Advanced Numerical Linear Algebra Homework 3
AMSC/CMSC 763 Due October 26, 2017

Problem 1. Let

$$A = \begin{pmatrix} \alpha & \beta \\ \epsilon & \gamma \end{pmatrix}$$

where α, β and γ are all $O(1)$ in magnitude, α and γ are not close to each other, and ϵ is small. Show that if one step of the QR-iteration based on Givens rotations is applied to the matrix $A-\gamma I$, then the resulting matrix has an entry of magnitude $O(\epsilon^2)$. What does this tell you about the shifted QR-iteration?

Problem 2. Given a square matrix A, show that there is an orthogonal matrix Q such that the transformed matrix

$$\hat{A} = Q^T A Q$$

has upper-Hessenberg structure.

Problem 3. Let

$$AV_m = V_m H_m + v_{m+1} h_{m+1,m} e_m^T,$$

and let p be a polynomial of degree $j < m$. Show that

$$p(A)V_m = V_m p(H_m) + E_j$$

where $E_j \in \mathbb{C}^{n\times m}$ is identically zero except in the last j columns.

Problem 4. Write your own version of Arnoldi's method for computing the eigenvalues of a general matrix and explore its performance for computing the eigenvalues of the matrix A given in the Matlab mat-file `hw3.mat`. Test this algorithm with Krylov spaces of various dimensions, including 5, 10, 25 and 35, and describe what happens.

You should use the Matlab function `eig` to compute the eigenvalues of the Hessenberg matrix that is constructed by the algorithm. You may also feel free to use `eig` to look at all the eigenvalues of A. You might also find it interesting to use Matlab's function `eigs` to compute just some of the eigenvalues of A and see how well it does in comparison to your code.