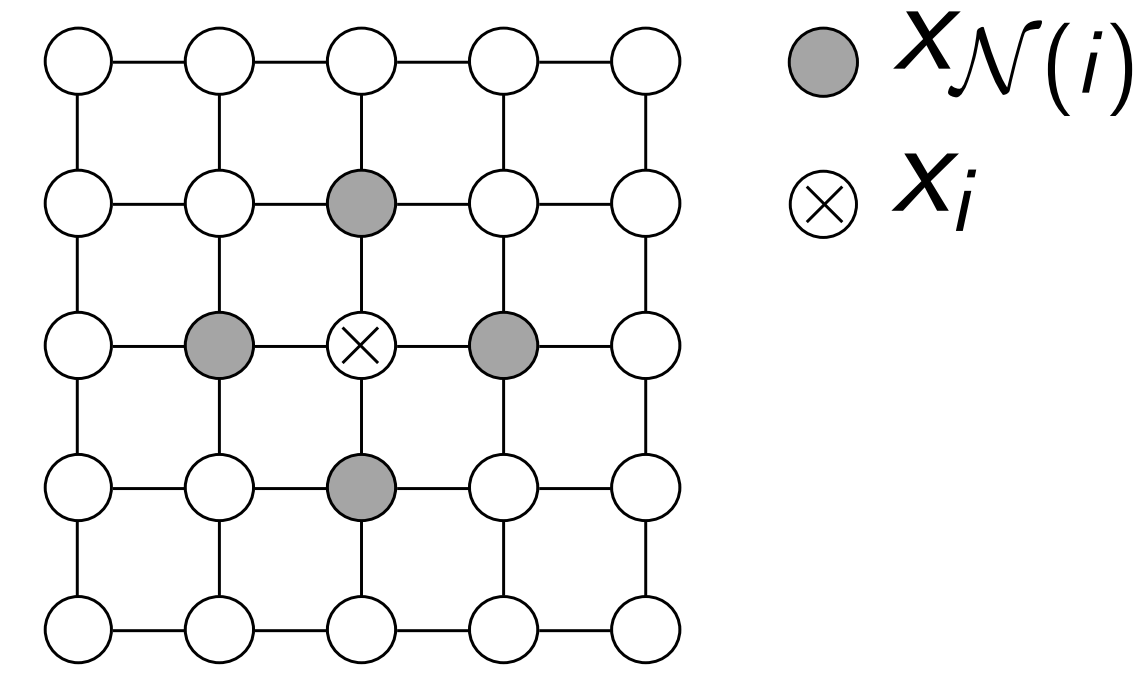


Who Killed the Directed Model (of Natural Images)?

Justin Domke, Alap Karapurkar, and Yiannis Aloimonos, University of Maryland

Background

Undirected Models:



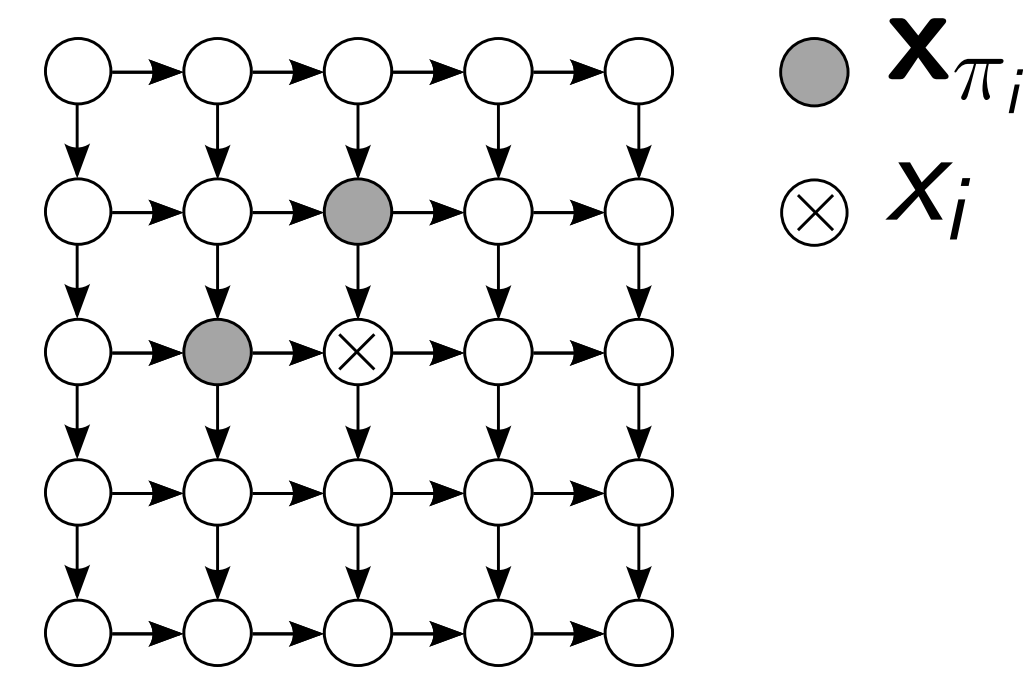
$$p(\mathbf{x}) = \frac{1}{Z} \exp\left(\sum_C f_C(\mathbf{x}_C)\right)$$

$$Z = \sum_{\mathbf{x}} \exp\left(\sum_C f_C(\mathbf{x}_C)\right)$$

Maximum likelihood learning is hard.

$$l = \sum_{\hat{\mathbf{x}}} \left(\sum_C f_C(\hat{\mathbf{x}}_C) - \log Z\right)$$

Directed Models:



$$p(\mathbf{x}) = \prod_i p(x_i | \mathbf{x}_{\pi_i})$$

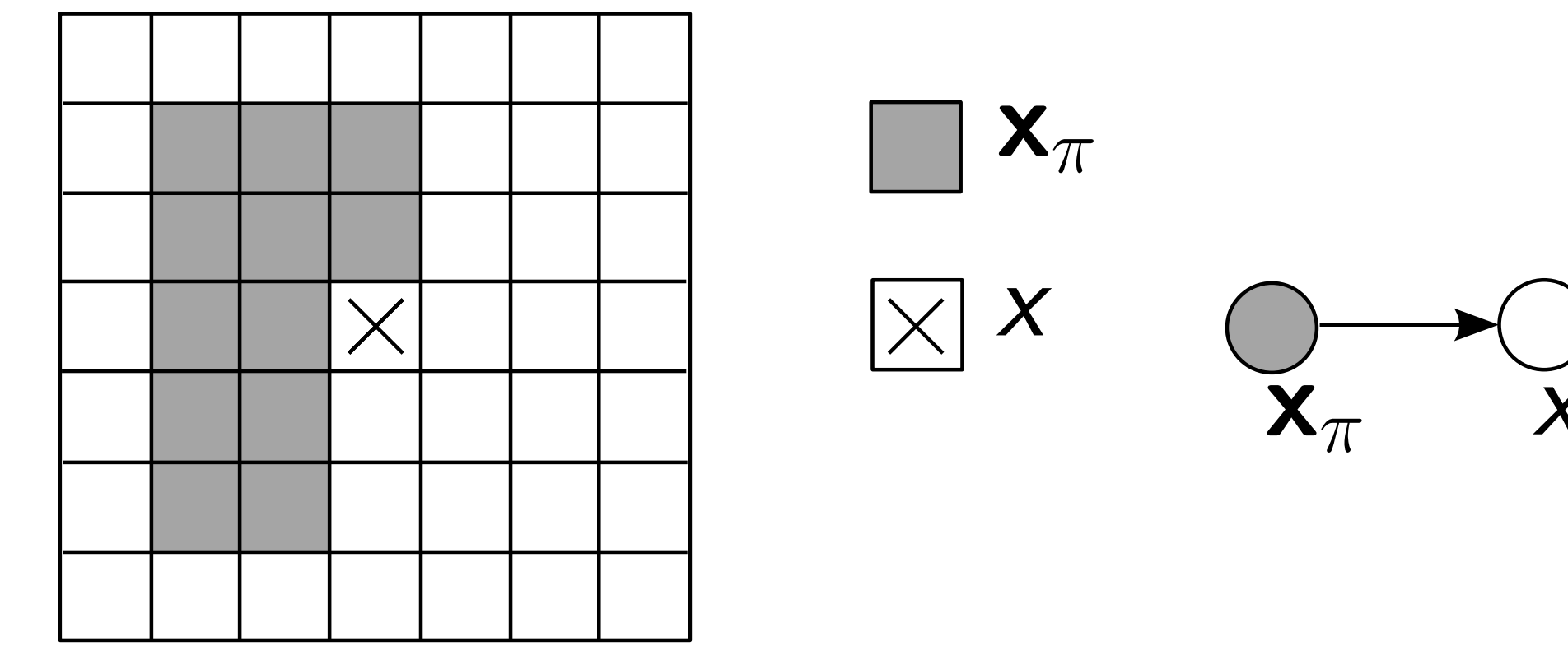
Maximum likelihood learning is easy.

$$l = \sum_{\hat{\mathbf{x}}} \sum_i \log p(\hat{x}_i | \hat{\mathbf{x}}_{\pi_i})$$

No theoretical reason to prefer either type of model.
This paper tests experimentally how directed models compare.

Natural Image Representation

Assume translation invariance.



Represent $p(x | \mathbf{x}_{\pi})$ using a mixture of gaussians.

$$p(x | \mathbf{x}_{\pi}) = \frac{p(x, \mathbf{x}_{\pi})}{p(\mathbf{x}_{\pi})}$$

$$p(x, \mathbf{x}_{\pi}) = \sum_m \alpha_m \mathcal{N}\left(\begin{bmatrix} x \\ \mathbf{x}_{\pi} \end{bmatrix}, \Sigma_m, \mu_m\right)$$

$$p(\mathbf{x}_{\pi}) = \sum_m \alpha_m \mathcal{N}(\mathbf{x}_{\pi}, \Sigma_m^*, \mu_m^*)$$

Natural Image Learning

Get 1,000,000 5x5 patches from the Berkeley segmentation database.
Fit by stochastic gradient descent (100 passes).
Learn priors with 1, 2, and 5 components.
Takes about an hour in Matlab on a 2.79 GHz PC.

Denoising

Given “noisy” image \mathbf{w} , find “clean” \mathbf{x} .

$$p(\mathbf{x} | \mathbf{w}) \propto p(\mathbf{w} | \mathbf{x}) p(\mathbf{x})$$

Assuming Gaussian noise,

$$p(\mathbf{w} | \mathbf{x}) \propto \exp(-\|\mathbf{w} - \mathbf{x}\|^2 / (2\sigma^2))$$

Find \mathbf{x} by L-BFGS using the 1, 2, and 5 mixture priors.

Inpainting

Just maximize $p(\mathbf{x})$ over the hidden pixels.
Again, use the 1, 2, and 5 mixture priors, in that order.

Denoising Results

σ	$\lambda(\sigma)$	Lena	Barbara	Boats	House	Peppers	Mean	R&B
1	1.5	48.2	48.2	48.1	48.9	48.2	48.3	47.90
2	1.5	42.8	42.8	42.2	43.9	42.9	42.9	43.02
5	2	37.7	36.7	36.1	37.9	37.5	37.2	37.49
10	2.5	34.4	32.2	32.8	34.8	33.8	33.6	34.05
15	3	32.6	29.7	31.0	33.3	31.7	31.7	32.04
20	3.5	31.4	27.9	29.8	32.1	30.3	30.3	30.57
25	4.5	30.4	26.5	28.8	31.1	29.2	29.2	29.38
50	9	27.5	23.2	25.7	27.9	25.6	26.0	25.09
75	12.5	25.7	22.3	24.1	25.9	23.4	24.3	22.76
100	15.5	24.5	21.7	22.9	24.4	22.0	23.1	20.74

σ	Lena	Barbara	Boats	House	Peppers	Mean	W&F
1	47.7	47.8	47.8	48.7	47.8	47.9	43.6
2	42.2	42.5	41.6	43.5	42.4	42.4	40.2
5	37.1	36.2	35.4	37.0	36.9	36.5	36.3
10	33.7	31.3	31.8	34.4	33.0	32.8	33.3
15	31.8	28.3	29.9	32.6	30.6	30.7	31.1
20	30.5	26.0	28.6	31.2	28.8	29.0	29.4
25	29.5	24.4	27.5	30.2	27.4	27.8	27.9
50	26.6	22.7	24.6	26.8	23.9	24.9	23.1
75	25.2	22.0	23.3	25.0	22.1	23.5	20.0
100	24.1	21.5	22.5	23.9	21.1	22.6	17.8



Inpainting Results

