Problem Set 1: Nonlinear Diffusion

Note that there is a test image on the class web page. This has been generated with the Matlab commands:

```
I = checkerboard(8,5,5);
sigma = 10/256;
I = I + sigma*randn(size(I,1), size(I,2));
```

Please include in your write-up images that show the result of your smoothing using this image.

Note that when you apply these algorithms to any image, you will need to scale the intensities in the image to lie in the range 0 to 1 in order for the parameters given below to be reasonable.

1. Gaussian Smoothing

   Implement a function to smooth an image using convolution with a Gaussian filter. Show the results of this filter on the test image, using $\sigma = 1.5$. You do not have to write your own convolution code. For example, if you are using Matlab you can use built in functions like imfilter.

2. Perona-Malik

   Implement the Perona-Malik diffusion method, as described by Equations (23) and (21) in Weickert's paper. Show the results of applying this diffusion to the test image, using $\lambda = .01$ and and $\sigma = 1$. Weickert does not indicate a diffusivity. Use a diffusivity of .1, so that $u_{new} = u + .1u_t$. Apply this diffusion process for 200 iterations.

3. Non-linear diffusion

   Implement the diffusion method in Weickert's paper, in which the diffusion tensor D is formed with eigenvectors given in Equations (27) and (28), with $\lambda_1 = g(\|\nabla u_\sigma\|^2)$ and $\lambda_2 = 1$. Using this D, Equation (3) is then applied. Apply this to the test image using the same parameters as for Perona-Malik.

4. Non-local Means Implement the non-local means method described in Buades et al. Based on Section 5.3, initially use the following parameters. Use a 9x9 similarity window. Use $h = 12sigma$, where sigma is given above as 10/256. Only compare the window around a pixel to windows within a 21x21 square centered around the current pixel. Use a value for a of 2.

   Buades et al. do not describe how to treat the boundary of the image in which the similarity window does not fit entirely in the image. Feel free to handle this in any way you wish. You may just set crop out all such pixels, for example.

5. Experimentation

   Find or create one or two test images that you think highlight the differences between these three diffusion processes. Show the results of applying all four to your test images, perhaps varying the parameters to produce a range of results. Explain in one or two paragraphs what you see as the main differences in these results, and why they occur.