

## Practice Sheet for CMSC 828J Final Spring 2006

Topics to be covered on the final.

Since the midterm:

- Features
  - Corner Detection
  - Scale Selection
- Hidden Markov Models
  - Markov chains and their asymptotic behavior.
  - Hidden Markov Model
  - Finding the probability of a set of observations in an HMM.
  - Baum-Welch algorithm for learning an HMM
- Linear Separators
  - Perceptron algorithm
  - Support vector machines
  - Winnow

Before the Midterm

1. Invariants
  - Affine
  - Projective
2. Linear subspaces
  - Linear combinations of images
  - PCA
  - LDA
  - Linear illumination subspaces
3. Morphometrics
  - Shape space
  - Procrustes distance
  - Thin-plate splines
4. Template Matching
  - Chamfer matching
  - Hough Transform
  - Interpretation Tree
  - Transformation Space methods

I am allowed to base a challenge problem on material in one of the readings.

### Practice Problems

These problems are meant to give you examples of some things that might be asked on the final. It should be helpful to do these while studying. These are NOT comprehensive, and the final may include questions on any of the above topics.

### 1. HMMs

Suppose we have a two state HMM, with transitions:

$P(1 \rightarrow 2) = \frac{1}{4}$  (ie, the probability of moving from state 1 to state 2 is  $\frac{1}{4}$ ).

$P(1 \rightarrow 1) = \frac{3}{4}$

$P(2 \rightarrow 1) = \frac{1}{2}$

$P(2 \rightarrow 2) = \frac{1}{2}$ .

- a. Suppose we are equally likely to start in either state. After a very large number of transitions, what is the probability that we will wind up in state 1?
- b. Suppose we have possible observations A and B. The probability of observing A in state 1 is  $\frac{3}{4}$ , and the probability of observing A in state 2 is  $\frac{2}{3}$ . What is the probability that we observe ABA?
- c. What is the most likely sequence of 10 observations?
- d. What is the probability that we observe AABAABAAB... over  $3k$  observations, for any  $k$ ?
- e. Suppose we observe AABB. Compute an iteration of the Baum-Welch algorithm given this observation, and see what the new model will be.

### 2. Corners

- a. Create a set of image gradients in a  $5 \times 5$  window so that we will have a maximum response with a corner detector. The  $x$  and  $y$  components of the gradients should range between  $-1$  and  $1$ . Describe the matrix that you produce to detect the corner, and the eigenvalues of the matrix.
- b. **Challenge problem:** Prove that if we have a set of gradients, and rotate their directions while keeping their magnitudes the same we will not alter the magnitude of the gradient.

### 3. Linear Separators

- a. Suppose we use a circle to separate two classes in two dimensions. What is the VC dimension of this classifier?
- b. **Challenge problem:** What is the VC dimension for a hypersphere in  $n$  dimensions?
- c. Suppose we have elements of class 1 located at positions  $(3,3)$ ,  $(4,1)$ ,  $(0,4)$ , and elements of class 2 located at positions  $(5,9)$ ,  $(8,6)$  and  $(7,8)$ . What will be the maximum margin linear separator for these two classes? What will be the support vectors.
- d. Suppose we initialize the perceptron algorithm with a classifier such that if  $x - 6 > 0$  then  $(x,y)$  is assigned to class 2. Describe how this linear classifier will be updated by further iterations of the algorithm.

### 4. Geometry

- a. Consider affine transformations, written as  $Ax + t$ , where  $x$  is a point,  $A$  is a  $2 \times 2$  matrix, and  $t$  is a translation. Suppose we restrict ourselves to affine transformations in which  $A$  has a determinant of 1. Describe an image property of three points that is invariant to such a transformation.
- b. **Challenge problem:** Suppose we consider affine transformations that have a determinant of 2. Can you produce an invariant property for three points?
- c. Of course, you should think about projective geometry too.