

(This is a WRITTEN HW)

- (25 points) Let $k, n \in \mathbf{N}$. An (n, k) -Grey Code is a list of all elements of $\{1, 2, \dots, k\}^n$ such that every two adjacent elements differ in only one place, and in that one place they differ by one. (EXAMPLE: For $k = 3, n = 2$
GREY CODE: 00,01,02,12,11,10,20,21,22.
NOT a GREY CODE: 00,01,02,12,10,11,21,20,22)
Prove that, for all k, n , there exists a Grey Code.
- (25 points) Fill in the following sentence:
If you want to do Matrix Multiplication of $n \times n$ matrices in $O(n^3/\log n)$ steps over Z_k then you need to have a listing of $\{0, 1, 2, \dots, k - 1\}^n$ that has property YYY. (Property YYY should NOT be a grey code, but something that includes grey codes.)
- (25 points) Prove the following rigorously: if (G, s, t, c) is an instance of the network flow problem, c maps edges to naturals, f is a flow, and G_f is the residual graph, and if G_f has a path from s to t with the least edge having L on it, then there exists a flow with value $|f| + L$.
- (25 points) Give an example of an instance of the flow problem (G, s, t, c) , and a flow f , such that all the paths from s to t in G_f use an edge $e = (u, v)$ such that (u, v) is not in G (but (v, u) is).