Review Notes for Final
CMSC828J, Fall 2012

The final will be based on material covered in lectures. Notes are posted on the class web page for each lecture. These provide an outline of the material that was covered. You are responsible for any material covered in these notes. References are also provided on the web page that will contain a more detailed and lucid description of material in these notes. Lecture notes are not provided that describe the graph cuts algorithm for finding the MAP estimate of an MRF or for the interactive graph cuts algorithm. You are referred instead to the papers by Boykov, Veksler and Zabih, and Boykov and Jolly.

Below is a list of topics covered in the class. I've included some hints about what is important in this topic, but questions may not be entirely limited to these points. For the topics covered since the midterm, I've included some sample problems to use in studying. These are generally more complex, and broader than questions I would ask in a final.

Fourier Transforms

1. Understand what the Fourier Transform is as well as the Fourier Series. Know how to take a Fourier Transform.

Convolutions and the Convolution Theorem

2. Understand what a convolution is. Know important examples, such as convolving to remove noise. Understand the convolution theorem and how to use it.

Linear, Homogenous Diffusion Processes

3. Know the diffusion equation. What does it mean to say that diffusion simplifies an image? Understand numerical methods for solving diffusion equations.

Edge Detection

4. Know the Canny Edge Detector. Understand that it determines a pixel to be an edge if the magnitude of the gradient is above some threshold, and is a local extrema in the direction of the gradient. Also understand hysteresis.

Non-linear Diffusion

5. Understand the basic ideas of: 1) modulating the amount of diffusion by the local gradient patterns, as in Perona-Malik; 2) modulating the amount of diffusion in different directions, as in Weichert's algorithms.

Dynamic Programming and Markov Models for Grouping

6. You should know how the Intelligent Scissors approach performs segmentation using a shortest path algorithm.

Normalized Cut

7. You should understand the Shi and Malik algorithm for Normalized cut, as well as knowing what the normalized cut criteria is.

Markov models, Markov Random Fields

8. You should understand what a Markov chain is. Given an initial distribution for a Markov chain, you should be able to figure out what the distribution is at the next time step, or asympototically.
9. You should understand what a Markov Random Field is, and what the relationship is between this and a Gibbs Random Field. You should understand ICM, simulated annealing and graph cuts as algorithms for solving these. You should understand how to formulate segmentation as an MRF.
10. You should understand the difference between an MRF and a Conditional Random Field, and how CRFs can be used in segmentation.

Wavelets

1. You should know the basic ideas of what a wavelet is, what a wavelet transform is, and what an orthonormal wavelet basis is. You should be able to apply these ideas for the case of the Haar wavelet.
2. **Sample Problem:** Write the Haar transform of $y = x*x$.

Texture

3. Understand the idea of comparing two textures as comparing two probability distributions. Representation of texture as a vector of the output of oriented filters. Markov models for texture..

E-M, K-means

4. You should understand how these algorithms work. For example, if I gave you the current state of one of these algorithms, you should be able to work out the next iteration.
5. **Sample problem:** Implement E-M and K-Means to work on a set of 2D points. Just do E-M and K-Means so that it can find two clusters. Test it by generating two sets of points, one uniformly distributed in the range $0 < x < .6$, $0 < y < .6$, the other uniformly distributed in the range $.4 < x < 1$, $.4 < y < 1$. What would be a reasonable way of initializing each algorithm in this case?

Motion

6. Understand how affine structure-from-motion leads to a rank 4 measurement matrix. Understand why n independently moving rigid objects leads to a rank 4n matrix. Understand the basic idea of how the measurement matrix can be separated into independently moving objects.
7. **Sample Problem:** Suppose we have two sticks that are attached at a single point. They can rotate and translate independently, as long as they remain attached at this point. If we take video of the sticks moving, with affine motion, what is the maximum rank of the measurement matrix.

Level Sets

8. You should understand the boundary value formulation of level sets, including how to write down a curve evolution as a boundary value problem and to solve it with finite differences. This requires understanding upwind differencing.
9. **Sample Problem:** Consider a curve that begins in the location, x*x + y*y = 1. The curve evolves by just scaling, so that it becomes an ever larger circle centered at the origin. Write equations that describe the curve evolution as a Boundary Value formulation. Write down update equations to solve the boundary value formulation using finite differences.

Some Key Equations

$$f = \sum_{k=-\infty}^{\infty} c_k e^{ikt}$$

$$c_k = \frac{1}{2\pi} \int_0^{2\pi} f(s) e^{-ikt} ds$$

$$h(t) = \int_{-\infty}^{\infty} f(t-u) g(u) du$$

$$f * g = h \Longleftrightarrow FG = H.$$

$$J(x,t) = -D \frac{\partial C}{\partial x}. \qquad \frac{\partial C}{\partial t} = -\frac{\partial J}{\partial x} \qquad \frac{\partial C}{\partial t} = D \frac{\partial^2 C}{\partial x^2}$$

$$j = -D \nabla u \qquad \frac{\partial u}{\partial t} = \mathrm{div}(D \nabla u)$$

$$\frac{\partial u}{\partial t} = \mathrm{div}(g(x) \nabla u)$$

$$P(f) = \frac{1}{Z} e^{-U(f)/T} \quad U(f) = \sum_{c \in C} V_c(f) \quad Z = \sum_{f \in F} e^{-U(f)/T}$$

Min Ncut(A,B) = cut(A,B)/assoc(A,V) + cut(B,A)/assoc(B,V)

$$\frac{y^T (D-W) y}{y^T D y} \qquad D^{-\frac{1}{2}} (D-W) D^{-\frac{1}{2}} z = \lambda z$$

$$p^{(i)}(k\,|\,n) = \frac{p_k^{(i)}\, g(\mathrm{x}_n;\, \mathbf{m}_k^{(i)}, \sigma_k^{(i)})}{\sum_{m=1}^{K} p_k^{(i)}\, g(\mathrm{x}_n;\, \mathbf{m}_k^{(i)}, \sigma_k^{(i)})}$$

$$\begin{aligned}
\mathbf{m}_k^{(i+1)} &= \frac{\sum_{n=1}^{N} p^{(i)}(k\,|\,n)\, \mathrm{x}_n}{\sum_{n=1}^{N} p^{(i)}(k\,|\,n)} \\
\sigma_k^{(i+1)} &= \sqrt{\frac{1}{D}\, \frac{\sum_{n=1}^{N} p^{(i)}(k\,|\,n)\, \|\mathrm{x}_n - \mathbf{m}_k^{(i+1)}\|^2}{\sum_{n=1}^{N} p^{(i)}(k\,|\,n)}} \\
p_k^{(i+1)} &= \frac{1}{N} \sum_{n=1}^{N} p^{(i)}(k\,|\,n) \,.
\end{aligned}$$

$$\|\nabla T\| F = 1 \qquad \phi_t + F\|\nabla \phi\| = 0,$$

$\Psi^{u,s}$=1/sqrt(s) Ψ((t-u)/s)

Ψ (t) = -1 if 0<=t<=1/2

Ψ (t) = 1 if ½<=t<=1