Fall 2007 **CMSC 652: Homework 3** William Gasarch

Due Sep 24

COURSE WEBSITE: "http://www.cs.umd.edu/~gasarch/652/652.html"
READING: Read the notes on P, NP, PH and also on Randomized Computation.

1. (10 points) Write your name clearly. Staple the HW. Where and when will the midterm be? Where and when will the final be? HW must be TYPED or NEAT.

2. (30 points) A graph is k-colorable if there is a way to assign to every vertex a color in the set $\{1, \ldots, k\}$ such that no two neighbors have the same color.

   Let

   $$COL_k = \{G \mid G \text{ is } k\text{-colorable}\}.$$

   Assume that $COL_2 \in P$, COL_3 is NP-complete, and $P \neq NP$.

   For which pairs $(a, b) \in \mathsf{N} \times \mathsf{N}$ is $COL_a \leq_{\mathrm{m}}^{\mathrm{p}} COL_b$. Justify your answer.

   (HINT: This problem does NOT involve incredibly hard reductions.) (STUPID NOTE: $\mathsf{N} = \{0, 1, 2, 3, \ldots, \}$. The only graph that is 0-colorable is the graph with no vertices.)

3. (30 points) Let QBF_2 be the following set of Boolean Formulas.

   $$\{\phi(x_1, \ldots, x_n, y_1, \ldots, y_m) \mid (\exists b_1, \ldots, b_n)(\forall c_1, \ldots, c_m)[\phi(b_1, \ldots, b_n, c_1, \ldots, c_m)]\}.$$

   Show that, for all $A \in \Sigma_2^p$, $A \leq_{\mathrm{m}}^{\mathrm{p}} QBF_2$. (HINT: Write $A \in \Sigma_2^p$ as

   $$A = \{x \mid (\exists^p y)(\forall^p z)[(x, y, z) \in B]\} = \{x \mid (\exists^p y)\neg(\exists^p z)[(x, y, z) \notin B]\}.$$

   HINT2: Use Cook's theorem.

4. (30 points) Prove each of the following.

   (a) If $A \in P$ and $B \leq_{\mathrm{m}}^{\mathrm{p}} A$ then $B \in P$.

   (b) If $A \in NP$ then $A^* \in NP$. (Recall that $A^* = A^0 \cup A^1 \cup A^2 \cup \cdots$ where A^i is the set of all strings that are the concatenation of i strings from A. Formally $A^i = \{w_1 w_2 \cdots w_i \mid w_1, \ldots, w_i \in A\}$ STUPID NOTE: $A^0 = \{\lambda\}$, the empty string.)

5. EXTRA CREDIT: Show that if $A \in P$ then $A^* \in P$.