Fall 2006 **CMSC 651: Homework 8** **Due: Mon Dec 11**

(This is a WRITTEN HW)

1. (25 points) Let
$$HC = \{G \mid \ G \text{ has a Hamiltonian Cycle } \}.$$
Let
$$HP = \{(G, a, b) \mid \ G \text{ has a Hamitonian Path from } a \text{ to } b \ \}.$$
Show that $HC \leq_m HP$.

2. (25 points)
Let
$$GI = \{(G, H) \mid \ G \text{ and } H \text{ are graphs and are isomorphic } \}.$$
Let
$$DGI = \{(G, H) \mid \ G \text{ and } H \text{ are directed graphs and are isomorphic } \}.$$
(NOTE- for two directed graphs to be isomorphic you need to have that edges map to edges in the correct orientation.)
Show that $GI \leq_m DGI$.

3. (25 points) Let
$$VC_{17} = \{G \mid G \text{ has a vertex cover of size} \leq 17 \ \}.$$
Let $f(G)$ output NO if there is no VC of size ≤ 17, and output an actual VC of size ≤ 17 if there is one.
Show that if $VC_{17} \in P$ then f can be computed in poly time.

4. (25 points) Given an LP problem in STANDARD FORM (that is max $c \cdot x$ with constaints $Ax = b$ and all $x_i \geq 0$) how do you find a Basic Feasible Solution (or show that none exist). Give a clear exposition. Since this should be very easy to look up, and we expect that you will do so. You may be graded on clarity. In particular, if your exposition is not understandable you will lose points even if it is correct.

5. (0 points, but do it anyway) Let $(X, \leq_1)$ and $(Y, \leq_2)$ be two wqo's (Well Quasi Orders). We define the ordering $\leq$ on $X \times Y$ as follows:
$(a, b) \leq (c, d)$ iff $a \leq_1 c$ and $b \leq_2 d$
Show that $(X \times Y, \leq)$ is a wqo.