Fall 2006 **CMSC 651: Homework 6** **Due: Wed Nov 29**

(This is an ORAL HW)

1. (50 points) Consider a variant of the Ford-Fulkerson algorithm where for every interation an augmenting flow of minimum length is chosen. Give an outline of the algorithm, and prove that it acheives a runtime of $O(VE^2)$. (This is the Edmonds-Karp algorithm, and you may use wikipedia, the CLRS textbook or any other cited source to help.) As usual, we will be asking question throughout your presentation.

2. (50 points) **The main changes are: a) clarifying that the network is larger than just the 6 nodes of interest b) We give an example. of how cost and capacity on an edge work.**

   Consider the following word problem:
   A company owns three factories. Each one produces a different product. The amounts produced per day are fixed. There are three stores that each sell a different product. All six of these locations are connected by a single transportation system that has other nodes in it. Note that it may well be the case that there is no edge from a particular factory to store but there will be a path (of many edges) from the factory to the store. Note that the network is much larger than the 6 locations owned by the company! Every edge is directed in one direction. Each edge in the network has a maximum capacity (total of all products) and a cost per unit of each product. For example, if main street (an edge) moves 3 units of product A, 2 of product B, and 1 of product C then: main street must have a capacity of at least 6, and the cost is

   $$3 * \text{ main street's cost for A } + 2 * \text{ main street's cost for B } + \text{ main street's cost for C.}$$

   The costs for A, B, and C will be different on other edges.

   (a) Formally translate the problem of how to find the lowest cost per day to transport all of the three products from their destinations to their outlets as a variant of the network flow problem. Specifically describe the graph, the flow restrictions, and what you are minimizing in a mathematical way (note, this is not a max-flow, in fact the total flow for each product is fixed).

   (b) Redefine the problem as a linear programming problem. Specifially define the variables, give the constraints, and give the objective function.