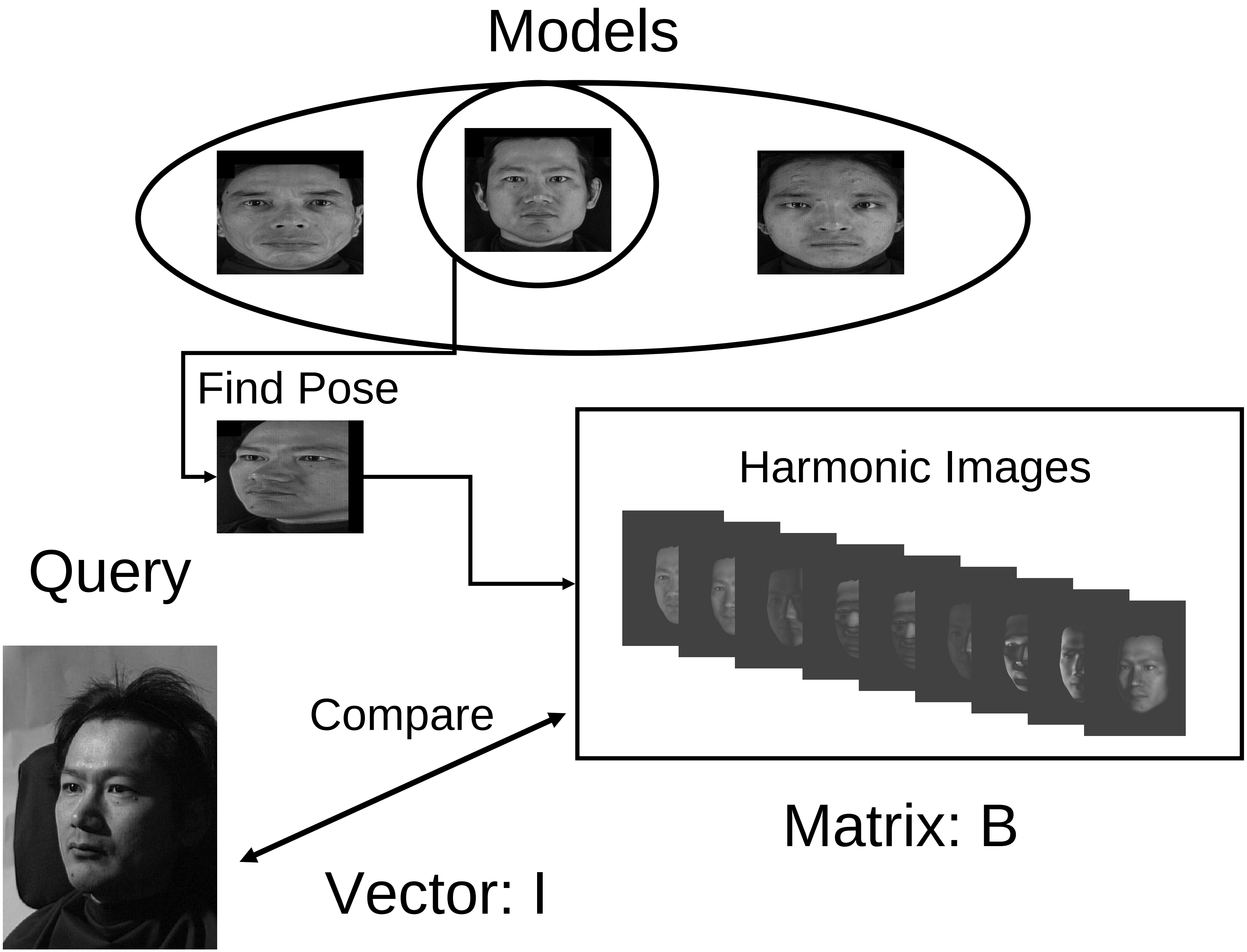






Accuracy of Approximation of Images

- Normals present to varying amounts.
- Albedo makes some pixels more important.
- *Worst case approximation arbitrarily bad.*
- *“Average” case approximation should be good.*


Models
Find Pose
Harmonic Images
Query
Compare
Matrix: B
Vector: I