

Due Oct 12

1. (10 points) Write your name clearly. Staple. Where and when will the midterm be? HW must be TYPED or NEAT.
2. (30 points)
 - Let HI-MID-LO-SAT be the following PROMISE problem: You are given a formula $\phi(x)$ on n variable and you are PROMISED that EITHER
 - $\#\phi \leq 2^{\sqrt{n}}$, OR
 - $\#\phi = 2^{n/2}$, OR
 - $\#\phi \geq 2^{n-\sqrt{n}}$.

You want to be able to tell WHICH type ϕ is.

You have the ability to FLIP COINS and QUERY SAT. Show that there is an algorithm that will output the correct answer with probability $\geq 3/4$.

3. (30 points) Let H be a $k \times n$ matrix of 0's and 1's. Let $h : \{0, 1\}^n \rightarrow \{0, 1\}^k$ be the function which is $h(x) = Hx \pmod{2}$. Show how to construct a **Boolean Circuit** C such that $h(x) = \vec{0}$ iff $C(x) = 1$. The construction should be in polynomial time. We only allow AND, OR, and NOT gates of fan-in 2. How many total gates are used- you may use order notation (it will be a function of k and n).
4. (30 points)

Show how to construct, in polynomial time for each n , a family of boolean formula $\psi(x_1, \dots, x_n, y_1, \dots, y_n)$

such that

$\psi(x_1, \dots, x_n, y_1, \dots, y_n)$ is True iff

$(x_1, \dots, x_n) \preceq (y_1, \dots, y_n)$ (the order \preceq is lexicographic ordering).
5. (Extra Credit) Consider the following problem: HIHI-LOLO-SAT: You are given a formula $\phi(x)$ on n variable and you are PROMISED that EITHER
 - $\#\phi \leq n^2$, OR
 - $\#\phi \geq 2^n - n^2$.

You want to determine which type ϕ is. MAKE AN INTELLIGENT STATEMENT ABOUT THIS PROBLEM, WITH PROOF.