Sample Problems

1. All HW's and the midterm are good sample problems.

2. Let g_1 and g_2 be two functions. Let BPP_{g_1,g_2} be defined as $A \in BPP_{g_1,g_2}$ if there exists a poly p and a poly predicate B such that

   $x \in A \Rightarrow Prob_{r \in \{0,1\}^{p(n)}} B(x,r) \geq g_1(n)$.

   $x \notin A \Rightarrow Prob_{r \in \{0,1\}^{p(n)}} B(x,r) \leq g_2(n)$.

   For which values of (g_1, g_2) can you show that $BPP_{g_1,g_2} = BPP$? For which values of (g_1, g_2) can you show that $BPP_{g_1,g_2} = P$? Open ended- is there anything else you can show it equals?

3. Find an infinite set of pair of functions $(g_1(n), g_2(n))$ such that the following occurs:

   IF the below holds THEN P=NP.

   There is a poly time algorithm that will, given ϕ, produce a formula ϕ' such that

   - If ϕ has n variables then ϕ' has n^2 variables.
   - $\phi \in SAT \Rightarrow \phi'$ has at least $g_1(n)$ satisfying assignments.
   - $\phi \notin SAT \Rightarrow \phi'$ has at most $g_2(n)$ satisfying assignments.

4. Assume $SAT \in PCP(c \log n, d \log n, e/n)$. This is the only PCP result you are allowed to use in this problem. Make a strong statement about not being able to approx clique, and prove it.

5. Define P/poly. Show that if $SAT \in$ P/poly then PH collapses.

6. Recall that $AM = AM_1$ (and I mentioned that $MA = MA_1$). However, it is not know if $BPP = R$. Why is it the case that in one case we can show that 1-sided error = 2-sided error, and in one case we cannot. (NOTE- this is an ill defined question which would never be on the exam, but I want you to think about it.)