Fall 2006 **CMSC 651: Homework 7** **Due: Mon Nov 20**

(This is a WRITTEN HW)

1. (50 points) A set A is *3-free* if there is no arithmetic sequence of length 3 in A. For example $\{1, 2, 5, 6\}$ is 3-free, where as $\{10, 13, 16, 20, 24\}$ is not. Dr. Gasarch wants to find the largest 3-free set of $\{1, \ldots, 100\}$.

   (a) Formulate this as an integer programming problem of the form $Ax \le b$ and either MAX or MIN some c DOT x. (The matrix A may be quite large. You do NOT have to write it all out but you should describe it enough so that from your description one COULD write it out.)

   (b) How many variables does it have? How many equations does it have?

2. (50 points) The STANDARD FORM for an LP is a problem of the form

   MAX $\sum_{i=1}^{m} c_i x_i$

   Subject to

   $A\vec{x} = \vec{b}$

   and

   $x_1 \ge 0, \ldots, x_m \ge 0$

   The DUBOIS FORM for a linear programming problem is the same except that it does not have the last contraint. That is, it has no constraint that the x_i be positive. Show that GIVEN a problem P in DUBOIS FORM there exists a problem P' in STANDARD FORM such that an optimal solution answer to P' easily yields an optimal solution for P.

3. (THIS IS NOT BEING ASKED ON THIS HW BUT WILL BE ASKED ON A LATER ONE SO I WANT YOU TO BEGIN THINKING ABOUT IT.) Given an LP problem in STANDARD FORM how do you find a Basic Feasible Solution (or show that none exist). Give a clear exposition. Since this should be very easy to look up, and we expect that you will do so, you may be graded on clarity. In particular, if your exposition is not understandable you will lose points even if it is correct.