Fall 2007 **CMSC 652: Midterm Sample** William Gasarch

Please Due by Oct 29

COURSE WEBSITE: "http://www.cs.umd.edu/~gasarch/652/652.html"
READING: Read all notes that I gave out and when over in class.

1. Assume that there is a machine with the following behaviour:

   INPUT: (G, k) where G is a graph and k is a number

   OUTPUT: If G has a clique of size k, then the output IS that clique of size k. If G DOES NOT have a clique of size k then the output is some k vertices.

   If such a machine exists that runs in poly time then what complexity classes will collapse?

2. Let A be a set such that $SAT \leq_T^p A$. Show that there exists a function $f \leq_T^p A$ such that the following hold:

   (a) If ϕ is a formula on n variables then either $f(\phi) \in \{0,1\}^n$ or $f(\phi) = NO$.

   (b) If $\phi \in SAT$ then $f(\phi)$ is a satisfying assignment for ϕ.

   (c) If $\phi \notin SAT$ then $f(\phi) = NO$.

3. Show that if $A \leq_m^p B$ and $B \in \Pi_3^{\text{p,SPARSE}}$ then $A \in \Pi_3^{\text{p,SPARSE}}$.

4. (20 points) Let f be such that the following hold.

   PROPERTIES OF f

   (a) $f : \{0,1\}^* \to \{0,1\}^*$.

   (b) f is 1-1 and onto. Hence f has an inverse.

   (c) f is length preserving. That is, for all x, $|x| = |f(x)|$.

   (d) f is computable in polynomial time.

   END OF PROPERTIES OF f

   Assume P = NP. Show that f^{-1} (the inverse of f) can be computed in polynomial time. (That is, there is a poly time Turing Machine that, on input x, outputs $f^{-1}(x)$.)

   (RECALL: if g is 1-1 and onto then the inverse of g, denoted g^{-1}, is the function such that $g(g^{-1}(x)) = g^{-1}(g(x)) = x$.)