

(This is an ORAL HW.)

1. (50 points) We want to solve the Single source shortest path problem where all of the edges are positive integers that are $\leq C$. In class we did this in $O(V \log C + E)$. Improve this. In particular, find an algorithm that is $O((V \log C) / \log \log C + E)$. Use the ref *Faster Algorithms for Shortest Path Problems* by Ahuja, Mehlorn, Orlin, Tarjan which is posted on the course website. Your oral explanation must be very clear and you need to be able to answer questions about it.
2. (50 points) Consider the following alternative to the Path Compression UNION-FIND algorithm: When you do a FIND, instead of attaching every element encountered to the root, attach every element you find to its grandfather. Show that if the structure never gets more than n elements in it then m operations take $O(m \log^* n)$. Use the ref *A Potential-Based Amortized Analysis of the Union-Find Data Structure* that is posted to the course website; Your oral explanation must be very clear and you need to be able to answer questions about it.